CHAPTER 8
TOWARD A UNIFIED THEORY

At a number of points in our discussion we've glimpsed an underlying unity in physics. We find disparate concepts are actually branches off the same tree.

Historically there have been several great unifications in our understanding of the natural forces. Newton demonstrated that gravity on earth, as evidenced, for instance, in the fall of apples, behaves the same as gravity at the scale of the solar system, as evidenced by the moon's orbit and the orbits of the planets. In the mid-1800's, Ampere, Faraday, and Maxwell showed that electricity and magnetism are inter-related: a moving electric charge creates a magnetic field, and a changing magnetic field accelerates an electric charge. Einstein proved mass and energy are related. He also developed a new paradigm describing the forces in terms of geometry, and he demonstrated the connection between space and time. More recently, in the 1950's and 1960's, Steven Weinberg, Abdus Salam, and Sheldon Glashow showed that the electromagnetic force is related to the weak force.

Many physicists believe all four forces of nature are related, manifestations of a single underlying Force. They seek a "grand unified theory" linking the strong force to the electroweak force and, ultimately, a "theory of everything," linking all the forces of nature, including gravity. They are encouraged because all the forces can be described by local gauge theories.

In this chapter we outline recent endeavors to explain the inter-relations between fermions and forces. Our purpose here is not to present the theory of everything -- it still eludes us -- but to describe methods physicists are using to develop such a theory. We will review evidence that the fermions and bosons are inter-related. Then we will discuss the concepts of symmetry, multiple dimensions, and group theory that physicists use to explain those inter-relations. Finally, we will consider gauge theory, the most promising avenue for connecting all the fermions and forces.

EVIDENCE FOR UNIFICATION AT THE SCALE OF PARTICLES

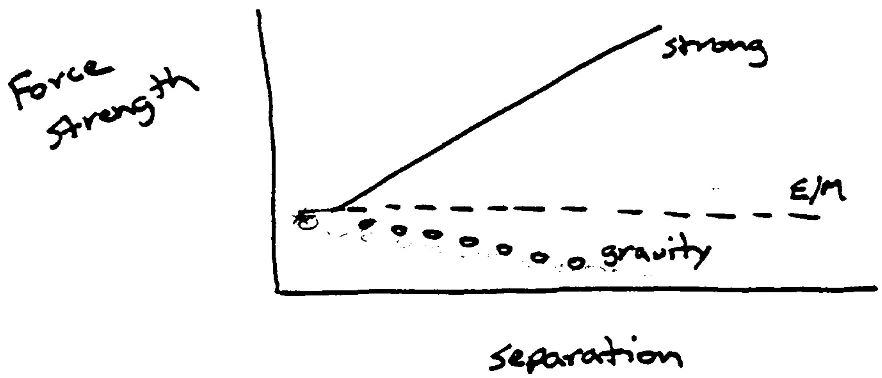
If there is, indeed, a unified description of nature, it must manifest itself at the scale of particles. In Ch. 6, we cited evidence that the fermions are related to each other:

-- The products of processes such as neutron decay and pion decay include leptons, where before decay there were only quarks.

-- The electric charge of the electron (a lepton) exactly equals, but is opposite in sign, to the charge of the proton (composed of quarks).

In Ch.7 we cited evidence linking the forces:

-- At high energies (close interaction distances) all four forces approach each other in strength.



-- Theory and experiment find the electromagnetic force indistinguishable from the weak force at interaction energies above about 90 Gev.

Finally, there is evidence (cited in Ch.7) relating fermions (the building blocks of matter) to bosons (the force carriers):

-- particle/antiparticle pairs (fermions) annihilate producing gamma rays (bosons).

-- W vector bosons decay to leptons, and gluons decay to quarks.

$$W^+ \rightarrow e^+ + \nu$$

Such evidence does not <u>prove</u> all the fermions and forces are related, but it is compelling.

<u>QUESTIONS</u>

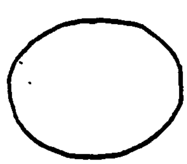
A theory of everything must answer the following questions:

What are the underlying qualities common to the fermions and bosons?

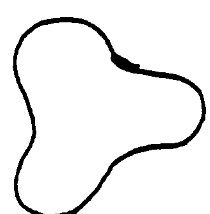
What distinguishes one particle (fermion or boson) from another?

What is the mechanism by which fermions and bosons interact?

We already described, in Ch. 6, one model linking fermions and bosons, heterotic string. In that model, all particles share a loop structure, and they are distinguished by the modes of vibration on the loop. In this chapter, we shall explore some mathematical approaches to a theory of everything. The mathematics complement the heterotic string model and in fact are incorporated into formal string theory.



The fundamental loop of heterotic string theory.



Standing waves on the loop — the string representation of a fermion

GEOMETRY

Much of physics, and much of the argument that follows, is based on geometry. Perhaps there's a psychological prediliction, wired into the human mind, that seeks geometric patterns; perhaps we've evolved to analyze our (primarily visual) world in terms of lines and edges, squares and triangles.

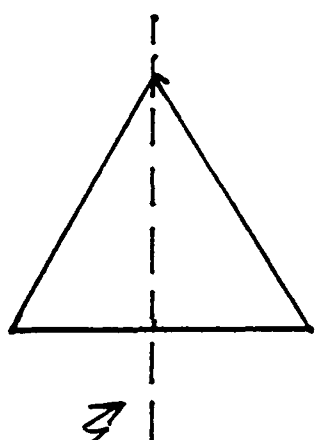
Certainly geometry is a powerful tool: witness the triumph of Einstein's geometric interpretation of gravity. But is geometry the "correct" way to analyze Nature? We don't know.

The conceptual tools of modern physics -- symmetry, multiple dimensions, and group theory -- are geometric. Geometric arguments are compelling, because they make predictions that have been verified experimentally, but keep in mind that they are tentative probes into Nature's whys and wherefores.

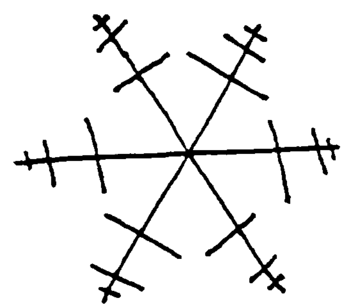
SYMMETRY

The concept of symmetry is a cornerstone of modern physics. It explains the conservation laws, and it helps model the fermions and their interactions.

Everyone is familiar with symmetric geometric figures such as equilateral triangles, squares, and snowflakes.

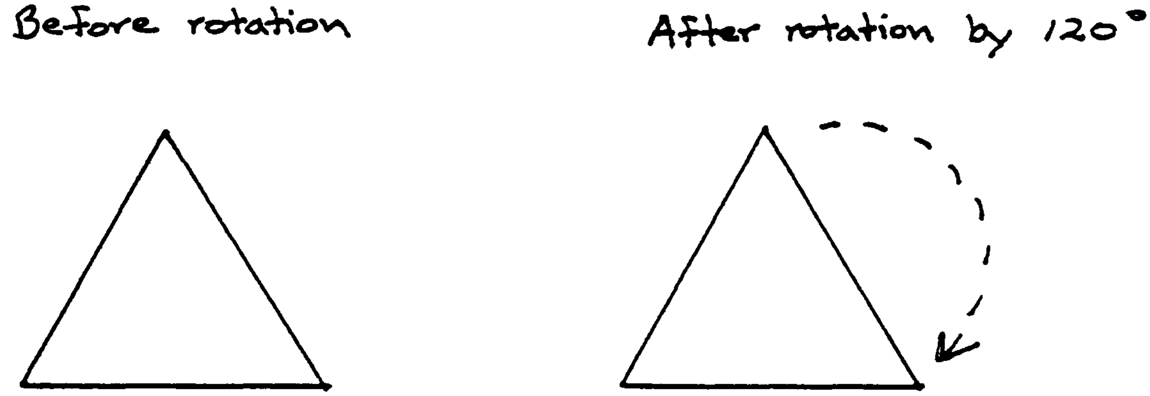


one axis of symmetry of an equilateral triangle: parts of the triangle to the right of the axis are a mirror reflection of parts to the left.

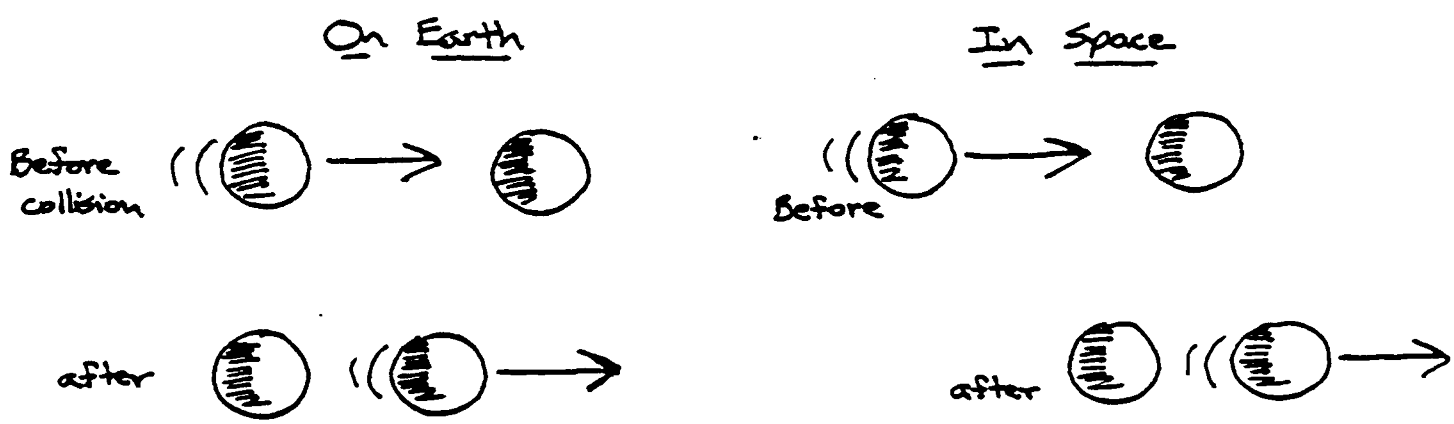


Snowflake can be rotated by multiples of 60° and still appear the same.

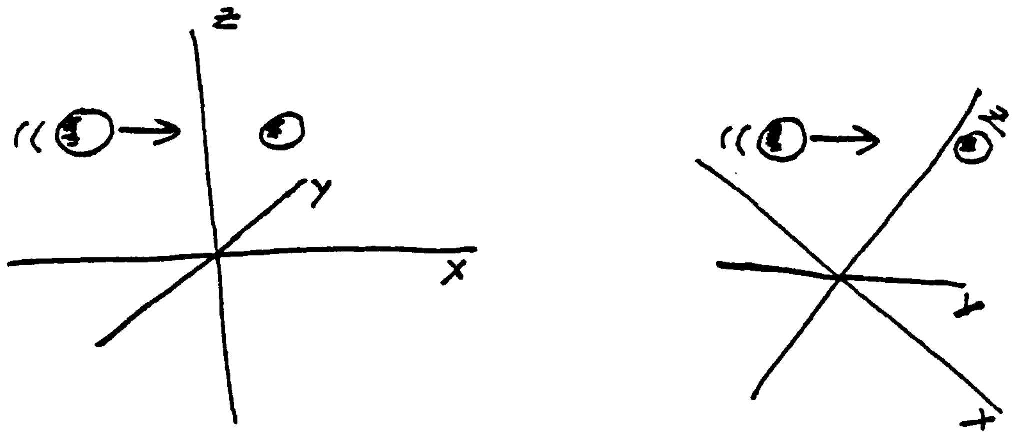
Physicists use another, related definition of symmetry: a physical system is symmetric under an operation if it remains unchanged by that operation. For example, an equilateral triangle is unchanged if it is rotated 120 degrees. Hence the equilateral triangle is symmetric under the operation of a 120 degree rotation.



As an example of symmetry in a physical system, consider elastic recoil of two billiard balls. The balls recoil from each other just the same if we transport them into outer space as they do on a billiard table here on Earth: the laws governing the collision of billiard balls are symmetric under the operation of translation (change in location).



The conservation laws are, in fact, consequences of symmetry. Demonstrating symmetry under translation, for example, proves the law of conservation of momentum. Symmetry under translation in time (i.e. an experiment produces the same results even if performed at different times) proves conservation of energy, and symmetry under rotation in space proves conservation of angular momentum.



Interaction measured in one frame of reference.

Same interaction measured in a rotated frame of reference. That this observer measures same results of interaction is consequence of conservation of angular momentu

Particle interactions provide many other examples of symmetry. Besides obeying the familiar conservation laws,

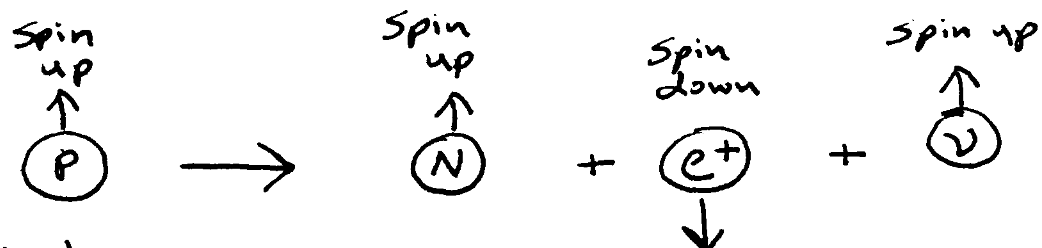
-- Particle interactions are symmetric in electric charge. That is, the final charge of a particle system is the same as the initial charge.

Proton		Neutron		positron		neutrino
p^+	$\longrightarrow$	N	+	e^+	+	ν

Net +1 charge before proton decay

Net +1 charge after proton decay

-- Particle interactions are symmetric with regard to spin.



Initial state spin up

Final state net spin up
(up + down + up = net spin $\uparrow$)

-- The number~~s~~ of baryons ~~are~~ is conserved in any particle interaction~~s~~. So is the lepton number.

$$N \rightarrow p^{+} + e^{-} + \bar{\nu}$$

1 baryon
0 leptons

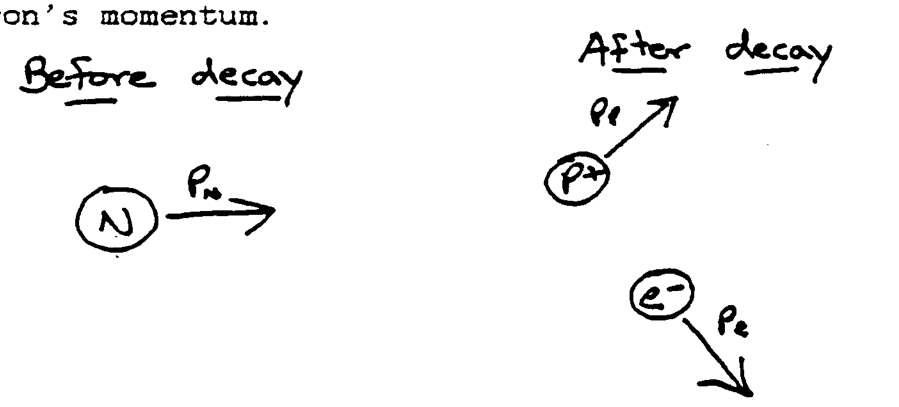
1 baryon
"lepton number" = 0
(electron carries lepton number +1
anti-neutrino carries lepton number −1)

As far as is known, only the weak force violates symmetry (see Ch. 7). Beta decay, for example, preferentially produces neutrinos with left-handed spin. In fact all weak interactions evince spin asymmetry. As we shall see in the next chapter, this asymmetry -- the exception to the rule -- may explain the particle/anti-particle ~~ratio~~ and the fact that the Universe is not, after all, perfectly homogeneous.

PRACTICAL APPLICATIONS

Nature's symmetry provides a powerful tool for learning how she operates. For example, the neutrino was discovered on the basis of symmetry principles:

When it became possible to probe the particulars of beta decay, physicists discovered the total momentum of the products -- proton and electron -- was less than the neutron's momentum.



$$p_P + p_e < p_N$$

Enrico Fermi hypothesized an unobserved particle which carried the missing momentum, and he was able to predict its properties -- charge and spin -- on the basis of symmetry. Later, in more sophisticated particle detectors, neutrinos were measured directly and their properties confirmed.

SUPERSYMMETRY

Some theoreticians have extrapolated known principles of symmetry into a theoretical construct called "sypersymmetry." Supersymmetry relates the fermions to the bosons: it postulates "supersymmetric partners" for the known particles. For example, according to supersymmetry the photon (a boson) has a fermion partner called the photino, and quarks (fermions) have supersymmetric boson partners called "squarks."

	Known particles	Hypothesized supersymmetric partners
fermions	electron	selectron
	quarks	squarks
	neutrino	sneutrino
bosons	photon	photino
	W	wino
	Z	zino
	gluon	gluino
	graviton	gravitino

No supersymmetric partners have been found experimentally, but more powerful accelerators may reveal them.

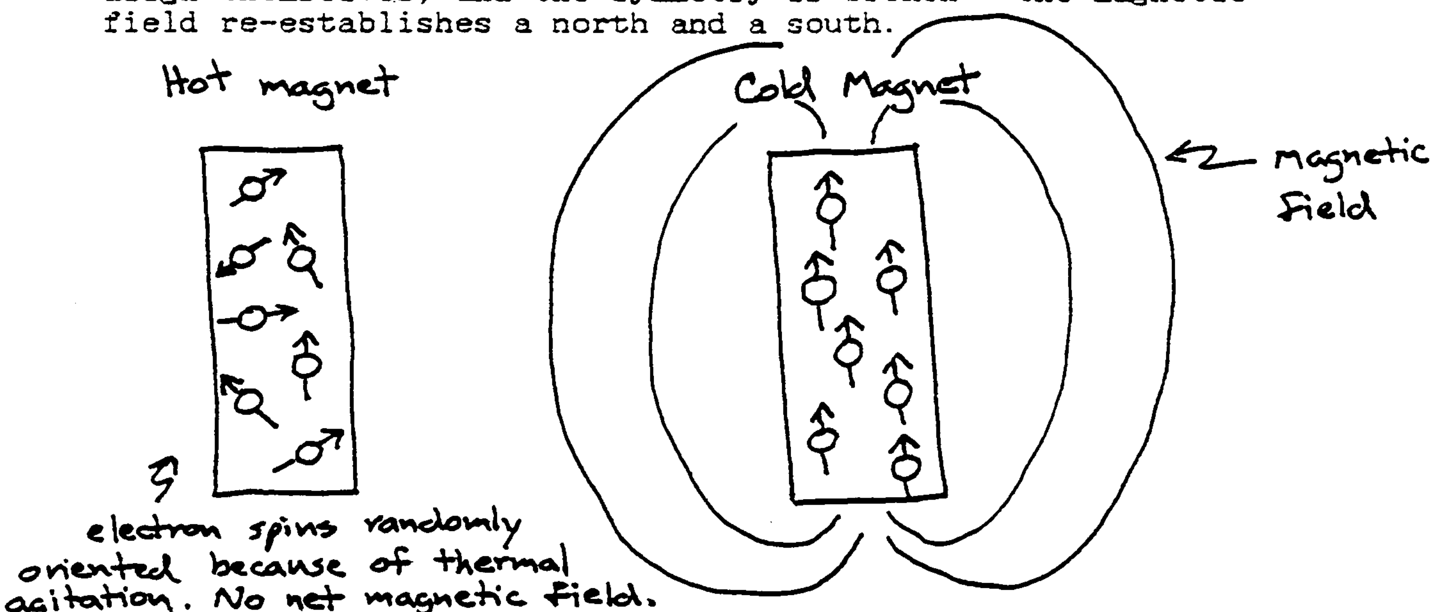
SPONTANEOUS SYMMETRY-BREAKING

If the particles and forces are symmetric and interchangeable, why does the Universe look the way it does? A perfectly symmetric Universe would be dull, indeed -- a perfectly uniform soup. But the real Universe obviously includes a variety of structures: galaxies drift across vast voids, and bright stars sparkle against black sky.

Physicists believe the very early Universe was completely symmetric (everywhere the same, all particles and forces interchangeable) but that the original symmetry was "broken" as the Universe evolved.

Clouds ~~model~~ the early Universe and demonstrate ~~the~~ process of "spontaneous symmetry breaking." They are symmetric accumulations of water droplets: ~~the~~ inside ~~of a~~ cloud looks the same in all directions -- gray mist everywhere. Cool the cloud, though, and the symmetry breaks: raindrops coalesce and fall earthward, defining direction.

Magnets also demonstrate spontaneous symmetry breaking and model the appearance of forces in the Universe. Around a hot magnet, the magnetic field is symmetric: there _is no field_ because high temperature jiggles the magnet's electrons out of alignment. Let the magnet cool (the electrons re-align themselves) and the symmetry is broken: the magnetic field re-establishes a north and a south.



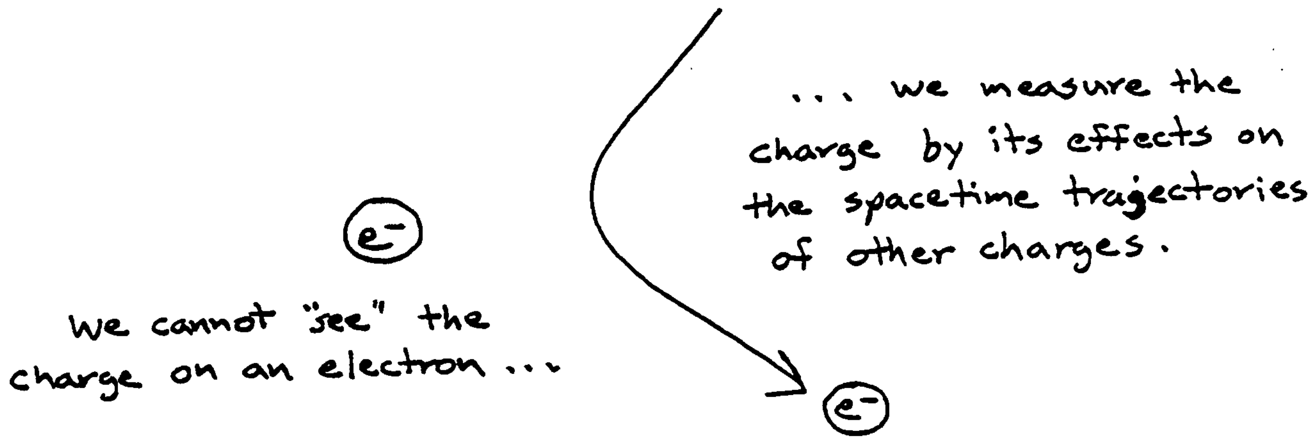
Physicists believe the early Universe (within 10^-43 seconds of its origin -- early indeed!) was perfectly symmetric, comprising pure energy at incomprehensibly high temperature and density. As it expanded and cooled, the symmetry broke. The fermions precipitated out, and so did the bosons governing their interactions -- just as raindrops precipitate out of a cloud and the field out of a magnet. (We will develop this idea further in the next chapter.)

MULTIPLE DIMENSIONS

By now, the reader is familiar with the idea of 4 dimensions -- three of space and one of time. To model the interrelations of particles and forces, physicists postulate even higher-order geometries -- a Universe of ten dimensions, maybe more.

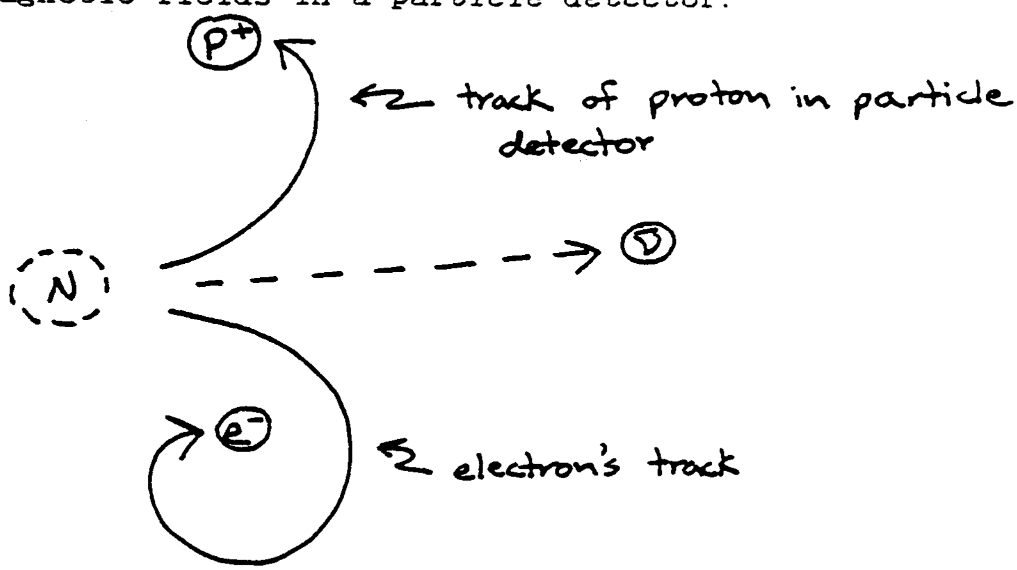
Physicists ~~require~~ multiple dimensions to explain how qualities such as charge, color, and spin can affect the observable universe even though they cannot be observed directly. We cannot "see" charge, for example: we observe its _effects_ on the spacetime trajectories of other charges. The unobservable qualities -- charge, color, spin, etc. --

presumably exist in dimensions that are hidden from direct observation.



As an analogy, we experience effects of "hidden dimensions" when we develop a common cold. The runny nose, sore throat, and congestion represent the visible manifestations of hidden invaders -- viruses which are many orders of magnitude smaller than ourselves, far too small to see directly.

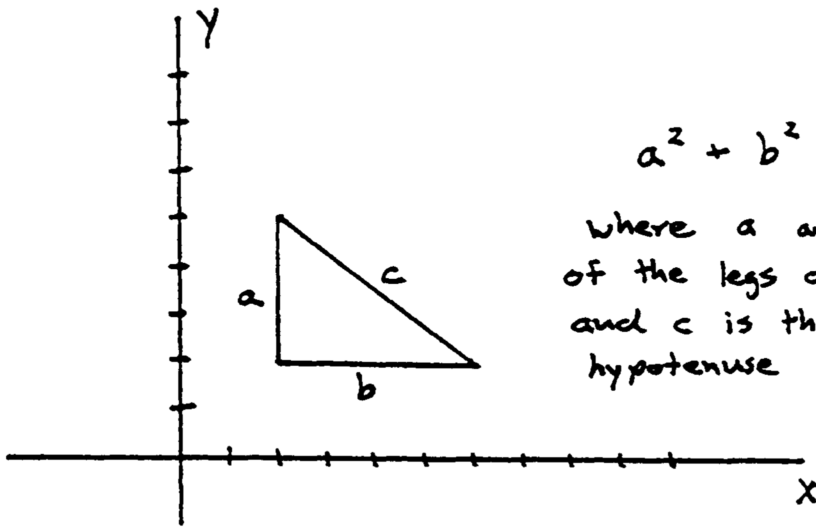
Describing the physical Universe, physicists say hidden dimensions are "compactified," i.e. embedded in the observable four dimensions of spacetime but too small to observe directly. Events in those hidden dimensions twist and shake the four dimensions of spacetime, and we observe the resulting distortions. For example, the events of beta decay -- exchange of W particles -- are hidden from our direct view, but we can observe the results of the weak interaction by tracking the spacetime trajectory of the beta particle (electron) and proton as they interact with the electric and magnetic fields in a particle detector.



MATHEMATICS OF MULTIPLE DIMENSIONS

Mathematicians label the hidden dimensions in their equations with extra "variables." We normally locate events in the visible Universe using four variables -- the three space coordinates, x, y, and z, and one of time, t. If the Universe harbors other "hidden" dimensions, we need more variables to describe them. Heterotic string theory, for example, postulates ten dimensions described by ten variables.

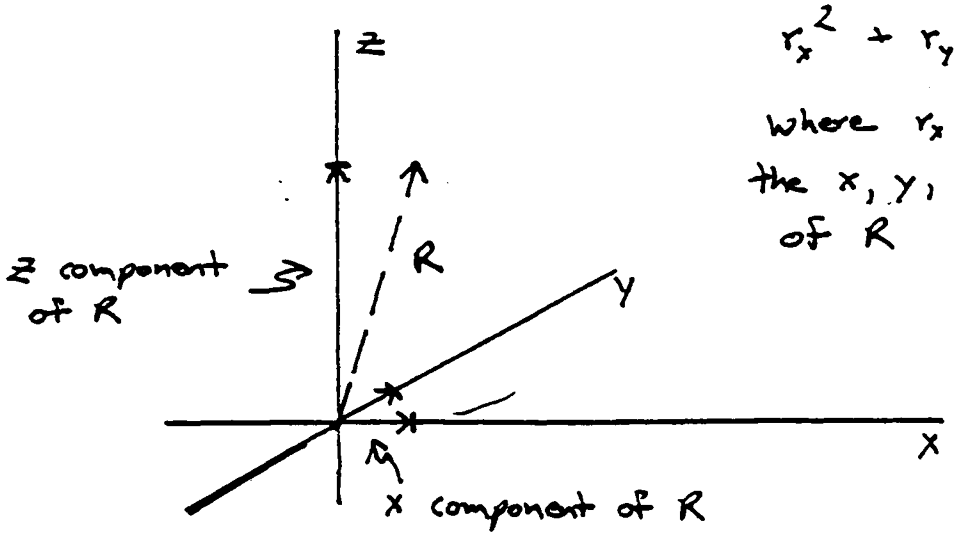
It is difficult to imagine the geometry of 5 or more dimensions, because our senses are confined to three spatial dimensions and one of time, but the extra dimensions behave mathematically just like the four we know. For example, we can define length in two dimensions on Cartesian coordinates according to the Pythagorean theorem:



$$a^2 + b^2 = c^2$$

where a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse

We can measure with a similar methodology ~~in three~~ dimensions:



$$r_x^2 + r_y^2 + r_z^2 = R^2$$

where r_x, r_y, and r_z are the x, y, and z components of R

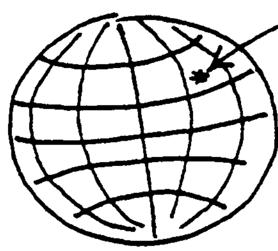
In four spatial dimensions:

$$r_1^{\,2} + r_2^{\,2} + r_3^{\,2} + r_4^{\,2} = R^2$$

where r_1, r_2 etc. are the components (orthogonal projections) of R in each dimension.

And so on in higher dimensions.

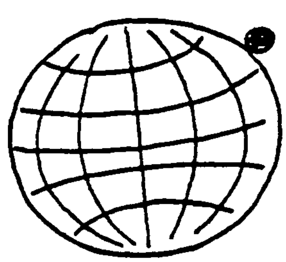
The following hypothetical model [illegible] ~~serves~~ to illustrate the physical effects of one dimension on another. Suppose we describe physics in terms of "spheres embedded in spheres," i.e. imagine at each point in spacetime there is a sphere, too small to be observed. We can locate an event on the surface of any sphere with two variables, latitude and longitude. (The surface is two dimensional.)



each point on a sphere can be identified by its latitude and longitude

Hypothetical sphere embedded at each point in spacetime.

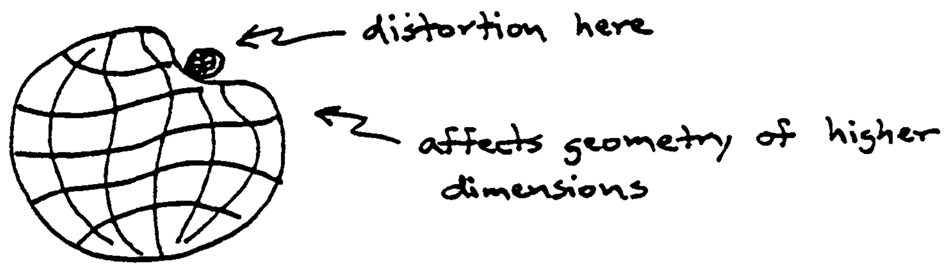
Now suppose at every point on this sphere is another compactified sphere, too small to be resolved but comprising two more dimensions.



secondary sphere

Imagine a secondary sphere at each point on the surface of the primary sphere.

And suppose, further, at each point on that second sphere is another, smaller sphere comprising two <u>more</u> dimensions . . . and so on. Fleas on fleas. If the surfaces are connected, distortions of one dimension affect the geometry of all higher dimensions.



So, perhaps -- and this is sheer speculation for purposes of illustration -- the weak force, in dimensions 7 and 8, affects fermion flavor and electric charge embedded in dimensions 5 and 6, which in turn affect the observable structure of 4 dimensional spacetime.

That's the essence of multiple dimensions: the qualities we associate with fermions -- charge, spin, color, etc. -- exist in hidden dimensions, and those hidden dimensions are embedded in and connected to the geometry of observable spacetime. Hidden dimensions -- the dimensions of the fermions and their interactions -- wiggle and warp the structure of spacetime, which is what we see. We glimpse the <u>tracks</u> of fermions and forces across spacetime, not the beasties themselves.

GROUP THEORY

Given evidence that the various fermions are related to each other and that fermions are related to bosons, we assume they share certain underlying qualities. Given evidence that they inter-convert, one particle becoming another, we assume the particles can exchange those qualities. A mathematical system called "group theory" offers a method to account for such inter-relations. Group theory classifies things according to their components, and it defines how different entities (such as particles) can interconvert if they are built from similar parts. (or exist in an interconnected geometry).

Mathematically, a group is any system consisting of a set, G, and an operation, +, on the elements of G such that:

1. If x and y are both elements of G, $x + y$ is also an element of G.

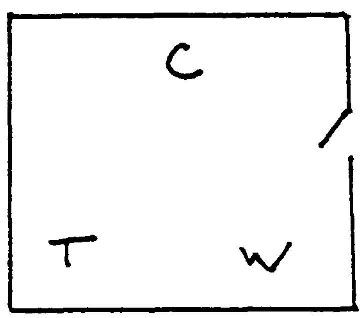
2. If x, y, and z are all elements of G, $x + (y + z) = (x + y) + z$.

3. There is an identity element, e, such that $x + e = x$.

4. Each element, x, has an inverse element, $-x$, such that $x + (-x) = (-x) + x = e$.

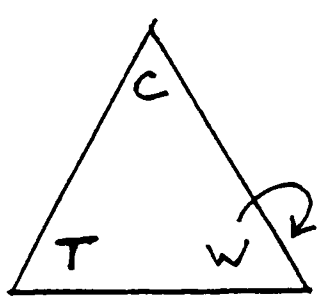
Group theory is an algebra of <u>operations.</u> It enables us to predict the final state of a system after an operation or series of operations.

For example, suppose Grizelda, a music aficianado, is trying to set up her speaker system for best listening. She has a tweeter, T, and a woofer, W, and she always listens from her favorite chair, C. She can place the speakers and chair in any of three locations in her room, and she is trying to find the optimum arrangement.



Grizelda's room, with initial arrangement of Chair, Tweeter, and Woofer

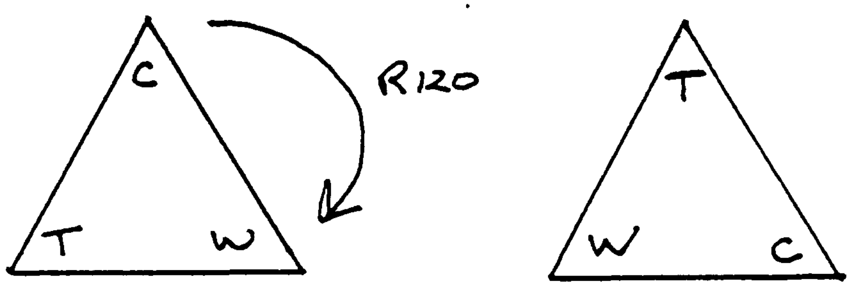
Determined to test every possible configuration of speakers and chair, Grizelda expects a long day of moving furniture, but she can exploit group theory to minimize her effort. (To follow the discussion, cut out a paper triangle and label the corners T, C, and W, as shown below.)



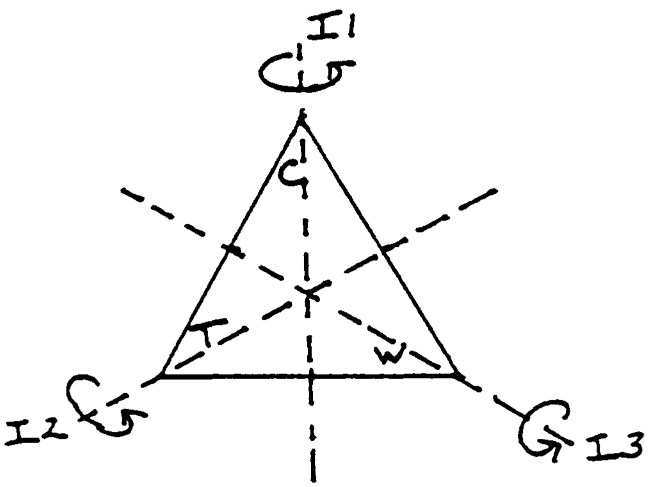
label both sides

Grizelda can perform two kinds of operation on her system -- rotations, R, and inversions, I. Let us define the possible rotations clockwise as seen from above:

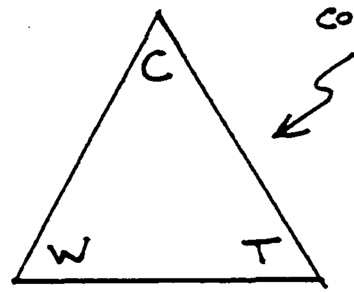
Rotation by 120 degrees



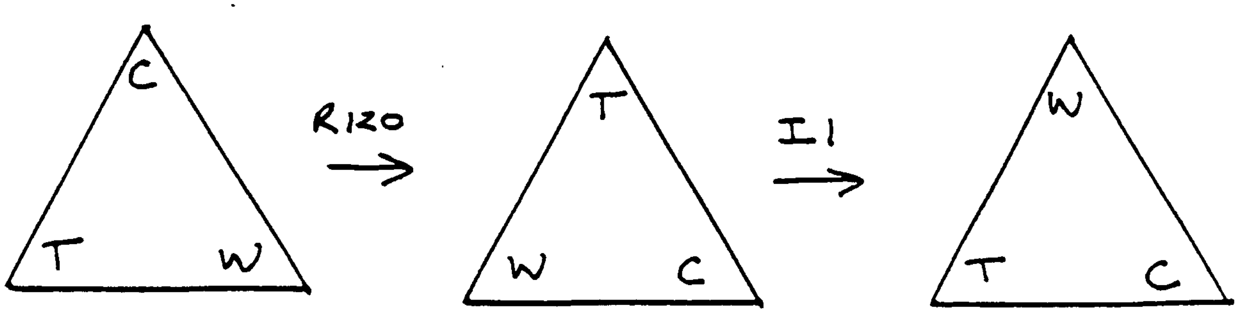
Inversions are defined about the axes drawn in the illustrations. ~~Thus~~



Inversion I1 transforms initial configuration to this configuration

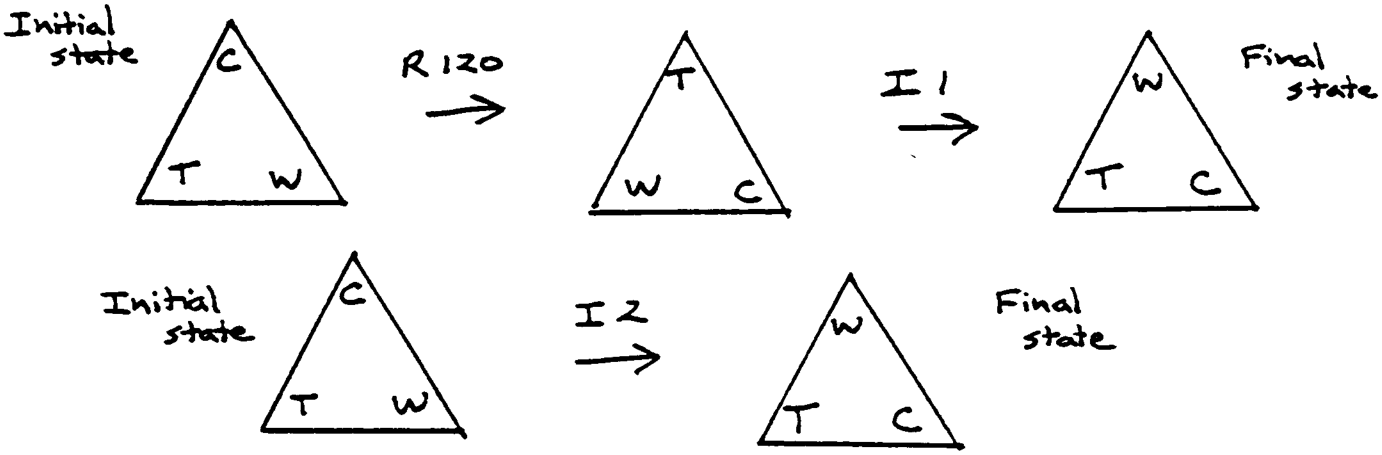


Of course, Grizelda may perform a series of operations. We will call the starting configuration the "initial state," and the final configuration the "final state." For example



Notice that the states are defined in the frame of reference of the room. In general, there must be some outside frame of reference in which to define a state. This becomes an important consideration when, later, we define particle states such as phase and isospin.

After hefting furniture around the room in a number of trials, Grizelda finds a pattern to her work: a single operation can produce the same final state as a series of operations. For example, R120 + I1 = I2



The operation R(120) followed by I(1) produces the same final state as operation I(2) alone.

With this discovery, Grizelda realizes she can save lots of effort. Instead of trying every possible series of operations, she devises the following table:

Second operation

First operation

	R360	R120	R240	I1	I2	I3
R360	R360	R120	R240	I1	I2	I3
R120	R120	R240	R360	I2	I3	I1
R240	R240	R360	R120	I3	I1	I2
I1	I1	I2	I3	R360	R240	R120
I2	I2	I3	I1	R120	R360	R240
I3	I3	I1	I2	R240	R120	R360

i.e. R240 + I1 = I3

I2 + I3 = R240

etc.

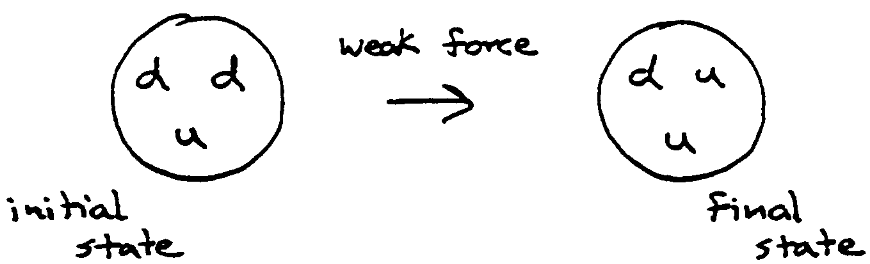
Equivalent single operation

Grizelda has discovered an algebra of <u>operations</u>. Whereas the algebra of our high school days involves numerical quantities, such as 4x + 3y = 10, the algebra of group theory analyzes operations.

GROUP THEORY AND THE FORCES

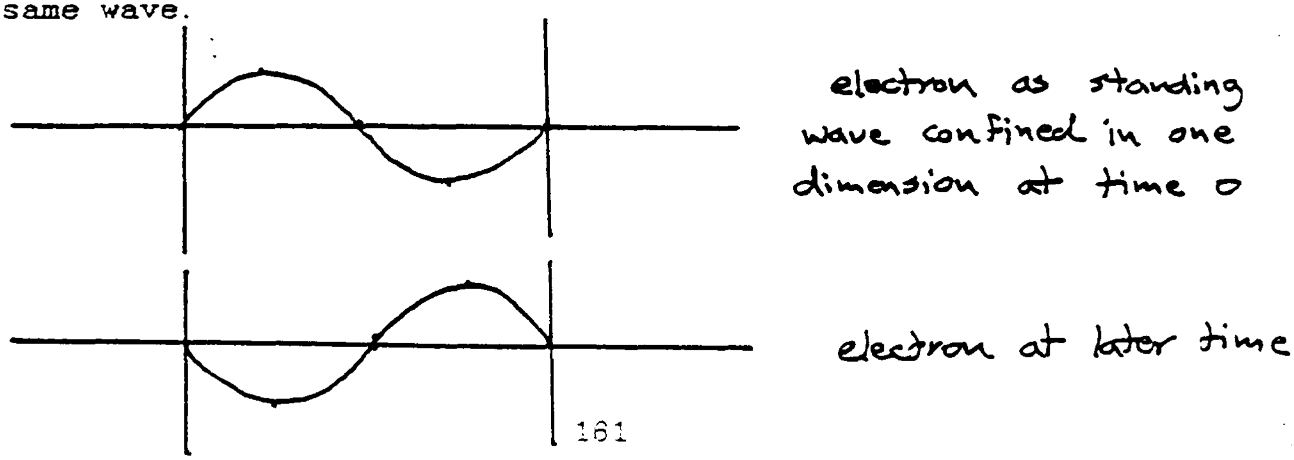
So how does Grizelda's stereo system relate to fermions and forces?

It turns out the electromagnetic force, the weak force, and the strong force all can be described mathematically by group theory. Modelled according to group theory, the forces themselves are operations, and the fermion states are elements of a group. For example, in neutron decay, the initial state includes two down quarks and one up quark. The weak force (operation) converts one down quark (initial state) into an up quark (final state).

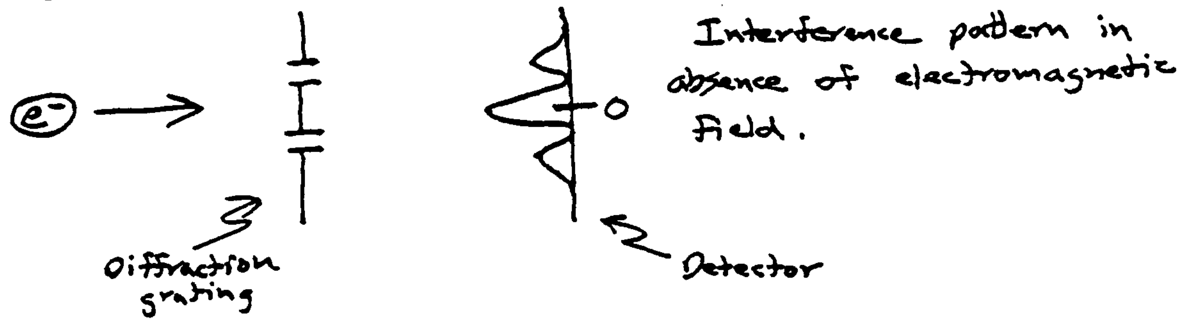


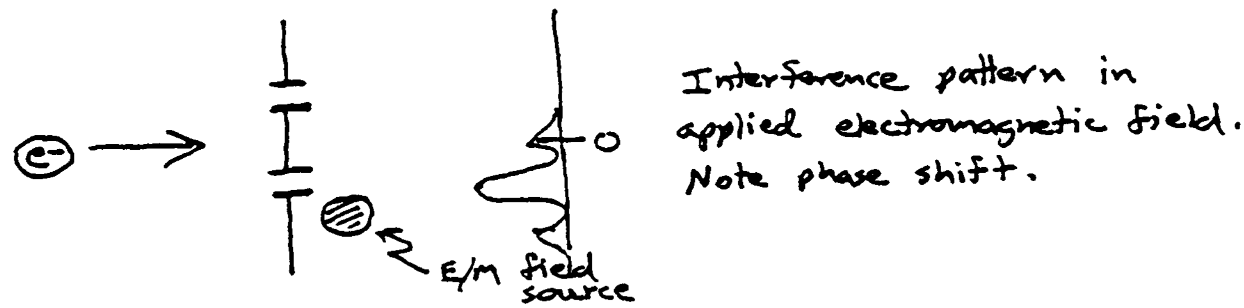
More specifically, physicists model the electromagnetic force as the group of operations called U(1), which stands for Unitary Group 1. U(1) is the group of operations that produce any of the continuous phase transitions from 0 to 360 degrees (0 to 2 pi radians).

We can see the relation of the mathematical group, U(1) to the electromagnetic force on an electron, for example, if we model an electron as a probability wave. The electron wave may change phase, just as any other wave, and remain the same wave.

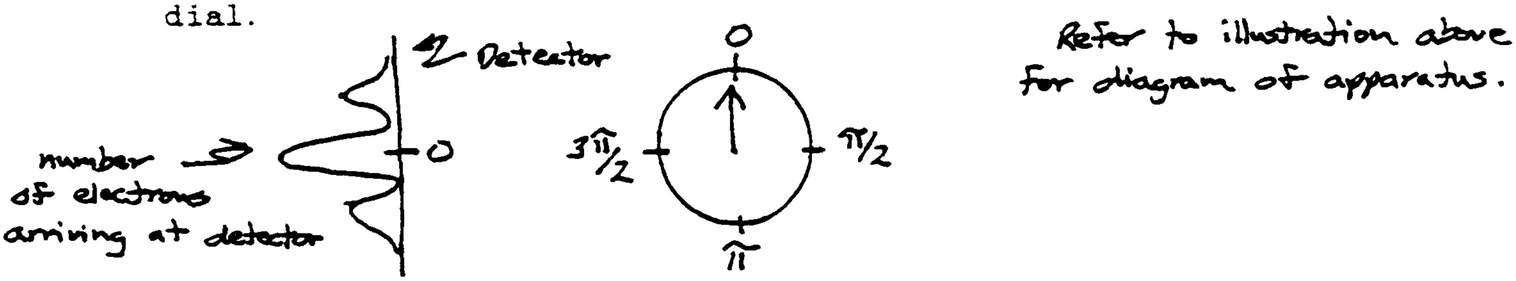


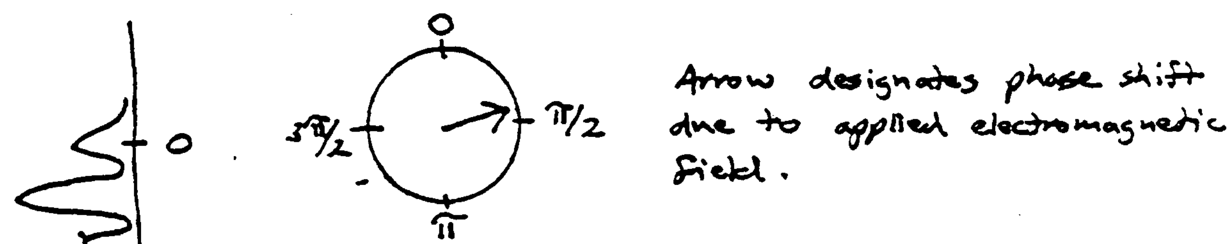
We know the electromagnetic force changes the phase of an electron, because it shifts the interference pattern in electron diffraction experiments, and the amount of shift is proportional to the strength of the field.





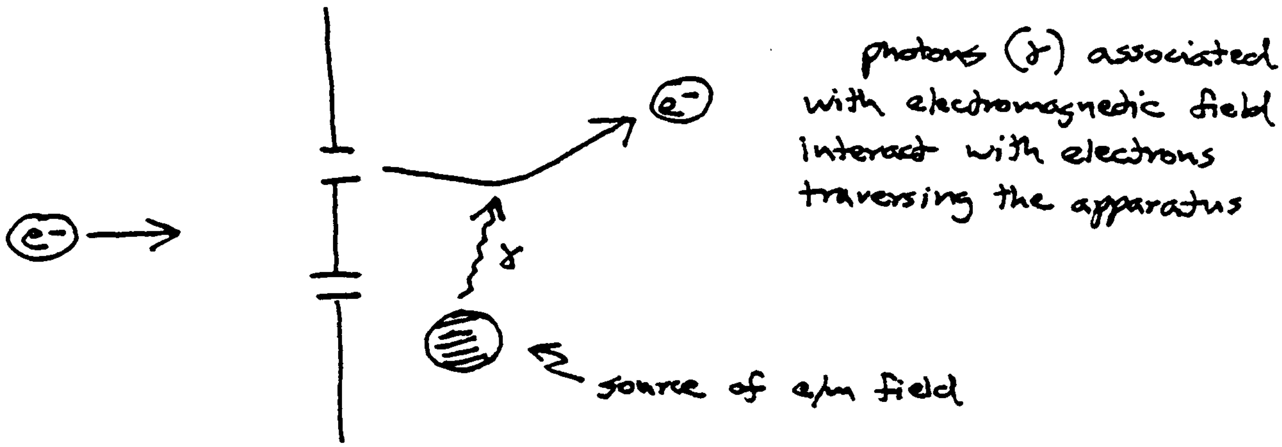
This relation of the electromagnetic _force_ to _geometry_ (phase shift) translates nicely to the mathematical terms of U(1). The electromagnetic force can be _represented by_ the geometric phase shift. By convention, we will represent phase as a pointer on a dial, like a single hand on a clock dial.



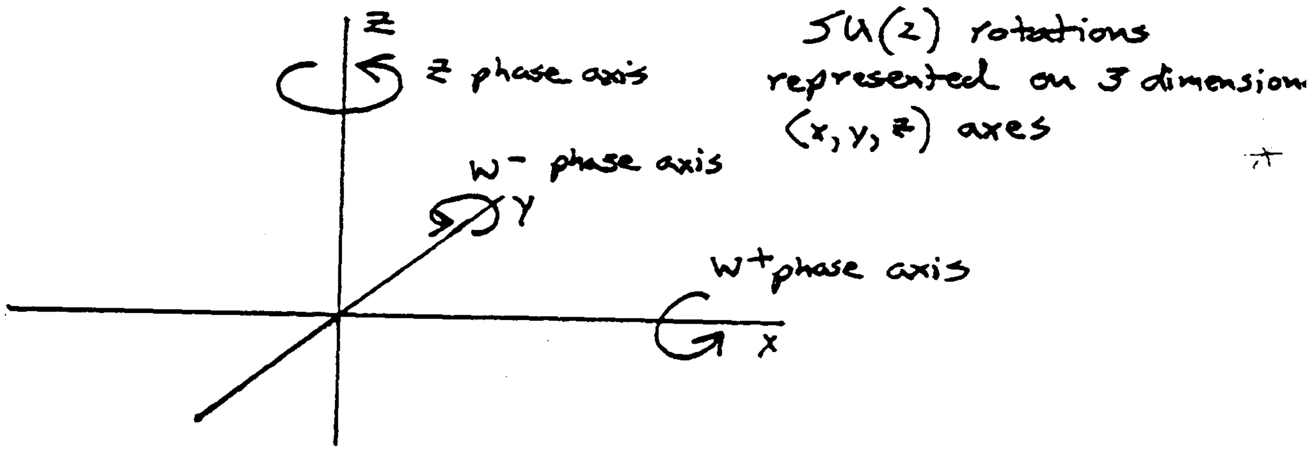


Notice that the operation (force) must deflect the pointer in relation to some frame of reference -- in this case the face of the dial.

If we compare U(1) with Grizelda's stereo system, the electromagnetic force behaves as a group of operations which can rotate the speakers and chair around the room by _any_ angle between 0 and 360 degrees. The agent, in the real world, that actually performs the electromagnetic operation is the photon.



Physicists model the weak force mathematically by the group SU(2) (special unitary group 2). SU(2) describes a geometry with <u>three</u> phase angles, corresponding to the three vector bosons, W-, W+, and Z. We might imagine, in analogy with Grizelda's stereo system, the final state is determined by some rotation around the room, as in U(1), mediated by the Z particle, which is the weak analogy to the photon, <u>plus</u> a rotation about the axis through the east and west walls of the room, <u>plus</u> some rotation through the north-south axis.



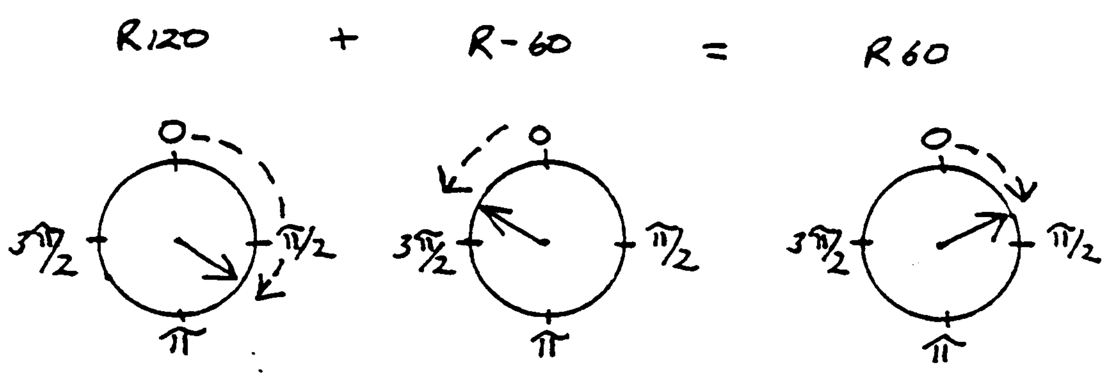
In the 1950's and 1960's, Steven Weinberg, Sheldon Glashow, and Abdus Salam showed that the weak force is related to the electromagnetic force: the weak force, with three phase axes, is a higher dimensional representation of the electromagnetic force, which has but one phase axis. "Electro-weak unification," that is, the underlying relationship between the two forces, becomes evident at high energies, about 90 GeV, with the production of the vector bosons. Experiments such as those by Carlo Rubbia and his group at CERN have comfirmed electroweak theory dramatically.

Physicists describe the strong force by the group SU(3), which includes eight independent phase angles corresponding to the eight possible gluons. The geometric representation of SU(3) requires 5 extra spatial dimensions: ~~I~~ won't ~~even~~ attempt to illustrate the possible rotations, since the paper limits ~~me~~ to but two dimensions in the three dimensional world of our experience. You might, however, try to imagine the possible rotations of Grizelda's system in higher dimensions.

Taken together, the combined groups U(1) X SU(2) X SU(3) represent the "Standard Model" of particle physics. Experimental tests in particle accelerators support the standard model, but it makes predictions such as the existence of the Higgs boson (to be discussed below) which have not yet been verified.

Most seriously, the Standard Model is incomplete in that it does not incorporate the force of gravity. As we shall discuss ~~below~~, finding the relation of gravity to the other forces is a major field of inquiry.

Two final observations ~~to~~ complete our discussion of group theory: notice that group theory incorporates concepts of symmetry and of multiple dimensions. The groups are symmetric in that each phase has an opposite, and the operations are symmetric in that series of operations can result in the same final state as a single operation (e.g. in U(1) a rotation by 120 degrees plus a rotation by minus 60 degrees results in the same phase as a single rotation by 60 degrees).



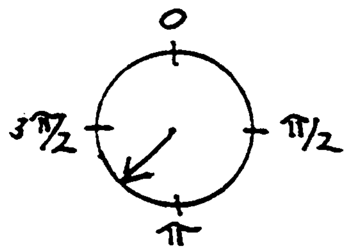
As we have seen, the groups SU(2) and SU(3) require multiple dimensions for their description, e.g. the eight independent phase angles of SU(3).

All these considerations of group theory are incorporated into the larger framework called "gauge theory," the current best mathematical system for understanding the fermions and forces.

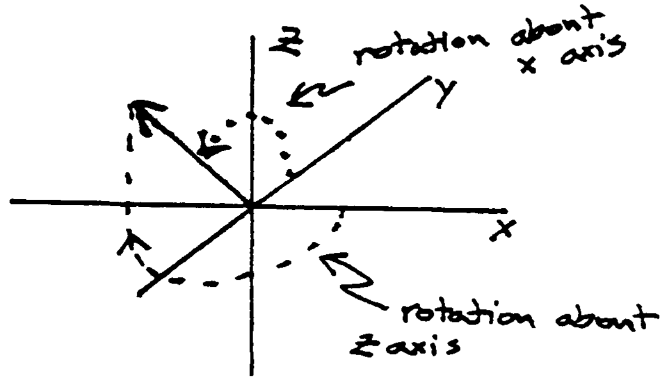
GAUGE THEORY

The Standard Model is one of the triumphs of modern physics. It accomodates known particles and their interactions, and it has predicted the existence of new particles which were subsequently found.

Mathematically, the Standard Model is a "local, non-Abelian gauge theory." In fact, we've already learned the foundations of gauge theory in our consideration of groups. Crudely defined, gauge theory describes the particles and forces as local measurements of state -- local gauges. A gauge, roughly, is a measuring device, like the dial on a clock, and the position of the hand represents a state. A clock, ~~for~~ example, is a gauge: it measures the state of time. As we discussed in the previous section, we can imagine other, similar gauges measuring the local electromagnetic state, the weak state, and the strong state.



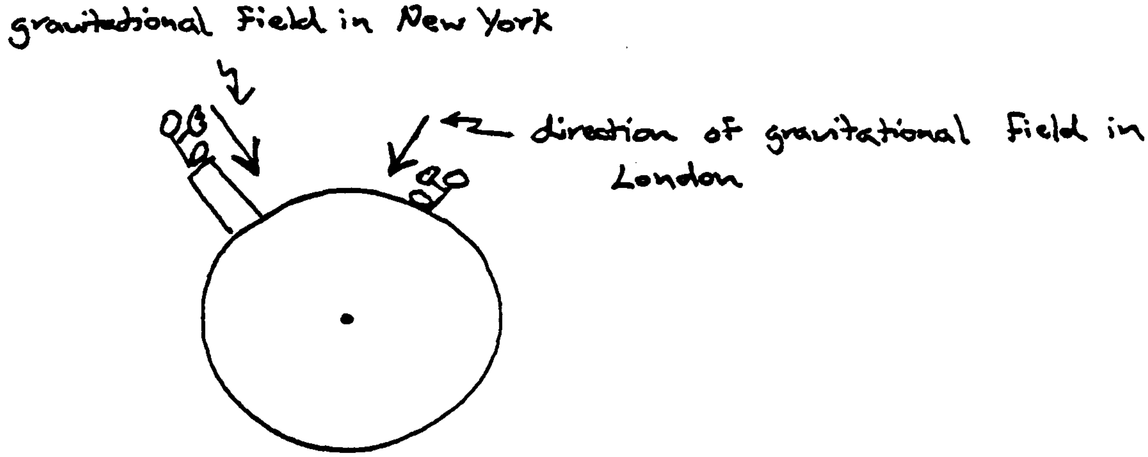
Electromagnetic state at a point in spacetime



Weak state at the same point in spacetime (solid arrow).

LOCAL GAUGE INVARIANCE

The concept, first recognized by Einstein, ~~that~~ all physics is local is central to gauge theory. Relativity theory says our measuring tools (gauges) change according to local spacetime curvature, i.e. the lengths of meter sticks and the rates of clocks depend on location. To cite an example from relativity, consider two observers, one on the banks of the Thames in London and the other atop the World Trade Center in New York City. The two observers experience different gravitational fields (different spacetime curvature): the field direction differs at the two locations, and the gravitational potential differs, since the observer atop the World Trade Center is farther removed from the Earth's center of mass.

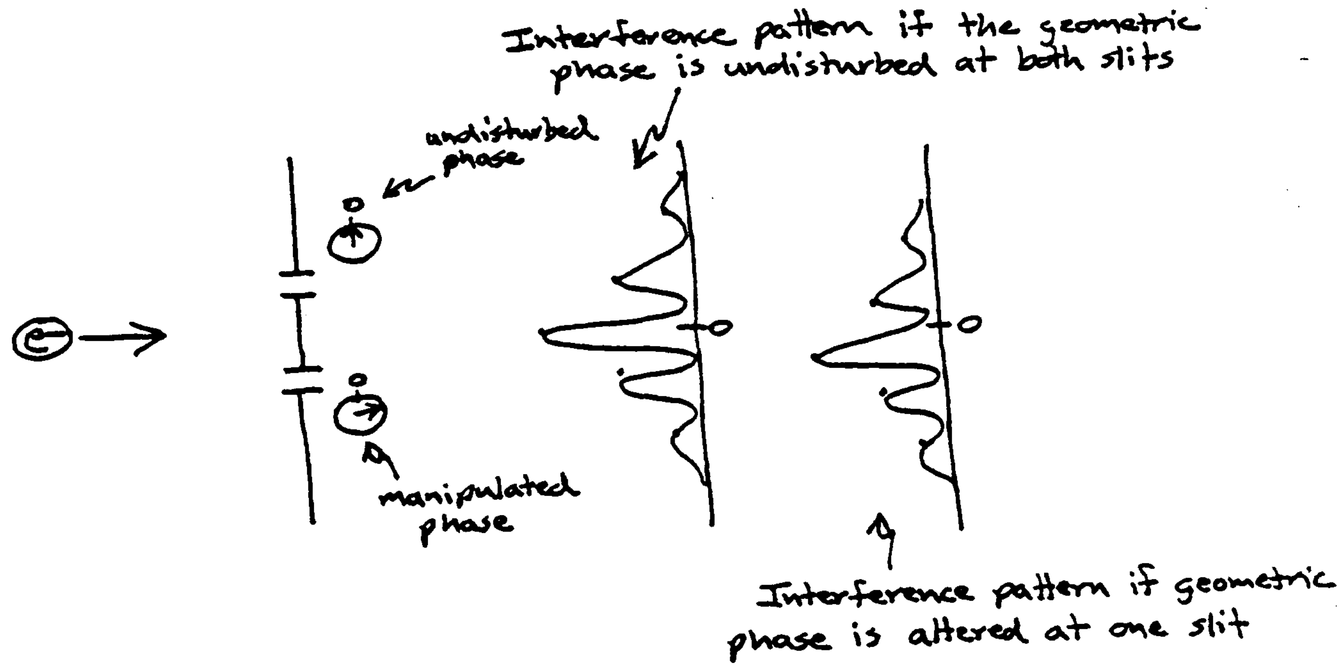


The gauges in spacetime physics are meter sticks and clocks. In our example, the gauges differ at the two locations: the clock ticks faster atop the Trade center, and a meter stick on the Thames is slightly longer, due to tidal effects.

THE GAUGE PRINCIPLE

The crux of the gauge theory of particle interactions is the gauge principle, which says that a force is indistinguishable from a local gauge transformation. We return to the double slit experiment to illustrate this concept.

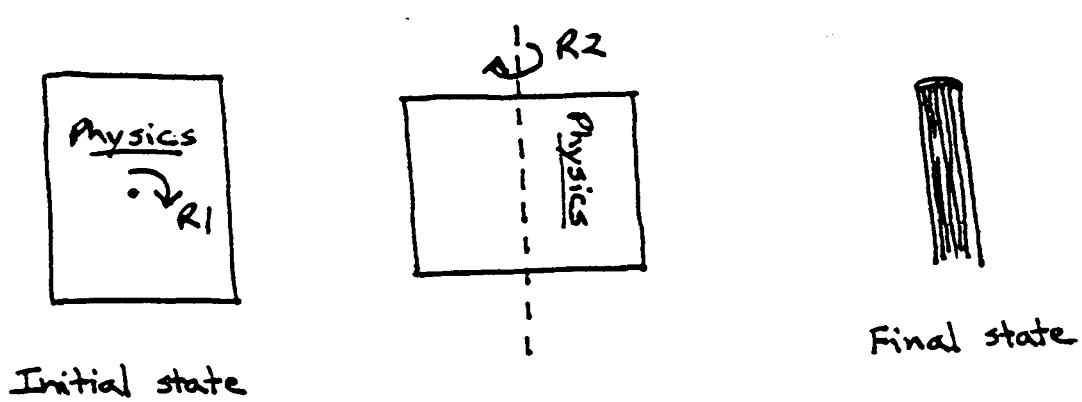
When electrons traverse a double slit, the interference pattern at the detector depends on the relative phase of the electron waves. If we can somehow change the geometric phase of the waves, the interference pattern changes.



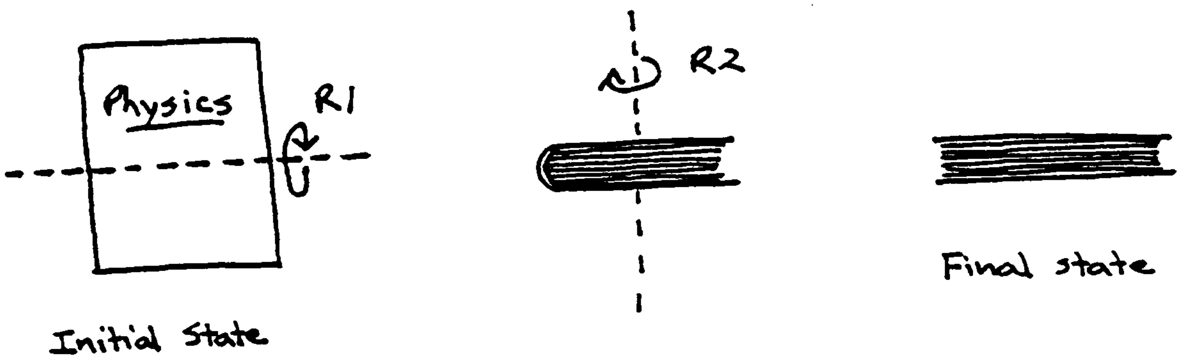
But changing the geometric phase of the waves is indistinguishable from applying an electromagnetic field at one of the slits (see p.162). That is, the abstract mathematical manipulation of electron phase is indistinguishable from the application of a "real" force.

The same principle applies to the weak and strong forces: the weak force is indistinguishable from the mathematical alteration of isospin (SU(2)) phase, and the strong force is indistinguishable from a rotation of color (SU(3)) phase.

Gauge theory is "non-Abelian," that is non-commutative, in that a series of rotations in multidimensional space may produce a different final state if the rotations occur in a different order. Operations on a textbook illustrate this idea: hold the text with its cover up, as if you were about to open it (initial state). Rotate it 90 degrees clockwise around the axis through the front and back covers, then rotate it around the axis running along the lines of text.



Now perform the same two rotations, but in reverse order.



This final state differs from the previous final state.

The non-Abelian nature of gauge theory is especially evident in the weak and strong interactions, where series of rotations in phase space may produce completely different particle states.

In the mathematical formalism of gauge theory, "forces" behave in such as way as to preserve the wavefunction of the Schrodinger equation (which describes the dynamics of a fermion -- see Appendix), except the force changes the phase of the wavefunction. Vice versa, changing the phase of the wavefunction demands the application of some "force."

$$\frac{1}{2m}\left(\frac{\partial^2\psi}{\partial x^2}\right) = i\hbar\frac{\partial\psi}{\partial t} + \psi V$$

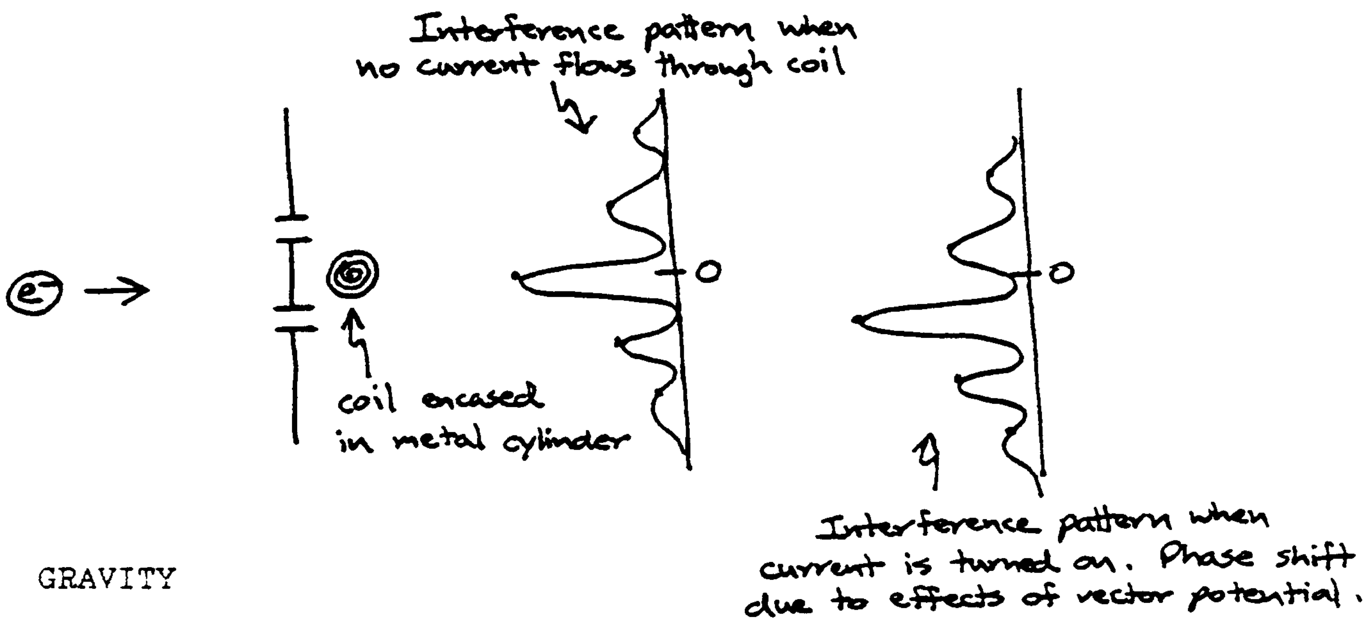
initial state, where ψ is the wavefunction of a fermion

If ψ changes phase, to $\psi' = e^{i\alpha}\psi$, a force term must be added to the Schrodinger equation

$$\frac{1}{2m}\left(\frac{\partial}{\partial x} - qA\right)^2\psi' = i\hbar\frac{\partial\psi'}{\partial t} + \left(V - \frac{\partial\nu}{\partial t}\right)\psi'$$

Note that this mathematical formalism implies a local symmetry: the change in phase leaves the amplitude of the wavefunction unchanged. The totality of a wave packet like an electron is unchanged even if it's phase changes, just as an ocean wave is still a wave whether we happen to be bobbing on its crest or in the trough.

Experimental evidence for ~~symmetry in~~ gauge interactions ~~is found~~ in the Bohm-Aharonov ~~effect~~, among others. In 1963, Aharanov and Bohm showed that ~~the~~ electron wave responds to the vector potential, a geometric measure of the electromagetic force in a region of spacetime, roughly analagous to the gravitational potential in descriptions of the force of gravity. In the experiment, they sent electrons through a double slit aparatus past a coil encased in a metal tube. ~~The metal~~ confines the magnetic field: i.e. there is no measurable field outside the tube. However, the vector potential remains, and its value depends on the current through the coil. If the experimenter imposes a change in the local gauge -- ~~which~~ is the vector potential -- by changing the current, the electron phase gauge compensates, as evidenced by ~~the~~ shift in the interference pattern. The phase shift is exactly the same as if a magnetic field was applied at one of the slits.



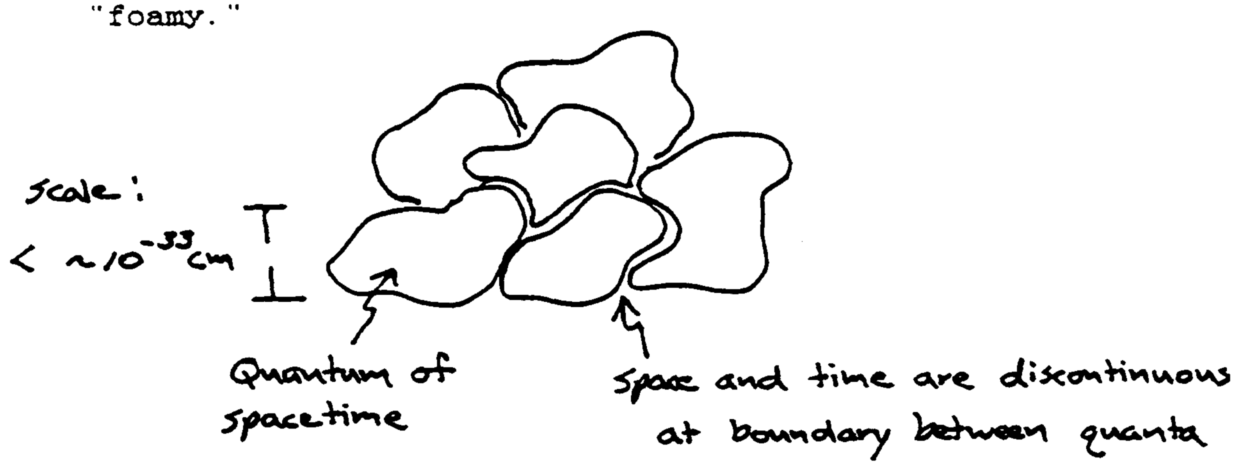
GRAVITY

The gauge theory of particle interactions, U(1) X SU(2) X SU(3), has been remarkably successful: it is supported spectacularly in accelerator experiments. But where does gravity fit into the picture? To phrase the question in terms that are more amenable to gauge theory, what is the origin of mass, which is the "charge" by which gravity is measured?

That it has proved difficult to reconcile gravitational theory with the gauge theory of particles is, in fact, rather ironic, since gravitational theory, as embodied in general relativity, was the first of the gauge theories: as described above, gravity can be understood in terms of local spacetime curvature measured by meter sticks and clocks -- the gauges of spacetime.

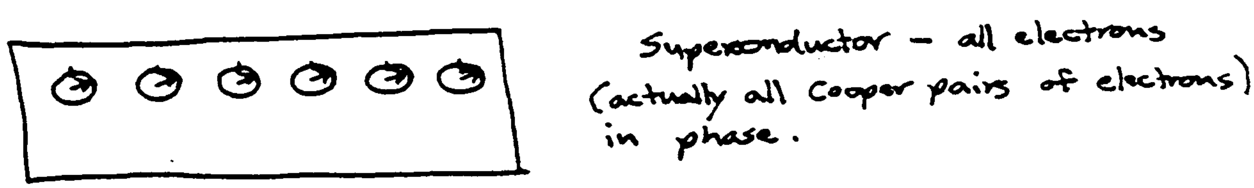
The problem is to reconcile the scale of stars and planets -- the realm of general relativity -- with the scale of particles. To incorporate gravity into a theory of everything requires a quantum theory of gravity: how does gravity behave at the scale of the particles?

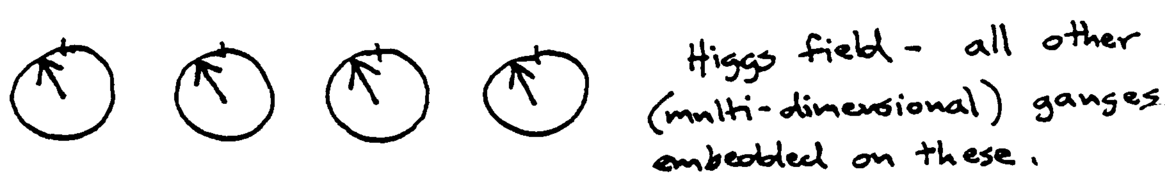
Presumably, at the particle scale spacetime itself is quantized: space and time are discontinuous, and there are quantum spacetime jumps from one point to the next -- as if you stepped out of your bathroom Tuesday morning and landed in the park the previous Wednesday afternoon. In the parlance of quantum gravity aficianados, spacetime is "foamy."



THE HIGGS FIELD

The most promising, but as yet unproved, quantum theory of gravity adduces a "Higgs field," a kind of background gauge against which all local gauges can be compared. It is a "self-coherent system:" that is, every local Higgs gauge is set identical to every other. The Higgs field is analogous to a superconductor, in which all the electrons exist in the same energy state.





By this model, mass is the measure of the difference between a local gauge and the Higgs background. But against which gauges is the Higgs gauge to be compared? The U(1) gauge of e/m? Or the SU(2) gauge of the weak force? Or is there another underlying gauge which must be evoked? The question remains open, but it is known that electric charge contributes to the mass of the electron. Whether the U(1) phase contributes <u>all</u> the electron's mass is not known.

If there is a Higgs gauge, there should be Higgs particles. None have been identified, as yet, in accelerator experiments, but particle physicists press the search in earnest. One of the goals of the superconducting supercollider is to seek Higgs particles.

<u>TYING IT ALL TOGETHER</u>: <u>HETEROTIC STRINGS</u>

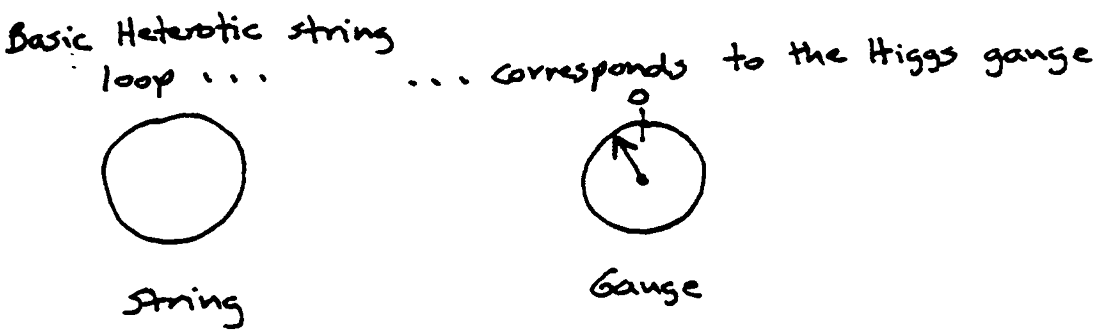
At the beginning of this chapter we posed three questions that any unified theory must address. Let's review those questions now, given our new tools.

How are the fermions and bosons related? We can describe the various fermions using group theory. Each fermion has a particular electromagnetic phase, weak phase, and strong phase described by the groups U(1), SU(2), and SU(3), respectively. The groups SU(2) and SU(3) require multiple dimensions for their evaluation.

What distinguishes one fermion from another? Differences in U(1), SU(2), and SU(3) phase.

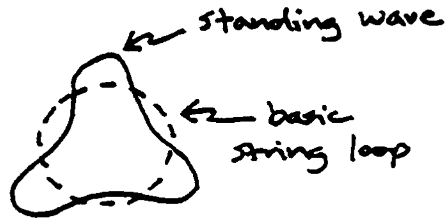
How do the fermions interact? According to the gauge principle, interactions can be described in geometric terms, as a change in phase.

Heterotic string theory incorporates all of the above concepts of multiple dimensions, symmetry, group, and gauge: It builds a Universe out of heterotic strings in which individual loops are the quanta of spacetime. Flat spacetime, the Higgs field, is the loop itself -- the basic, underlying gauge.

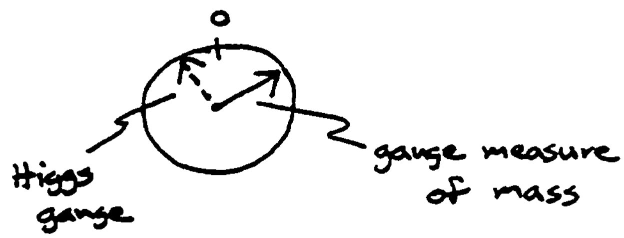


The loops exist in multidimensional spacetime (ten dimensions) and can oscillate in those dimensions.

Superimposed on the lowest dimension of each loop may be a standing wave. The wave frequency is the gauge measure of mass.



String



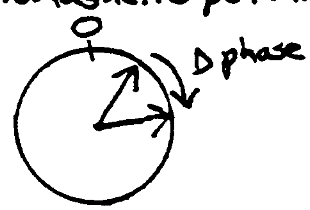
Gauge

Superimposed in a higher dimension is another wave. The phase difference between vibrations on adjacent loops in this dimension measures electromagnetic potential.

In a higher dimension . . .

. . . difference in phase measures electromagnetic potential

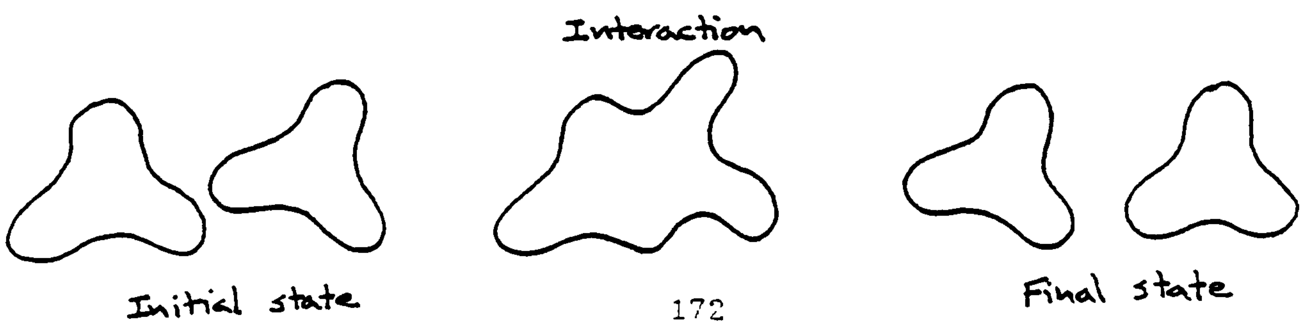
String



Gauge

Phase differences between oscillations in higher dimensions measure weak potential and strong potential.

Interactions occur because adjacent loops can unite and exchange modes of vibration before separating once again.



Interactions are local, between neighboring loops, and local gauge is conserved.

Heterotic string theory (or rather this interpretation thereof) thus provides a symmetric (fermions and forces made of the same loops), multidimensional (waves on loops of strings embedded in multiple dimensions), grouped (fermions defined by groups of vibration modes), gauge theory (geometric phase model for particle interactions).

SUMMARY

Physicists hypothesize that all fermions and forces are inter-related, and they seek a mathematical "theory of everything" describing the fermions and bosons. In this chapter we have outlined some of the key components of current theoretical endeavor.

The concept of symmetry provides a foundation for theory. The conservation laws and local gauge invariance follow from symmetry principles.

Multi-dimensional geometry provides a framework for linking the quantum characteristics of particles and forces. Qualities we call "spin," "charge," "color," etc., may be geometric structures -- "fleas on fleas" -- compactified in the four-dimensional grid of spacetime.

Group theory describes the fermions as members of the groups U(1), SU(2), and SU(3). The theory represents each fermion as a particular U(1) X SU(2) X SU(3) phase.

Gauge theory explains force as a change in local geometry, that is a force is equivalent, mathematically, to a change in gauge phase.

Heterotic string is one of several supersymmetric theories that incorporate these concepts and that may unify our understanding of the known particles and forces.