Draft, in progress

Feynman diagrams as quantum circuits
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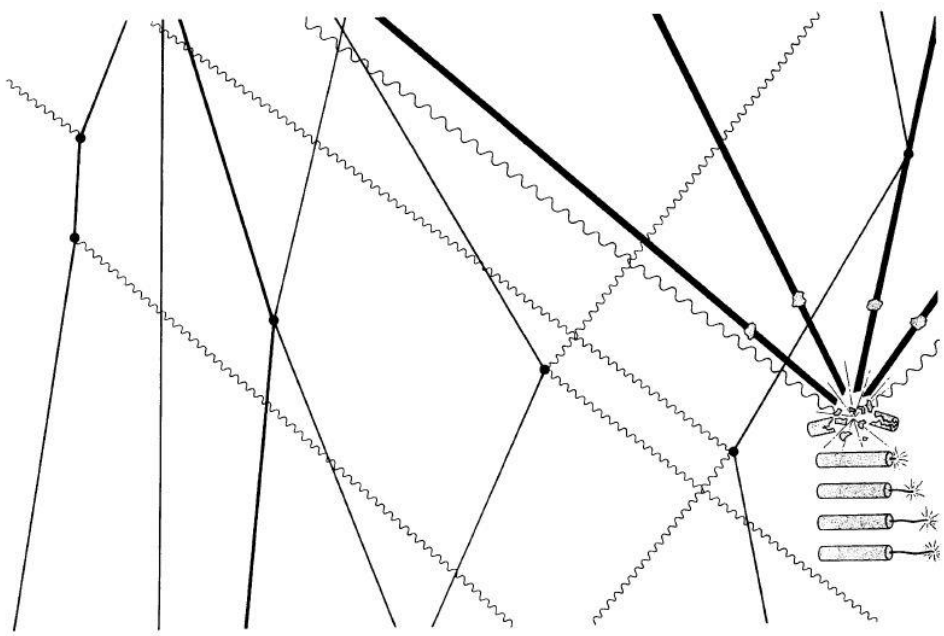
Abstract

Recent work describes spacetime in terms of tensor networks or quantum circuits and, more generally, in terms of quantum information (see e.g. van Raamsdonk, 2010; Brown et al 2017 and 2018) . Information requires a physical basis (Landauer, 1991). If spacetime is information, there must be some physical system that stores and processes that information. Here we propose that the computational circuit models of spacetime are analogous to Feynman diagrams. Lines representing fermion propagators in a Feynman diagram are analogous to wires in a quantum circuit, and particle interactions (mediated by bosons) are quantum logic gates.

Introduction

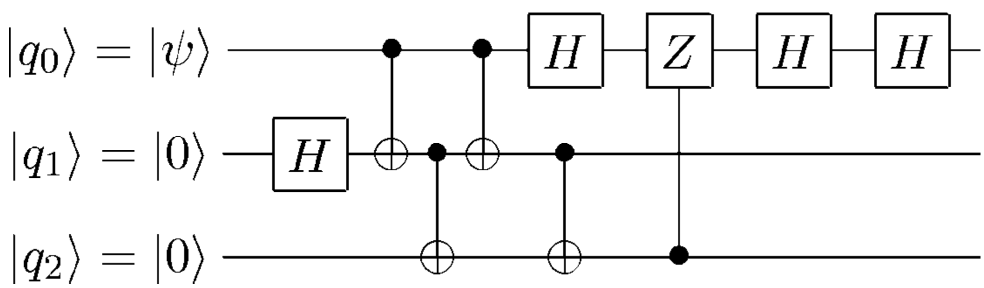
An early motivation for this picture is found in the classic text, *Gravitation* (MTW), (Misner et al, 1973, Figure 1.2.) which includes a figure showing spacetime as a web of events. There's no background on which the events occur. Events themselves build the structure of spacetime.

Figure 1. Spacetime as a web of events. Vertical axis is time, horizontal axis is position. Dots represent particles, wavy lines represent photons (light). A fire cracker goes off at bottom right; dark lines represent fragments from the explosion.



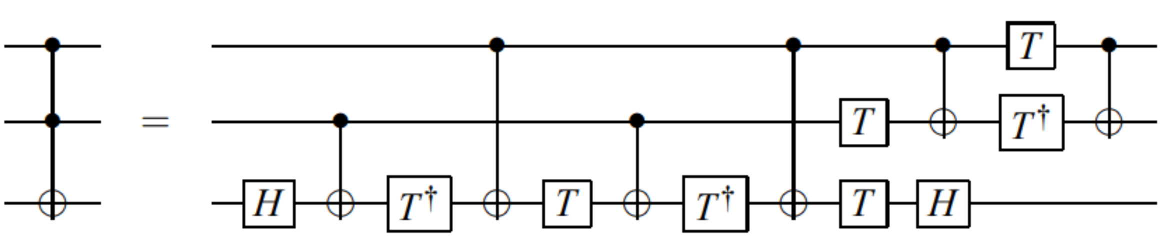
That essential idea has been expressed recently in the form of quantum networks and information processing. For example, van Raamsdonk models spacetime as quantum entanglement (van Raamsdonk, 2010), and John Preskill's group models the AdS/CFT bulk / boundary correspondence as a quantum error correcting code (Pastawski et al, 2015). Our present best understanding of the physical world rests on quantum field theory. Physicists are starting to model field theory as quantum circuitry (see e.g. Jefferson and Myers, 2018). Perhaps spacetime can be represented by the wires and logic gates of circuits in a quantum computer, as in the MTW picture. Wires represent field states, and logic gates represent operations on those states.

Figure 2. An example showing quantum interactions as a logic circuit. The figure shows a quantum teleportation circuit using Hadamard, CNOT, and Pauli Z gates. Source: Isaac Chuang, qasm quantum circuit demonstration, MIT.



Only a few quantum logic gates (e.g. the universal set of gates $\{H, R_X, R_Y, R_Y, Ph, CNOT\}$) are required to model any quantum calculation. (H is the Hadamard gate, $R's$ are rotation gates, Ph a phase change gate, and $CNOT$ is the controlled NOT gate.) The universal sets include reversible unitary gates that can transform the state of a system into any other state in a finite number of steps (see e.g. Preskill, 2017). Two- and three-qubit gates (and higher order) can be decomposed into combinations of other, simpler gates.

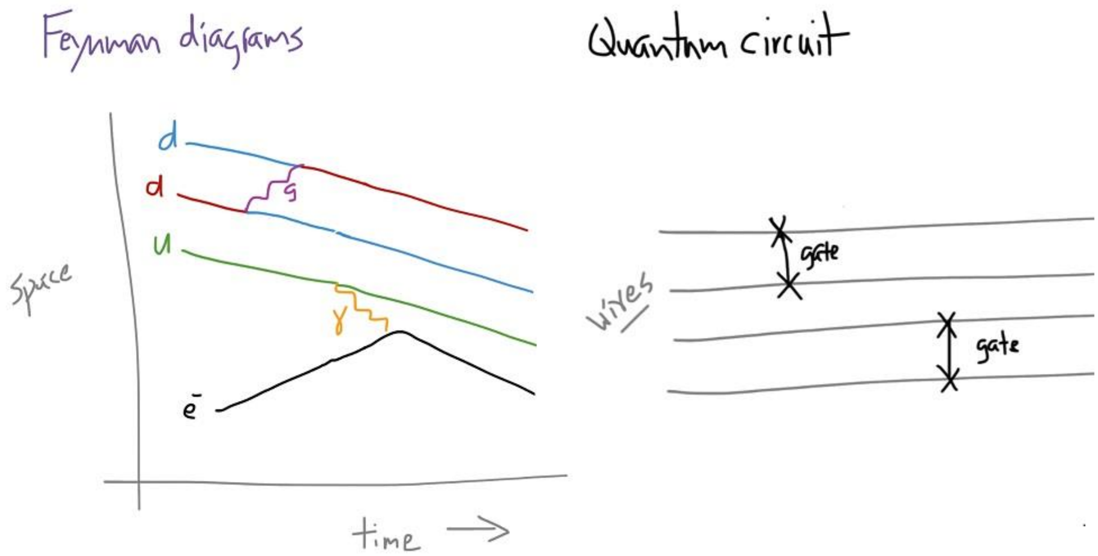
Figure 3. Decomposition of the Toffoli gate into a sequence of Hadamard, CNOT, and exponential gates. From Nielsen and Chuang, 2000.



Feynman diagrams = quantum circuits

On casual inspection, Feynman diagrams sure look like wires and gates. Maybe we can interpret them as such. In-coming and outgoing particles (fields) are the wires; bosons (interactions) are the gates. Such an extrapolation may not be too far-fetched. Any unitary matrix qualifies as a gate (Nielsen and Chuang, 2017), and field interactions can be described by such matrices, e.g. in S-matrix theory.

Figure 4. Feynman diagrams as quantum circuits. The Feynman diagram below is rotated from the standard representation, where time is the vertical axis and spatial displacement runs along the horizontal axis. The world is a network of events, field interactions. Perhaps it can be modeled as a quantum circuit; fields are the wires and interactions are mediated by logic gates.



A photon, for example, might be a swap gate exchanging spins when two electrons scatter. Or the exchange of a red / anti-blue gluon might be a rotation gate in "color space" exchanging color states of a red quark and a blue quark. A W^- gate transforms incoming d and ν (down quark and neutrino) wires to outgoing u and e^- (up quark and electron).

Figure 5. Bosons as gates in particle interactions. The weak interaction is modeled as a rotation in "flavor space" mediated by a Hadamard-like gate followed by CNOT. Or perhaps the "Hadamard-like" gate should be R_X and R_Z rotation gates on a Bell basis. See Figure 6.

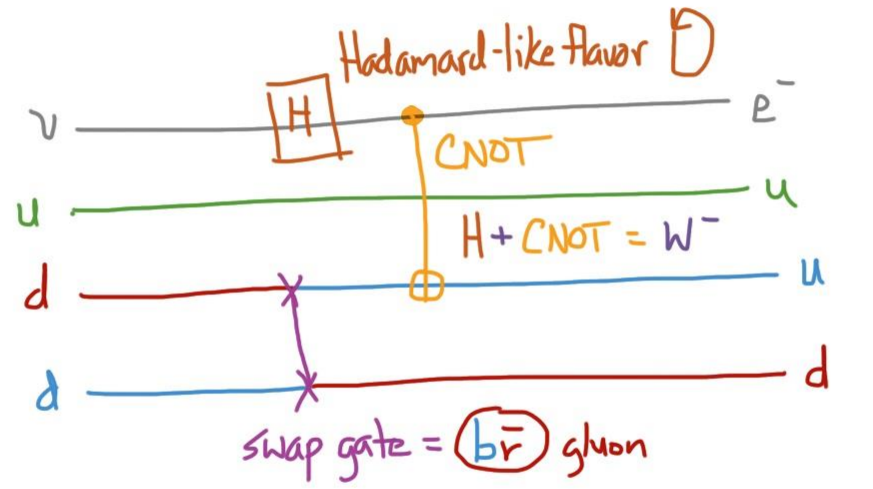
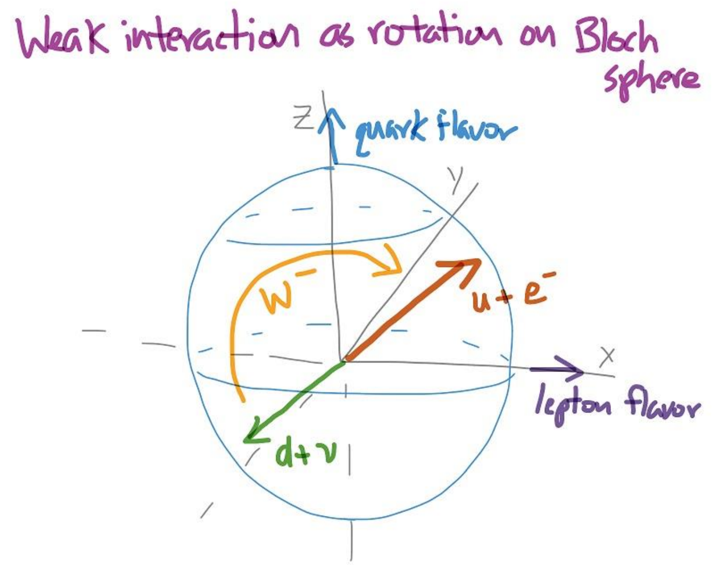


Figure 6. Another view of the weak interaction as rotation on a Bloch sphere in "flavor space." The vector transformation might be represented by a combination of rotation gates in a circuit.

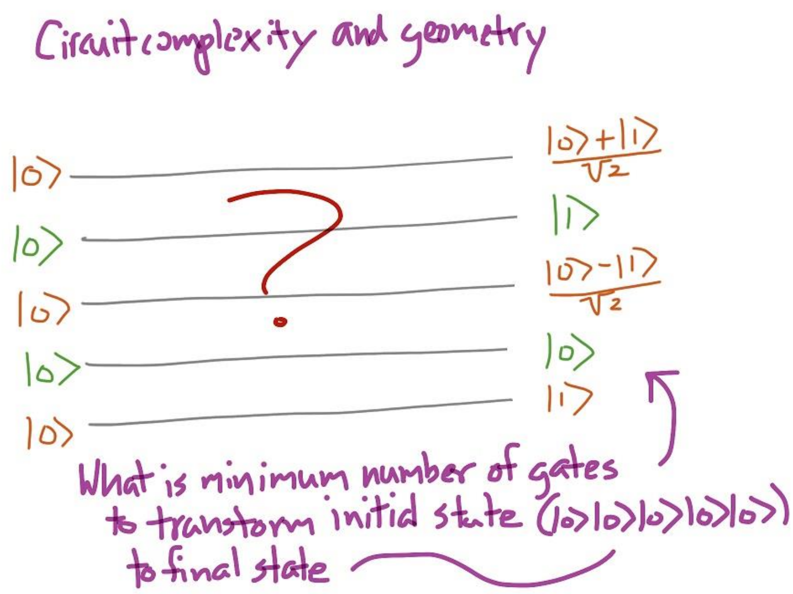


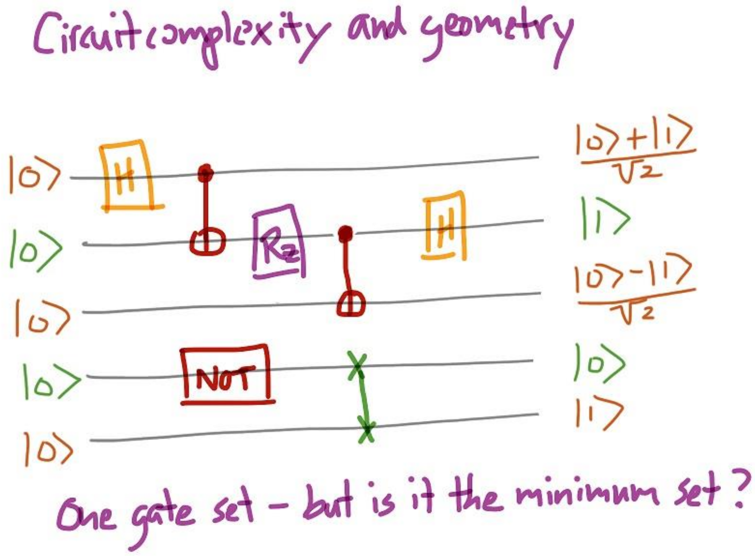
Are quantum logic gates sufficient to describe all Feynman diagrams? It seems plausible that the gates' capacity to change basis, change phase, and to rotate state vectors in Hilbert space might be sufficient to model the particle interactions. It is interesting to speculate, also, that the decomposition of various gates, e.g. the Deutsch gate, into series of simpler gates in a circuit might model the proliferation of loops in higher order Feynman diagrams (and maybe help determine a cutoff to those processes).

Emergent gravity

Where is gravity in this picture? Nielsen et al discovered that the complexity of a quantum circuit can be described in terms of geometry (Nielsen et al, 2006). Measure how many gates it takes to get from the initial state to a new equilibrium state. The minimum number of gates, it turns out, is analogous to a geodesic in good ol' Riemann geometry, and circuit complexity is a measure of geometric curvature (Brown et al, 2017). Roughly speaking, denser circuits with more gates – or more particles and interactions in a system – have greater curvature. In this picture, gravity is an emergent property of the circuit.

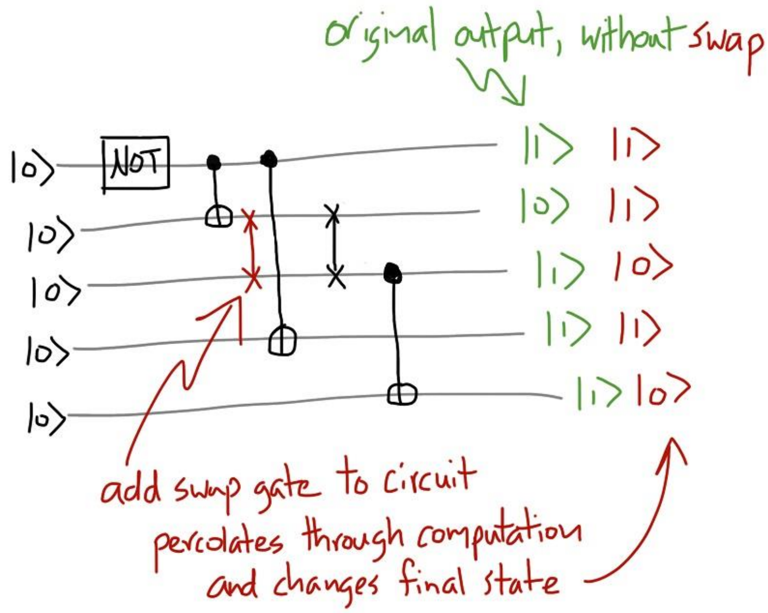
Figure 7. Complexity of a circuit can be measured as the minimum number of gates required to reach a given final state from initial, simpler state. That gate configuration has an associated geometry and curvature.





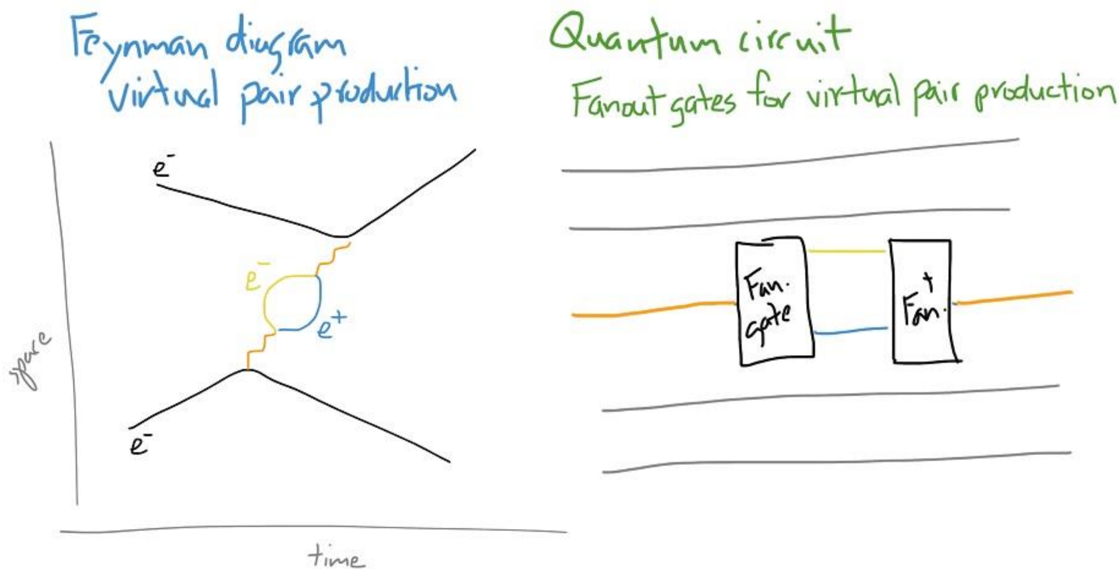
The Susskind group at Stanford proposed gravitational dynamics for such systems (Brown et al, 2015). Start with a stable circuit, in equilibrium. Add another gate to the circuit. The gate changes the states of the wires (particles) on which it acts. Information in those wires changes the output from the gates downstream in the circuit. For example, if a CNOT or Toffoli gate receives a $|0\rangle$ instead of $|1\rangle$ input, it no longer flips the bit in its target. The changes ripple through the circuit until it reaches a new equilibrium.

Figure 8. Inserting a new gate into a quantum circuit creates a ripple through the state space and results in a different final state. Such dynamics might model gravitational interactions.



What about the vacuum? How can the vacuum be represented as circuitry? Maybe as wires (fields) without gates. Every now and again a wire splits, mediated by one of Feynman's branching gates (Feynman, 1999), then merges – virtual pairs out of the vacuum. Unitarity, conservation of information with time reversal symmetry, requires that those branching gates return the system immediately to its vacuum state.

Figure 9. Virtual pair production out of the vacuum modeled with fanout gates (extrapolated from Feynman, 1999).



Discussion

So what's the use of this picture? How can circuitry help us understand spacetime? A couple or three ideas. First, maybe such a model can help build a quantum computer that replicates reality. If we can figure out a circuit analog to particle interactions, then we can model the world in a computer. Vice versa, if particle interactions represent circuitry, we can use particle systems to calculate. Also, maybe the circuit model can help get rid of infinities in scattering amplitudes. If particle interactions are gates and the gates can be decomposed into a finite assembly of simpler gates, then maybe we can figure out what loops to ignore in the Feynman diagrams.

Summary

This paper has attempted to relate various models for the underlying structure of spacetime. Those models include tensor networks, quantum entanglement, and other ideas based on information theory and quantum computation. We propose that Feynman diagrams might be dual to quantum circuitry; fields/particles are the wires, and bosons/forces are the gates. Such a model fulfills Landauer's requirement that information is physical, and it provides a plausible physical structure for spacetime.

Future work, most obviously, requires figuring out the various gates that mediate particular particle interactions. Tests include identification of which gates mediate known processes, mimicking those processes in a quantum computer, and prediction of new reaction channels resulting from different gate combinations. Such models might also provide means to calculate parameters such as particle mass and the fine structure constant from first principles. (Well, that might be a bit of a stretch, but maybe . . .)

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