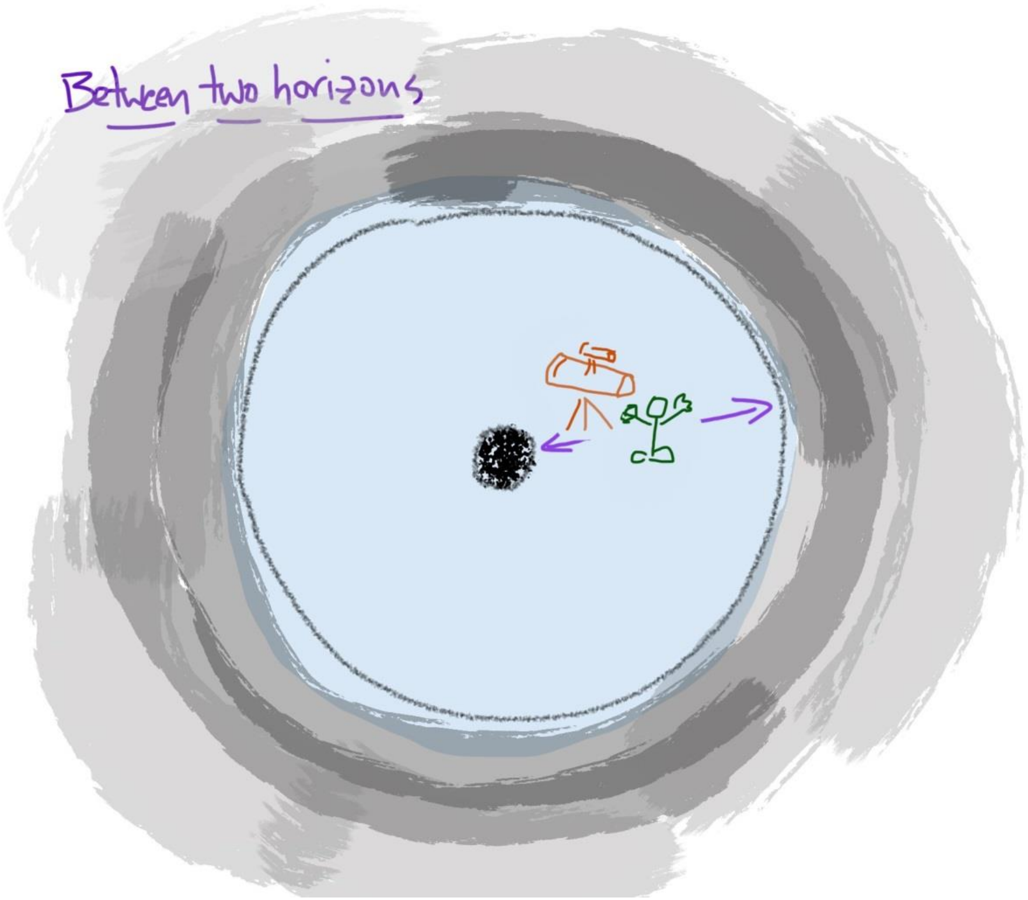
Chapter 16
Quantum Gravity



Introduction

Step outside on a clear night and look at the heavens. (Hopefully you can find a place away from lights and smog and wildfire smoke. It used to be easy.) What do you see? Of course, we're captivated by the sparkly stuff, the moon and stars and planets. A good telescope picks out the fuzzies, too. Planetary nebulae and galaxies and such. All fascinating. But what's there, mostly? Mostly it's the blackness. By far the most of what's out there in the night sky is the blackness. That's the remaining frontier. What's out there in between the galaxies? What are the great galaxy clusters swimming in? What is it out there in the blackness that's accelerating galaxies, masses of hundreds of billions of stars. Accelerating entire galaxies! Imagine your feet braced on the railroad ties trying to push a locomotive. What's doing that? Pushing galaxies, masses of zillions of locomotives, toward the speed of light away from each other and out beyond the cosmic horizon. That's what we're interested in here. Curious effects of gravity that create the sparklies and the voids and the pull and the push. What is it makes up the fabric and creates the dynamics of spacetime?

We've run into blank regions on the map trying to answer those questions with the familiar tools of general relativity and quantum field theory. We need new tools. And it turns out, just in time, the computer hackers have handed us a set of wonderful new tools in information theory.

What's the connection? Gravity is spacetime structure. Gravity is geometry. Information, on the other hand, is 1's and 0's manipulated in electronic circuits or mechanical devices or, these days, quantum computers. How do you get gravity from information, and vice versa?

Long answer is what follows in this chapter. Short summary is that gravitational systems behave like quantum computers. Drop the Oxford English Dictionary into a black hole. Black hole stores that information, processes it, eventually spits it back out (we think). Just like your laptop. Enter information. Computer stores it, processes it, calculates an answer to your query. Input, computation, output. Same for the universe at large and all its parts. Input fields. Algorithms, the laws of physics, process those fields. Output is large scale structure and stars and planets and brains to try to figure it all out. "It from bit" (now "it from qubit") is how John Wheeler summarized the program (Wheeler, 1989). All of reality from information. All of reality from the quantum.

Ideas from the realm of quantum information have enabled remarkable progress in efforts to develop a theory of quantum gravity. Among those ideas, tools provided by AdS/CFT continue to open new windows on the connections between general relativity, quantum mechanics, and information theory; recent reports claim to have resolved the black hole information paradox; and new work may have produced a model for quantum gravity in our (presumed) real de Sitter universe. Even more exciting, perhaps, laboratories are building quantum gravity systems on benchtops, black holes inside quantum computers. Marvelous stuff.

We'll review some of that recent progress, try to understand the underpinnings, and provide links to the research at its source. Most of what follows comes out of the East Coast / West Coast axis. Juan Maldacena at the Institute for Advanced Study and Leonard Susskind at the Stanford Institute for Theoretical Physics have added many new ideas and stirred the pot. They and their collaborators are weaving a compelling tapestry from quantum information theory, general relativity, and quantum mechanics that just may fill in the voids of our understanding what's out there in between the galaxies.

Part I of this chapter reviews key ideas in general relativity, quantum mechanics, thermodynamics, and information theory. It's by no means complete, just what we'll need to try to get a grasp on the new stuff. Part II summarizes recent progress. We briefly present the new concepts then provide links to relevant research articles and video lectures. There are lots of good video resources out there; we've catalogued the best of the best (well, our personal favorites, anyway). The field is progressing rapidly, but most researchers and their institutions try to make their work accessible. The internet links, especially, will help you keep up.

Part the First: Background

In which we provide the motivation and general background necessary to understand the information-gravity connection. We'll review essential ideas of general relativity, quantum mechanics, thermodynamics, information theory, and quantum computation. See previous chapters for more details. Here are just the essentials.

Why we need a theory of quantum gravity

We are approaching four hundred years since Newton formulated his universal law of gravitation. It has served us well. Besides holding us on the surface of Earth it landed humans on the moon and navigated robot probes to the planets. It has helped us understand the large scale structure of the universe, its origins and likely fate. Only in the most extreme concentrations of mass and energy do its predictions fail. In the vicinity of stars and galaxies and, certainly, black holes we need Einstein's improved description of gravity, the general theory of relativity (GR).

These days, however, we are bumping up against the limits of GR. General relativity assumes a smooth, connected spacetime where action is local. Time flows smoothly (though clock rates change depending on their distance from mass). Different events may or may not appear simultaneous to different observers, traveling at different speeds or located in different gravitational potentials, but all observers can agree on a system (a metric) to locate events in spacetime. And an event occurs because some neighbor jiggled it, not because of some spooky, instantaneous connection between it and a faraway joy stick. But in extreme environments, as near the singularity in a black hole or at the Big Bang singularity, these assumptions fail. GR can't tell us what happens. Even at the event horizon of a black hole, where solutions to Einstein's equations still (we think) give us an accurate description of what an intrepid astronaut would experience, events are occurring that we cannot (yet) explain.

Most obvious are the enigmas within black holes and cloaking them. Black holes are the hydrogen atoms of gravitational physics. Just as the physics of the hydrogen atom provided the inspiration and testing ground for the development of quantum mechanics in the early twentieth century, black holes offer conditions demanding an improved theory of gravity, a quantum theory of gravity. As Stephen Hawking and Jacob Bekenstein showed in the 1970's, black holes radiate. They emit particles, Hawking radiation. That is a quantum process. It is thought that the Hawking radiation results because tidal (gravitational) forces near the horizon separate virtual particles seething out of the vacuum. One falls into the black hole, the other flies out. Gravity and quantum mechanics both at work. Together.

Another, more recent example: the black hole information paradox. According to general relativity, if you drop an encyclopedia into a black hole, that information is lost from our universe. But quantum mechanics insists information is strictly conserved. There's compelling argument, and some persuasive laboratory evidence, that quantum mechanics is right. So what's the fix for gravity?

We need a quantum theory of gravity. As we'll see, black hole physics will shed a lot of light on the subject. So will quantum computers and information theory. So will good ol' classical thermodynamics. Let's get going.

General Relativity

General relativity (GR) is a classical theory, i.e. not quantum. It is Einstein's theory of gravity, built on the foundation of Newton's mechanics, Maxwell's fields, Boltzmann's thermodynamics, and Einstein's own special relativity. (For a more complete review, see Ch.5 of this book and, even more complete, Susskind and Cabannes, *General Relativity*, 2023.) General relativity, like the other classical theories, is local and it is causal. As we'll see, this puts it in apparent conflict with quantum experiments.

GR assumes that all physics is local. In Einstein's words, there is no "spooky action at a distance." If something suddenly moves, it must have been pushed by something right next to it. That something may have been another physical object touching it, or more generally a field. The moon orbits Earth not because there's a cable connecting them but because the moon is trapped in Earth's gravitational field.

Field? What field? You can't touch it, can't grab ahold of it. You only know it is there because you can see its effects on light and on masses. GR helps paint a picture. GR interprets the field as the curvature of spacetime. The presence of mass-energy distorts the surrounding space and time, like a bowling ball distorts the surface of a trampoline. Space stretches, clocks slow down in the vicinity of increased mass-energy density. There's no cable holding moon in its orbit. Instead the moon is rolling around the pit in spacetime created by Earth.

That's the essence of local action in GR. The moon feels the gravitational curvature right there, where it's at. And it is time-ordered causal, i.e. earth curves space and then at clock tick 1 the moon feels local curvature. At clock tick 2 curvature changes moon's path. At tick 3 moon feels different curvature at its new location. At clock tick 4 new curvature changes moon's path, etc. (There's more to consider, e.g. earth is moving and its motion changes the local curvature, the moon also curves spacetime and its motion induces a changing curvature . . . feedback loops on loops.)

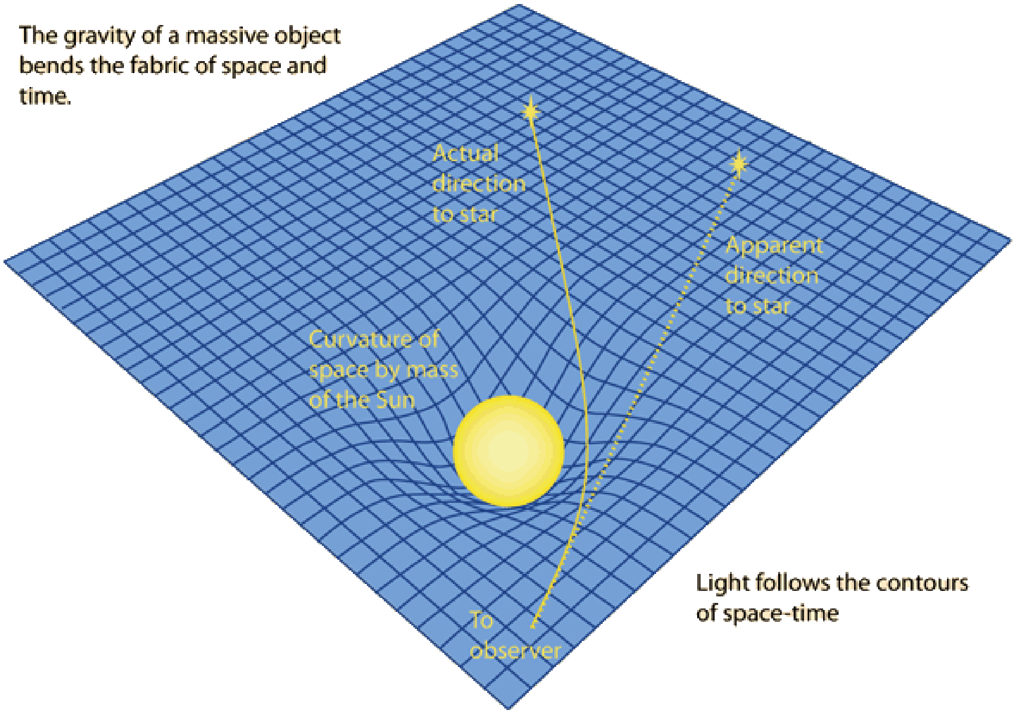


Figure 1. Not just planets, but even light follows the curvature of spacetime. First observed by Arthur Eddington during the solar eclipse of 1919, the apparent positions of stars are shifted around the sun. If earth was in the image, it is rolling around the pit at a tangential velocity with linear momentum that keeps it from falling into the sun. Image credit: HyperPhysics.

This essence of general relativity is captured in Einstein's field equations. The condensed version reads

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

It's a tensor equation. Sixteen equations in all, wrapped up neatly in the 4×4 matrixes $G_{\mu\nu}$ and $T_{\mu\nu}$. Left side of the equation is a geometric measure of spacetime curvature. Right side contains all the factors that contribute to that curvature: mass-energy density, pressure, and momentum. The distribution of mass-energy, pressure, and momentum determine the local curvature of space and the distortion of time.

It’s also worth noting, in passing, that the field equations are non-linear. Gravity produces more gravity. The presence of a gravitational field (spacetime curvature) changes the local mass-energy density, so changes the local field. Gravity chases its own tail. That’s another potential source of conflict between GR and (largely linear) quantum mechanics.

Just like polynomial equations may have multiple solutions (e.g. $x^2 = 1 \rightarrow x = \pm 1$), so the Einstein field equations allow multiple solutions for different mass-energy distributions. Those solutions can be captured in the metric, the equations that calibrate local meter sticks and clocks. In flat space, the Minkowski metric holds:

$$ds^2 = -dt^2 + (dx^2 + dy^2 + dz^2)$$

where ds is the metric, a measure of local curvature, dt is the unit time increment on a local clock, and the factors in parentheses are the unit length increments along each of the three spatial directions. Add mass-energy and the metric changes. We’ll run across another couple or three different metrics for curved spacetimes as we proceed.

In the vicinity of a black hole spacetime is described by the Schwarszchild metric. (This was the first of the solutions to the field equations, discovered by Karl Schwarszchild shortly after Einstein published his 1915 GR paper).

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \frac{1}{\left(1 - \frac{2GM}{r}\right)} dr^2 + r^2\Omega$$

Note that we’ve set the speed of light $c = 1$, so it doesn’t show up in the equation. We’re working in radial coordinates, convenient because non-rotating black holes are radially symmetric. r is radial distance away from the black hole. As they approach the (Schwarszchild) radius of the black hole, $r = 2GM$, the event horizon, clocks slow down and meter sticks stretch. Just as expected in standard GR.

It is obvious from the Schwarszchild metric why GR cannot be a complete description of nature: singularities. The bane of classical physics. There are conditions in which the equations just don’t make sense. If r goes to zero in the metric, as at the center of the black hole, then $\frac{2GM}{r}$ blows up to infinity. Space and time have no meaning. There’s no understanding that, except there be dragons. If we really want to understand how nature works, we think we should be able to figure out what’s going on inside the black hole. But that’s not the only singularity of interest. Our best cosmological model, the inflationary, big bang beginning, also posits a singularity at the origin of the universe. The great hope of a quantum theory of gravity is that it will avoid those singularities, so we can understand cosmic origins.

Two metrics we will encounter later: a metric for de-Sitter space and one for anti-de-Sitter space. These metrics apply at the scale of the universe. de-Sitter is a toy model, spacetime only, empty universe (no stars or galaxies or other matter content) with positive cosmological constant that drives an accelerating expansion. Our universe trends toward de-Sitter over time, as its matter density dilutes with expansion. Anti-de-Sitter, on the other hand, is a universe without matter content but with negative cosmological constant. As we'll see, AdS is the favorite playground for many of the advances in quantum gravity. (See Klauber, 2018, for nice reviews of dS and AdS.)

Quantum mechanics

That's general relativity in brief. What about quantum mechanics? What is it about QM that keeps it from meshing nicely with GR?

We'll work in the Heisenberg matrix formalism. It is handy because it relates nicely to calculations in information theory and computer science. It employs the mathematics of good ol' linear algebra. The alternative Schrodinger wave mechanics will creep into our discussion from time to time. Schrodinger waves are convenient for talking about the quantization of fields.

Matrix mechanics models the world with vectors in a state space. For example, we can represent the spin states of an electron thusly.

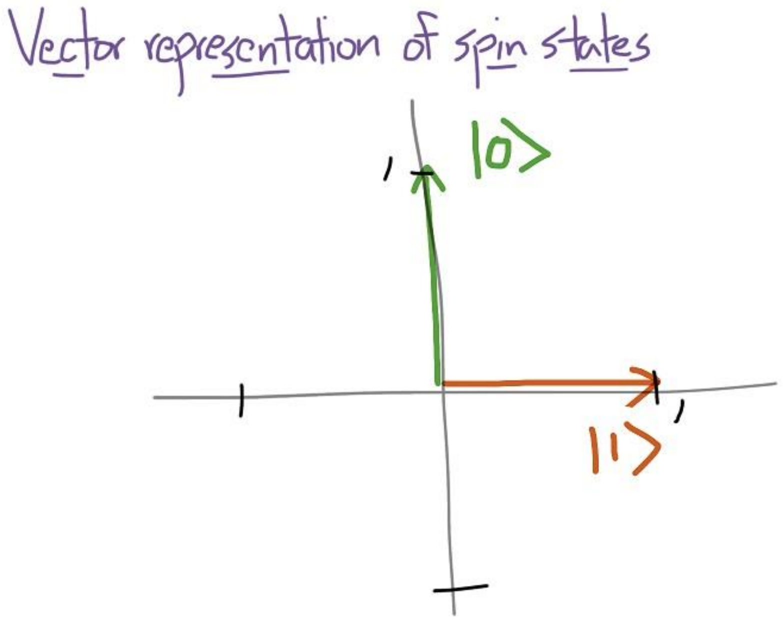


Figure 2. Vector representation of spins up $|0\rangle$ and down $|1\rangle$ on the coordinates representing the vector space of all possible spin states. Note a couple things. First, the coordinate axes are *not* the x, y axes we're used to in regular geometry. The axes here represent the direction of the electron's spin relative to some other, outside Cartesian (x, y, z) axes. For example, the vectors might represent spin direction relative to the spatial z axis. Second, note that the vector labels are arbitrary. By convention, we've chosen $|0\rangle$ as the spin up vector and $|1\rangle$ as spin down. We might have used $|\uparrow\rangle$ and $|\downarrow\rangle$ instead. Finally, note that these vectors are *orthogonal* (here represented by the $90°$ angle), *not* pointing opposite directions as we would expect in the regular world of ups and downs. This orthogonality in the vector representation follows the mathematical rules of linear algebra assuring that when we measure the spin of an electron it is either up or down – even though the real state of the electron, before any measurement, may be a mix of both!

Using Dirac's bra-ket convention, spin up is the ket (vector) $|0\rangle$, and spin down is $|1\rangle$. For calculating, they are written as column vectors.

$$|0\rangle \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and

$$|1\rangle \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Of course an electron in the real world might have its spin oriented in any which direction. We can take care of that easily.

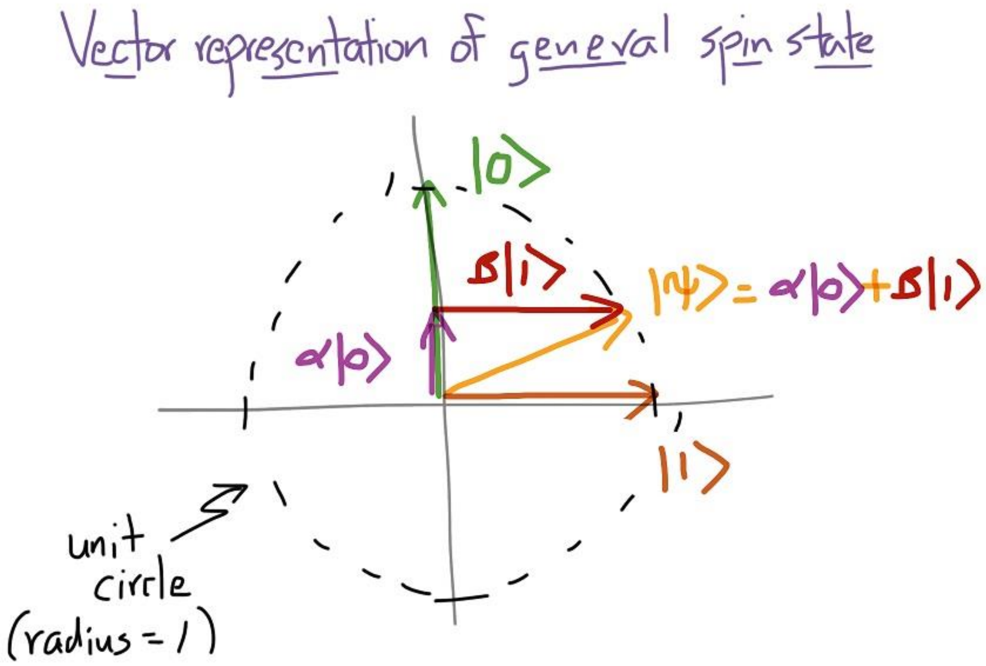


Figure 3. Vector representation of a general state vector, $|\psi\rangle$, showing its component vectors $\alpha|0\rangle$ and $\beta|1\rangle$. In this case, $|\psi\rangle$ is built from a (complex-valued) proportion α of $|0\rangle$ and proportion β of $|1\rangle$. Note that $|\psi\rangle$, like $|0\rangle$ and $|1\rangle$, is one unit in length. This is the requirement of unitarity, assuring that calculations always give probability = 1 when you add all possible vector components for a particular state.

This is our first example of quantum superposition. Any general state, $|\psi\rangle$, can be represented as a combination of basis states, a bit of this basis vector plus a tad of that other one. In our example, psi is built from a complex-valued portion α of $|0\rangle$ and an amount (complex-valued) β of $|1\rangle$. Straightforward linear algebra. It’s vector addition, a “superposition” of quantum states.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

Two dimensions suffice for electron spin, but to describe the universe we need a whole lot more. In order to include all the state parameters – position, momentum, spin, charge, etc. – for all the particles in all the universe, we need a Hilbert space. This “state space” is multidimensional, a dimension for each of the parameters and a coefficient for each parameter for each particle. Our discussion will focus on spin as a primary example of a quantum state. Physicists use the Bloch sphere to represent spin states our 3D physical space.

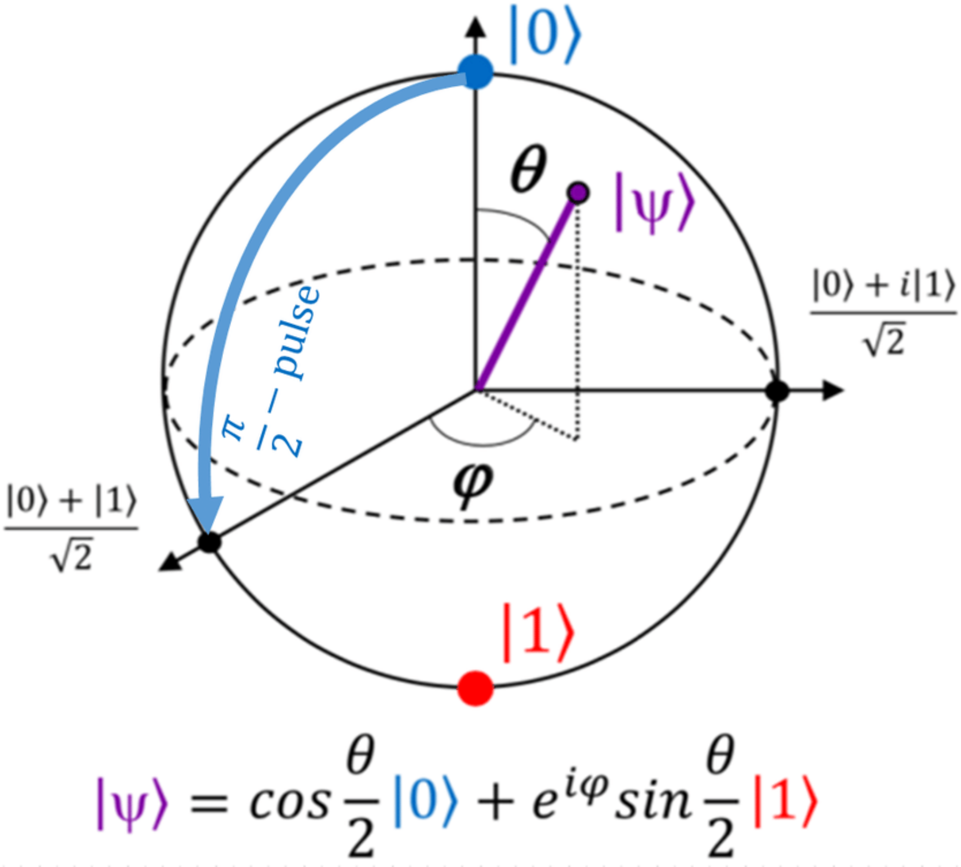


Figure 4. The Bloch sphere representation of spin in 3d space. Note that this representation requires complex numbers, a characteristic of quantum mechanics in general. The $\frac{\pi}{2}$ – pulse represents an operation on the spin, e.g. a photon interacting with an electron to re-orient its spin. We'll see more of those operations shortly when we discuss operators. Image credit: Jazaeri et al. 2019. A review on quantum computing. ArXiv: 190208656v1.

The enormous dimensions of Hilbert space are impossible to draw, but they're easy to represent as mathematical vectors. Columns of coefficients. Somewhere there's a vector in Hilbert space pointing to the state of the universe.

Of course, states change. Electrons move from here to there. Spins flip. How do we represent those changes? Well, matrix algebra readily accommodates vector transformations. Suppose a photon, for example, flips the spin of an electron from spin up to spin down. Here's what that looks like in matrix notation.

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ in this instance represents an electromagnetic field acting on an electron to flip its spin. We'll encounter a whole bunch of matrix "operators" shortly, each of which performs a particular vector rotation on the Bloch sphere. Nature's operators, the natural forces, operate on vectors in Hilbert space. Similar mathematical operators, maybe the same operators, act on qubits in quantum computers to process information.

All that could maybe fit into a classical theory. Discrete bits. Superposition of state vectors, or Schrodinger's waves, like waves on the ocean. But then quantum mechanics gets really weird. (It's not QM that's weird, it's just that our senses don't experience its weird effects directly. QM after all is the way the world works.)

It's mostly entanglement that generates the quantum weirdness. Here's the gist. Put two electrons in the same atomic orbital, as in helium. They interact such that one is spin up, the other spin down. Spins in the same energy state have to be opposite; that's Pauli's exclusion principle. We know that for certain. But what we don't know is which electron has which spin. Until we measure an electron, the probability is 0.5 spin up, 0.5 spin down. But when we <u>do</u> measure an electron, then we know for sure that the other electron has the opposite spin. We have certain knowledge about the state of the two-electron system – one is spin up and the other is spin down – but until we measure we don't know the spin of the individual electrons. After a measurement, we know for certain the spin of the other electron even though we haven't yet measured it. That's entanglement.

Here are the maths of an entangled state.

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

The kets with two numbers represent the possible states of the whole system, including both of the electrons in He. Green represents one of the electrons, red the other. If the green electron is up (0), then red is down (1). If green is down, then red is up. $|\psi\rangle$, the state of the He system before we measure, is a mix of both possibilities. (The minus sign is a convention to specify this particular Helium configuration, and the $\frac{1}{\sqrt{2}}$ assures that total probability comes out one.)

The spookiness occurs when we separate the electrons. If we're really careful not to disturb them, Alice can take the red electron on a starship ride to Alpha Centauri. Bob keeps the green electron carefully here at home on earth. One day he gets curious. He measures his electron and finds its spin is up. He knows immediately that Alice's red electron has spin down. She doesn't even know that yet, herself. She hasn't measured her electron. But when she does, she'll find its spin is down. Strange. Very strange.

Even spookier is the quantum eraser. This marvelous apparatus and the many experiments based on it have provided key insights into quantum reality. In particular, for our purposes, delayed choice experiments based on the quantum eraser require us to abandon our usual notions of classical causality. Quantum mechanics overthrows the classical paradigm required in general relativity. In the quantum eraser events at a later time appear to change measurements made earlier. As we'll see, these results are best explained in other terms and not actually the future affecting events in the past. Still, the experiments require us to replace notions of local causality with non-local, distributed entanglement.

The quantum eraser starts with an entangled pair of photons. Mirrors and beam splitters send the photons along separate paths. The optical system is configured so that one of the photons, far along on its path, can be measured to determine which path it took. That measurement can be performed at a time later than when the photon should have otherwise interfered with its partner and arrived at a detector. If no measurement is performed we see an interference pattern at the detector. But if the test photon is measured, even at a time later than when the photons arrived at the detector, the interference pattern disappears. See (Lincoln, 2021) for a nice summary of the experiment, and see (Ma et al, 2013) for a thorough description of the methodology and the implications.

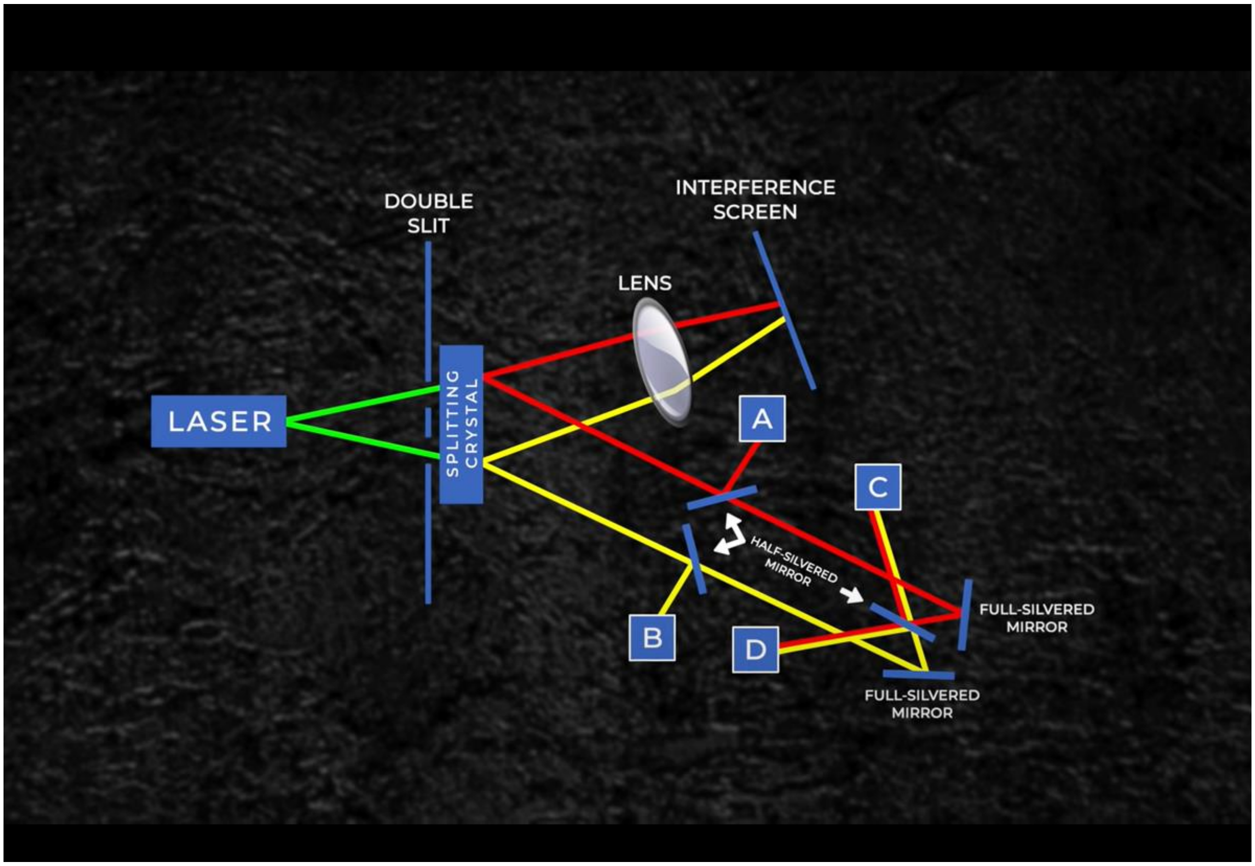


Figure 5. The quantum eraser / delayed choice apparatus. Entangled photons pass through a double slit. The splitting crystal sends each photon along two paths, upward or down. Along the bottom path, if you choose to measure the photons at detectors C and D you can't tell which slit the photon passed through and you'll see and interference pattern on the interference screen. On the other hand, if you choose to measure the photons at detectors A and B then you know which slit the photons came through. The interference pattern disappears. The detectors can be located far enough away from the splitting crystal and measurements made with switches fast enough, to measure AB vs. CD, such that the measurements can occur even after the photons should have arrived at the interference screen. Image credit: Fermilab: *The super bizarre quantum eraser experiment*.

To understand these results we have to consider the state of the entire apparatus, all the photons and all the paths, as one state vector. The no-measurement state includes the whole system $|\mathrm{no} - \mathrm{measurement} - \mathrm{therefore} - \mathrm{interference}\rangle$. The $|\ \mathrm{yes} - \mathrm{a} - \mathrm{measurement} - \mathrm{is} - \mathrm{performed}\ \rangle$ is the state including yes-a-measurement-and-no-interference, the complete package. Forget later or earlier. Consider the whole package of the experiment and its results, the state vector of entangled particles entangled with the apparatus.

Quantum information

There we have it, the tension between GR and QM. GR is local and causal. QM is non-local, entangled. The critical difference, though, is this. Quantum mechanics absolutely conserves information (a.k.a. "unitarity"). Among the great conservation laws – conservation of energy, of momentum, of angular momentum – information may be at the heart of them all. Time-ordered causality itself is informational. If we don't possess the information exactly what preceded what, then we have no way to determine this caused that.

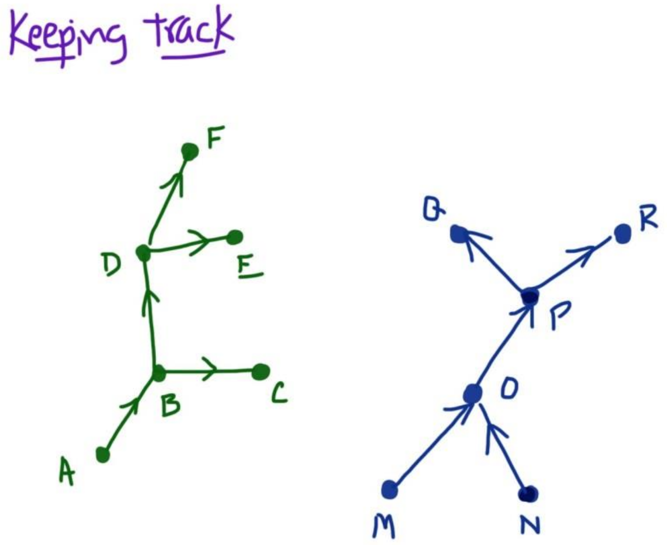


Figure 6 : Directed graphs representing the flow of information. Tree graph on the left conserves information. You always know from whence you came. If you're at E you must have come from D , and similarly for the other nodes. Information is lost, though, in the graph on the right. If you see an ant at O , you can't say whether it got there from M or from N.

QM keeps careful track, but, at least on first glance, GR blithely destroys information. Drop all of Shakespeare's works, all the plays and sonnets, all copies in all translations and all formats, into a black hole, and Shakespeare is lost forever. At least according to the classical theory. It's information we need to reconcile between GR and QM.

Other realms of classical physics accommodated to QM. Physicists have figured out recipes to "quantize" electromagnetism and the other forces of nature. That's the Standard Model of particle physics. But efforts to quantize gravity with similar methods have failed. It seems that an entirely new approach is required.

String theory has proven quite promising in this mission. We'll encounter its considerable successes. Other programs explore methods in complexity theory, e.g. emergent gravity (for example Verlinde, 2016), and serious efforts are in progress to dig into the foundations and find gravity in quantum mechanics (e.g. Carroll, 2020). The most promising path to reconcile GR and QM, though, appears to be information theory and quantum computation.

Two essential ideas underly quantum information theory. Rolf Landauer (Landauer, 1991) proved that all information is physical. That is, ones and zeros and qubits all have a hands-on, physical, observable realization in the world. In your computer, the one is the presence of an electron in a particular location in the memory register. Zero is the absence of an electron in that register. Spin up, spin down, or some of both, are registered physically right there by the electron's state.

The next great idea, from Alan Turing (Turing, 1936) and Alonzo Church, is that "universal" computers are all equivalent. Your new laptop is equivalent to Turing's original computational machine, an infinite paper tape, a print head that could move back and forth one step at a time along the tape, and instructions telling the print head what to do at each step depending on what it found at the previous step. The processing in your laptop is certainly different from Turing's machine. But the two are equivalent in the sense that any computation that can be performed on your laptop can be performed on Turing's. Turing's write-and-erase-and-move-along-the-paper-tape device will just take a whole lot longer.

Your Apple device and a PC both are universal Turing machines, so if you can perform an operation on one you can, with the appropriate software instructions, perform that same operation on the other. Church and Turing extended the idea. Nature itself is a computer. Natural processes, like photosynthesis and DNA replication and everything else in the natural world, operate on physical bits of information (electron configuration in chemical bonds or nucleotide sequences, etc.) using a set of instructions, the laws of nature. If that's the case, you should be able to model Nature's Turing machine on your own. How can you do that?

A universal computer requires a universal set of processors, i.e. a set of matrixes which, in combination, can perform any logical transformation and hence, by the Church-Turing Thesis, model any physical transformation. It turns out that there are many such sets, some of them quite simple. In computer science, those matrixes are called "gates." Table 1, in the Appendix, lists the common quantum gates. Among them, a standard universal set comprises the Hadamard, phase, CNOT, and $^{\pi}/_{8}$ gates. Imagine. A handful of gates, some qubits, and you can build a universe!

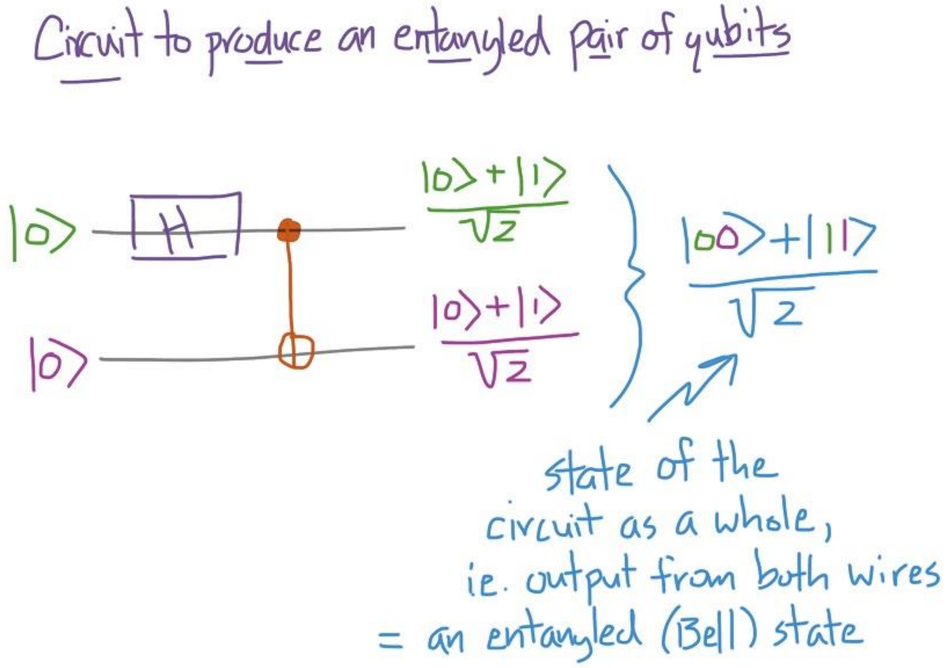


Figure 7. A simple quantum circuit to prepare an entangled pair of qubits. Input qubits in this example are both $|0\rangle$. Think of the lines as similar to wires in an electric circuit; time flows left to right as operators act on the states along those wires. A Hadamard gate produces a mixed state in the top qubit, and a CNOT transforms the lower qubit based on that mixed state. (See the Appendix for more information about the gates.) Note that CNOT acts on the bottom $|0\rangle$ twice, first with the $\frac{|0\rangle}{\sqrt{2}}$ as control and then with $\frac{|1\rangle}{\sqrt{2}}$ to produce the mixed state in the bottom wire. The output superposition of both wires is an entangled state referred to as B_{00}, the Bell state produced when both inputs are $|0\rangle$. See if you can figure out the other Bell states, B_{01}, B_{10}, and B_{11}.

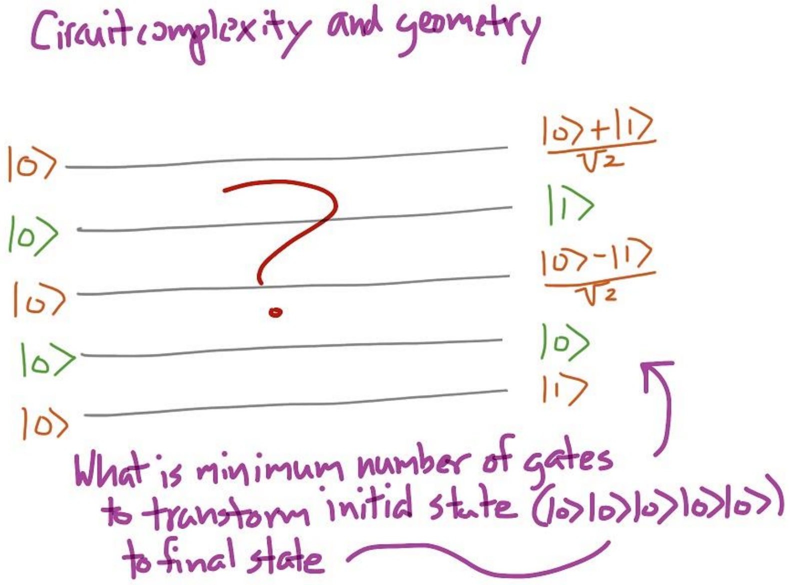
The implications of Landauer's observation and the Church-Turing thesis are mind-boggling, and they go to the very core of our work here. This is why we think we can understand gravity, hence spacetime structure and the origin and evolution of the universe, in terms of information. The universe processes qubits. It is a quantum computer. We can understand it if we can figure out its states and the operators that process those states.

Quantum computers and complexity

Computational complexity has burgeoned into a field of study in its own right, and it has provided helpful insights for a theory of quantum gravity. The study of computational complexity originated in the attempt to figure out how much memory and how much time it would take a computer to solve a particular problem. If you want to plan the trajectory of a mission to the moon, for example, you first have to build a computer with memory capacity to store all the necessary parameters – position, velocity, gravitational field, fuel, vehicle mass, etc. – and a processor fast enough to perform the calculations before time to launch the mission. How difficult is the calculation, and what computational resources do you need?

It turns out there's some deep mathematics in complexity theory. At its heart it's trying to determine the limits of knowledge. How much can we know about the world? What can we calculate about how Nature works, and what is beyond our capacity? The question whether P (polynomial time complexity) equals NP (non-deterministic polynomial time complexity) ranks among the most important open problems in mathematics. There's a million dollar prize for whomever figures out whether or not $P = NP$. (See Aaronson, 2013, for a general discussion of complexity theory, and see Roberts, 2021, for $P\ ?= NP$.)

For our purposes, the key contribution from complexity theory is the notion of circuit complexity. By its operational definition, circuit complexity is the minimum number of gates it takes to transform a given input state into a final output state. As we'll see, circuit complexity has a geometric interpretation that can be used to model spacetime curvature. We can derive Einstein's field equations from quantum circuit complexity (Nielsen et al, 2006). Patience, though. We have some other puzzle pieces to assemble before we get there.



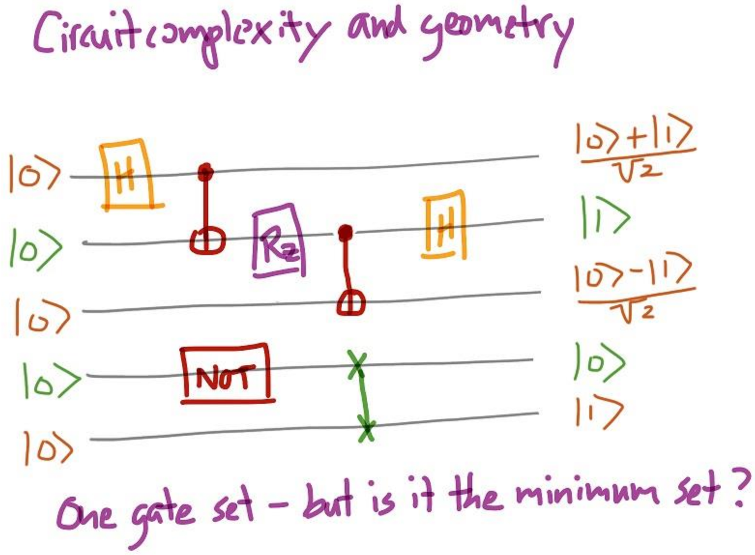


Figure 8. Complexity of a circuit can be measured as the minimum number of gates required to reach a target state from a given initial state, here $|00000\rangle$. That gate configuration has an associated geometry and curvature (Nielsen et al, 2006).

Information and thermodynamics

Thermodynamics provides a more substantial link between information and physics. In brief, information is entropy.

Classically, entropy counts how many microstates there are that can produce same observable macrostate. Consider a box divided by a partition into two chambers. Punch a hole in the partition that allows molecules to pass between the chambers. Add three molecules to the box. They are identical in all properties, but imagine we color them red, green, and blue just so we can talk about their distribution. What's the entropy of the system (box) if all three molecules are in the left chamber? Well there's only one configuration of the system having all three molecules on the left. That's low entropy, certain knowledge – we know where all the molecules are. On the other hand, what's the entropy of the system if one molecule is in the left chamber, two on the right? There are three different configurations that would give that state. Any one of the three molecules, red, green, or blue may be the one on the left, with the remaining two on the right. That's a high entropy state. We have less knowledge where the marbles are located. Until we look, we don't know which color is where.

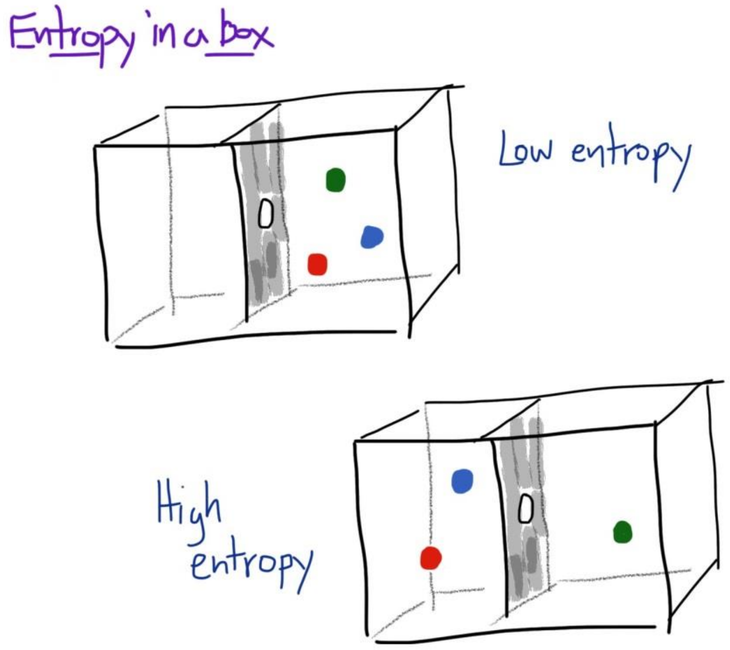


Figure 9. Three marbles in a container with two chambers. Marbles can pass through the hole in the divider. We color the marbles so we can keep track of all the different possible configurations, but imagine the marbles are indistinguishable particles. Top box is a low entropy system. We know all the marbles are in the right half of the box. So we know where's red, where's blue, where's green. Configuration in the bottom box, on the other hand, has more uncertainty. There's one particle on the right, but which color is it? There are three different possibilities for the same distribution of marbles, two left, one right. That's a higher entropy state. Greater uncertainty. More different ways to distribute indistinguishable particles into the same overall configuration.

Szilard's engine offers a nice model relating thermodynamics to information, and it teaches a surprising lesson about computer function (Maloney, 2009). The engine is an evacuated chamber with two pistons, one at each end. There's a slot in the middle for a sliding divider. All moving parts are friction-less. The chamber sits in a thermal bath that maintains constant temperature, and the chamber can exchange thermal energy (heat) with the bath. Maxwell's Demon (MD) can observe the system and report on its state.

Consider the following sequence of events. Pull both pistons to their extreme positions at the ends of the chamber. Add a single molecule to the interior. The molecule flies around in the chamber. MD is watching. When it observes the molecule in the left half, it drops the divider and reports "L." We've gained a bit of information about the system. That didn't cost any thermodynamic energy. No (classical) work was done.

Now move the right piston to the center and remove the divider. Because the molecule has kinetic energy it will push the piston back out to the right end. If we attached the piston to a machine, we could do work – lift a weight perhaps, or turn a crankshaft.

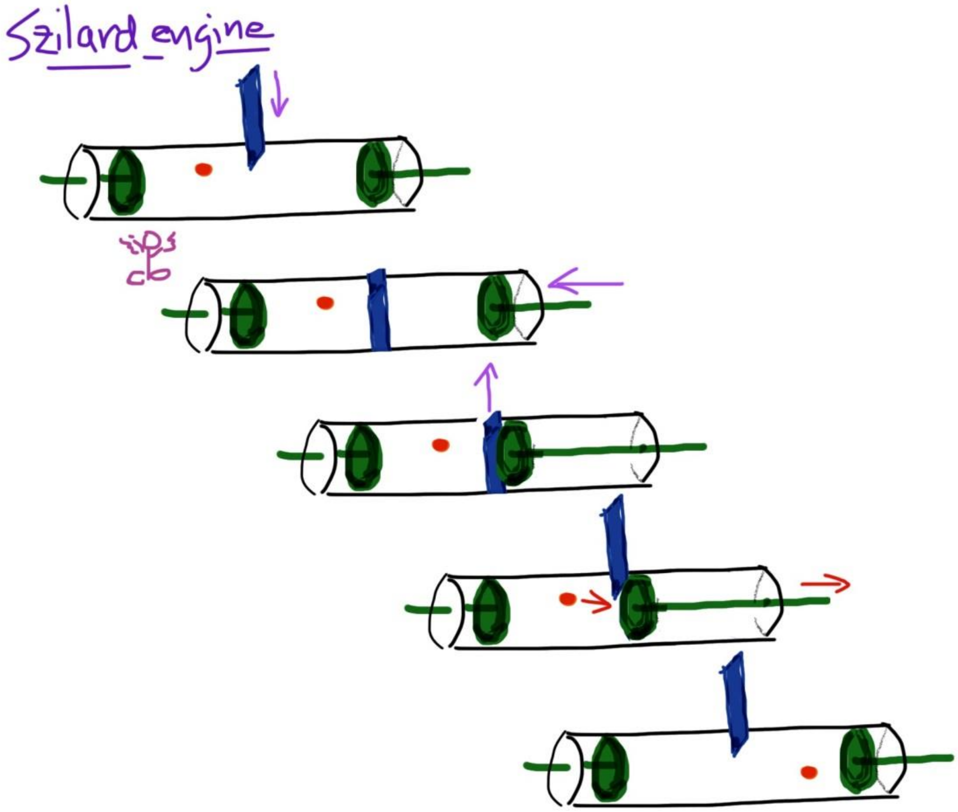


Figure 10. Szilard engine as modified by Rolf Landauer. Time runs from top to bottom, through one cycle of the engine. Steps are described in the text.

Two lessons here. First, information processing is thermodynamic. Measuring a bit of information is equivalent to adding energy to a system. Second, erasing that information transfers energy to the environment. That's the practical take-home here. That's why your computer processor heats up. It's not the number crunching that generates heat. It's erasing all those memory registers along the way that heats the computer chips.

Claude Shannon and John von Neumann made these thermodynamic connections rigorous. They proved that information is entropy. As an illustration, suppose meteorologist Alice sends pilot Bob a bit-wise weather report from the landing field. One means clear skies. Zero means cloudy. Until Bob receives the report he is uncertain. He has an information deficit. Clear? or cloudy?

Now suppose that Alice has been tracking the weather over the years. She has found that on this calendar day the probability for clear skies is $^3/_4$ and the probability for cloudy is $^1/_4$. Bob has access to those weather records. He hasn't yet received Alice's real-time report, so he looks up the probabilities. He gains some information from the probabilities. He is less uncertain. It's more likely to be clear than cloudy.

Shannon put rigor in those calculations. The uncertainty in information he called "entropy." It measures much information is missing, the likelihood that any particular one of a number of possibilities may happen. Entropy. The name was suggested to him because his formula for missing information turned out to be the same as for thermodynamic entropy. Working in bits,

$$S = -\sum_i P_i \log_2 P_i$$

S is entropy. P_i is the probability for the i-th event, e.g. clear or cloudy. The minus sign is a convention, so entropies come out positive.

Back to our weather report. Suppose the probabilities are equal, 50-50, clear vs. cloudy. Then

$$S = -\sum_i \frac{1}{2}\log_2\frac{1}{2} = -\left(\frac{1}{2}\times -1\right) - \left(\frac{1}{2}\times -1\right) = 1$$

Entropy equals one. That's maximum entropy in the digital world. You're completely uncertain about the outcome. Could be clear. Could be cloudy. A coin flip.

On the other hand, Bob checks the historical records. Now

$$S = -\left(\frac{3}{4}\log_2\frac{3}{4}\right) - \left(\frac{1}{4}\log_2\frac{1}{4}\right) \cong 0.811$$

Lower entropy, less uncertainty. Clear skies more likely than clouds. (For a consistency check, calculate the entropy if skies are always clear. Hint: then entropy $= 0$.)

Shannon and von Neumann extended the argument to entire messages, strings of bits. A string of three bits can accommodate eight possible different messages. 2^3 possible messages. One or the other, 0 or 1, in each of three slots in the message. 000, 001, 010, 011 . . . 111. If you know the probability for each of those eight possible messages, you can calculate the entropy of a three bit message. If Alice always sends 000 , Bob is certain what he'll receive. (Check out Shannon's formula.) If Alice sends digits at random, Bob has no clue. (Check that out, too.)

00000000

00000001

00000010

⋮

01001011

01001100

⋮

Figure 11. Some of the $2^8 = 256$ possible messages in an 8-bit register. We can calculate the entropy of the system if we know the probability for each of the messages.

The information content of relatively small bit strings boggles the mind. A string of 300 or so bits can hold all the information in all the universe. 2^{300} possible states. All the electrons and protons, galaxies and dark matter, their positions and momenta. Everything. Just the variations on that bit string. Imagine. And we haven't even got to the (enhanced) capacity of qubits yet.

Information and thermodynamics have surprising links. As we shall see, those links have proven extraordinarily productive.

Part the Second: Explorations

In which we review key ideas in the (as yet incomplete) structure of a quantum theory of gravity. Black holes provide the favorite models for generating and testing ideas. We'll spend extra time describing their information-theoretic and thermodynamic properties. Holography with all its ramifications, especially AdS/CFT, has proved enormously fruitful. Entanglement seems to weave the fabric of spacetime, and complexity drives its dynamics. Finally we'll take a look into the materials science and quantum computer labs that are testing the ideas. Most sections will introduce key concepts then provide links to more complete discussions by the gurus themselves. We're lucky to live in such an exciting time and a time where we can easily access the minds that are creating all the excitement.

Black holes as physics laboratories

Just as hydrogen atoms provided the test ground for ideas in quantum mechanics, black holes offer the test cases for figuring out quantum gravity. It's in black holes that general relativity most obviously meets quantum mechanics. Understanding black holes requires both.

Black holes first appeared in solutions to Einstein's field equations. The maths said that under conditions of extreme mass-energy density matter would collapse to a singularity. By the calculations in general relativity (GR) those black holes are very simple. A black hole could be completely characterized by just three parameters: its mass, its angular momentum, and its electric charge. All the stuff that collapsed into the black hole – entire stars with their forge-full of atomic elements, maybe entire civilizations with eons of accumulated knowledge – all reduced to just three numbers.

Then along came Jacob Bekenstein and Stephen Hawking with their outrageous notions that black holes have thermodynamic properties. They have entropy, a measure of information. And they radiate. After some (very, very long) time, they radiate away their mass. They evaporate.

Here's Bekenstein's argument simplified (Susskind, 2013). See his 1973 paper (Bekenstein, 1973) for the original. It starts with classical GR and thermodynamics plus a dash of well established quantum mechanics. It provides the first hint that the two great theories must be intimately related.

We're given a black hole with radius R . Idea is to see what happens to the structure of the black hole when we drop in one more bit of information. Where does that bit go? How does it change the black hole?

First take a look at that R . We're going to need its relation to mass-energy. That comes out of the metric for the black hole. In fact it comes straight out of the equation for the escape velocity off the event horizon of a black hole. By definition that escape velocity is the speed of light.

$$v_{esc} = c = \sqrt{\frac{2GM}{R}} \rightarrow c^2 = \frac{2GM}{R}$$

so

$$R = \frac{2GM}{c^2}$$

Now, we have to be careful about our bits. If we drop any old one or zero into the black hole we're actually including more than a bit of information. Where the bit enters the black hole is itself information, its location on the horizon. So we choose as our bit a photon with wavelength equal to the black hole radius. Then its location is fuzzed out over the horizon. Just a photon, one bit, and no further information.

We want to figure out how that additional photon changes the mass-energy of the black hole. Well, what's the photon's energy? Einstein figured that out from the photoelectric effect. Energy of the photon is quantized.

$$E = hf \rightarrow E = \frac{hc}{\lambda}$$

Photon energy is inversely proportional to its wavelength, and we know the wavelength. So

$$E = \frac{hc}{R}$$

That's the quantum of energy the photon adds to the black hole. Best call it the increment of energy:

$$dE = \frac{hc}{R}$$

Good. Now we can find how much the black hole mass changes with the addition of that photon.

$$dE = dMc^2 = \frac{hc}{R} \rightarrow dM = \frac{h}{Rc}$$

Just about there. We're curious to find by how that photon changes the geometry of the black hole. Substitute back into the relation between mass and radius.

$$R = \frac{2GM}{c^2} \rightarrow dM = \frac{dRc^2}{2G} = \frac{h}{Rc}$$

where the last equation is the one we just derived. After rearranging, we get

$$RdR = dA = \frac{2Gh}{c^3}$$

That is very cool. And unexpected. And the doorway to pretty much everything else that follows – holography, AdS/CFT, and all. What that equation tells us is that when you drop a bit of information into a black hole it changes the area of the event horizon of the black hole by a fixed amount. Each tiny patch of area represents one bit of information. $RdR = dA$, the change in area, and the right side of the equation is constant. A very small constant value, to be sure. You haven't changed the area by very much. But on the other hand, you can pack a whole lot of information onto the event horizon, tiling all those little bits!

Shortly after Bekenstein's discovery of the area / entropy relation, Stephen Hawking had his great aha! moment. Where there's entropy there's thermodynamics. And where there's thermodynamics there's temperature.

$$dE = TdS$$

Black holes have entropy, so they must have an associated temperature. Hawking found that temperature. It's one of the most beautiful equations in physics.

$$T_{BH} = \frac{\hbar c^3}{8\pi G k_B M}$$

Take a moment to appreciate that equation. All of physics is in there. Gravity there in Newton's constant. Quantum mechanics (QM) in Planck's constant. Thermodynamics in Boltzmann's constant. Gravity, quantum mechanics, and information all in one nice, neat package. There must be something deep to these notions.

It is important to point out that black holes aren't just mathematical constructs, solutions to the field equations of general relativity. Astronomers have accumulated lots of evidence for their existence. Stars in orbit around invisible masses and generate fireworks at the centers of galaxies that could only be powered by black holes. The LIGO and VIRGO gravitational wave detectors have recorded gravitational waves generated by in-spiraling, merging black holes. And in a marvelous technological coup, with radio telescopes effectively the size of Earth, we've seen the black hole at the center of the Milky Way and a black hole in the giant M87, a galaxy long ago and far away (EHT, 2021). We've seen the monster in its lair.

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Holography

The previous section on black holes introduced the key notion of holography. Just as a 2D sheet of film stores all the information needed to create a 3D image of Princess Leia when you shine a laser on the film, the horizon, a 2D (two spatial dimensions) surface, stores all the information that fell into the (presumably) 3D black hole. Gerard ‘t Hooft extended this idea to show that any quantum theory of gravity requires such a dimensional reduction (‘t Hooft, 1993).

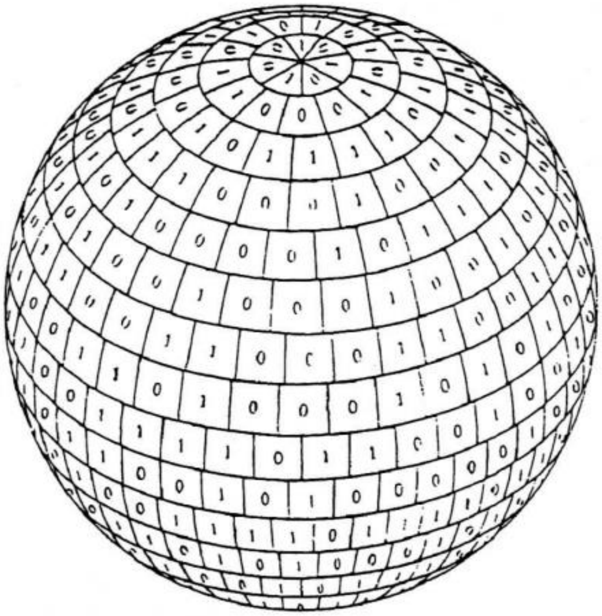

Figure 12 : John Wheeler’s original “it from bit” proposal. All information collected in the volume inside the sphere (3-dimensional here) is recorded on the sphere itself, which is 2-D. The volume might be a black hole, the surface its horizon. Or the volume might be the universe, its surface the cosmological horizon. Image from Wheeler, 1989.

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Advances in holography: AdS/CFT

Arguably the greatest advance in our efforts to understand quantum gravity is Juan Maldacena's discovery of the correspondence between a string theory with gravity in Anti-de-Sitter space and a conformal field theory in a lower dimensionality. As of 2018, Maldacena's 1997 paper introducing the formalism is the most cited paper of all time on the high energy physics ArXiv (Maldacena, 1997). It has inspired a wide range of discoveries in many fields, including quantum gravity, black hole physics, information theory, and solid state physics.

Maldacena himself prefers to call this the "quantum field theory / quantum gravity duality." This theory provides an interchangeable set of tools. It is not, per se, an answer to what is quantum gravity, but it gives us a wonderful mathematical construct to figure it out. On the one hand, on the "boundary" of the theory in D-1 dimensions, we have a quantum field theory. Good ol' quantum mechanics. We use those tools all the time. On the other hand, in the "bulk" of the theory in D spatial dimensions, is a string theory including gravity. We understand that string theory also. Its equations are familiar. What's marvelous is that you can use either mathematical formalism to solve the same problems. If it's easier to calculate in the quantum field theory of D-1 dimensions, then use those tools. If it's easier to calculate in the gravity theory of D dimensions, then use that. In this way, some gravity problems that are intractable in D dimensions might be easily solved in a quantum field theory in D-1 dimensions, or vice versa. This capacity to work with either tool set has proven enormously productive over a broad range of studies, including nuclear physics, condensed matter physics, information theory, black hole physics, and – of course – quantum gravity.

Maldacena’s original work, which has since been extended to other dimensions, studied a 5-dimensional anti-de-Sitter spacetime bulk with gravity inside a 4-dimensional boundary conformal field theory. Hence the AdS/CFT designation. From here on we’ll refer to the more general quantum-gravity-quantum-field-theory-duality also as AdS/CFT. Note that the boundary in AdS/CFT sits ‘way out there at infinity. Anti-de-Sitter space has no edge. So our “boundary” is a mathematical construct, not a physical barrier.

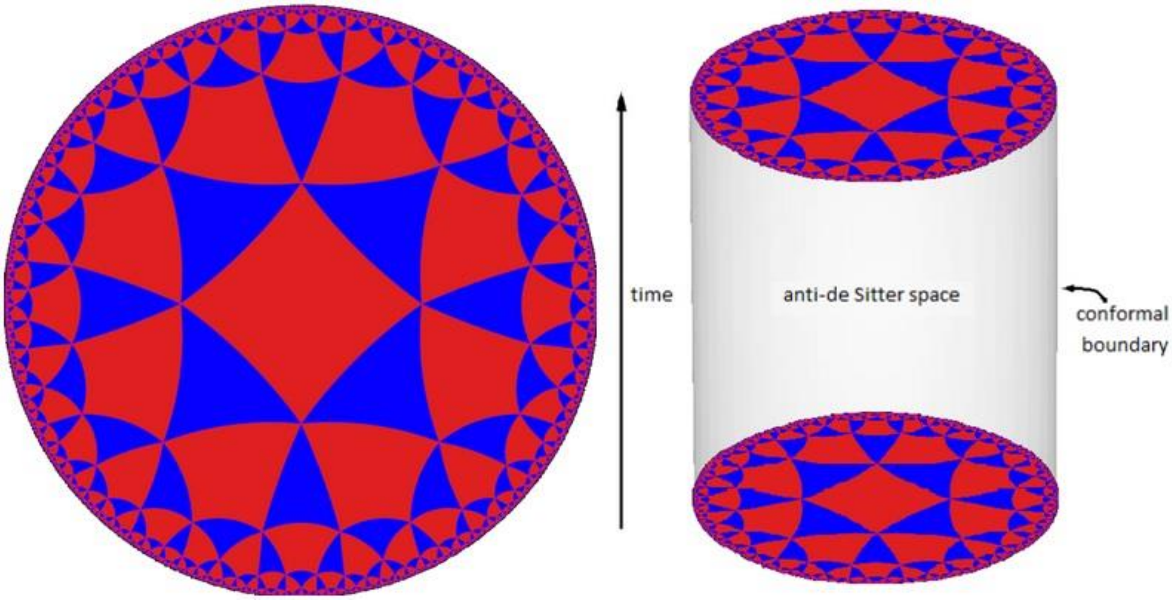

Figure 13. Anti-de-Sitter space with conformal field theory boundary. In left-hand image, AdS is represented by a hyperbolic disk, referred to as the bulk. Each triangle has the same area; imagine the edge curving off to infinitely far away, so distant triangles look smaller. The boundary is the edge of the disk. In this representation, the mathematics of quantum field theory on the 1-dimensional boundary circle derive the same results as 2-D spacetime math (e.g. general relativity) including gravity in the bulk. Right-hand image shows AdS evolving through time. Image from Wiki media.

Note the extension of the holographic principle in Maldacena’s discovery. In its original context, holography referred to the distribution of information. Information in a 3D spatial volume such as fell into a black hole is distributed on the 2D surface, the black hole horizon. In the AdS/CFT model we’re talking about more than information. Now we’ve included information processing. We’re considering two different mathematical systems. One applies in D dimensions. The other calculates in D-1 dimensions. Holography applies in the fact that calculations in the D-1 boundary maths produce the same results as calculations in the D-dimensional bulk maths. Operations on the boundary reproduce other operations in the bulk.

We should be all finished then. Right? We found a quantum theory of gravity right there in the quantum gravity/QFT duality. If a quantum field theory (QFT) is dual to a theory of gravity including general relativity, voila! There's our quantum theory of gravity. Problem is the AdS. The D-dimension string quantum gravity is a theory in anti-de-Sitter space. That's a hyperbolic space, a space with negative cosmological constant. That's not the actual (de-Sitter) space of our universe. Our universe, by the best of current measurements, is essentially flat (no curvature) and with a very very small positive cosmological constant.

The hunt is on for a QFT/quantum gravity duality that works in de-Sitter space. It may be that the original AdS/CFT is big enough to serve as a model for our essentially flat spacetime. Anti-de-Sitter is enormous, after all, and maybe asymptotically flat (teeny tiny essentially ignorable negative curvature) allows us to calculate for our real universe. But that's not very satisfying. As of 2023 a compelling holographic model of quantum gravity in de Sitter space still eludes us. Still, AdS/CFT has generated enormous progress.

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Discoveries in AdS/CFT: Ryu-Takayanagi and the bulk minimal surface

Among the first of the new insights glimpsed through the AdS/CFT window was the connection between bulk geometry and entanglement entropy on the boundary (Ryu and Takayanagi, 2006). RT showed that the entropy in a connected region on the CFT boundary is dual to the area of a minimal geodesic through the bulk connecting the endpoints of the geodesic on the boundary. "Geodesic." "Minimal." Sure sounds like something out of general relativity. Further work (to be discussed below) showed that's in fact the case.

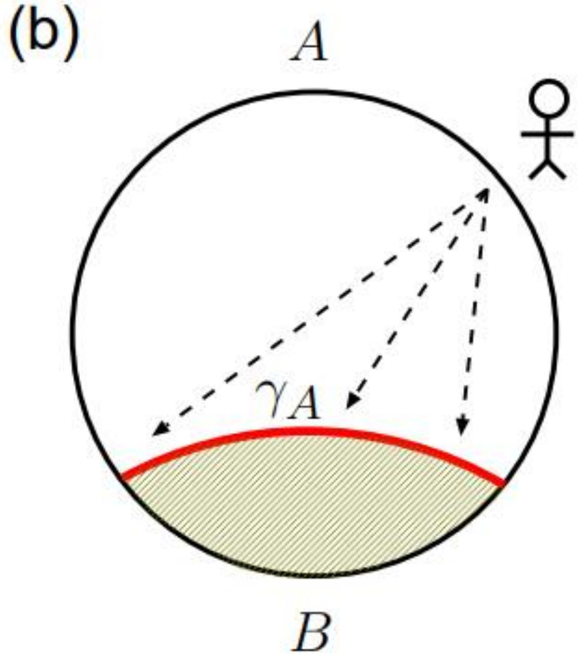

Figure 14: Boundary (circle) and bulk (disk) in AdS/ CFT. The entanglement in region B on the boundary is proportional to the area of the minimal surface γA connecting the endpoints of B through the bulk. In fact, as calculated by an "observer" in the boundary region A , γA looks like the event horizon of a black hole with the shaded region as the black hole interior. The value of the entropy of region B calculated by RT turns out the same as the Bekenstein entropy, $S_B = \frac{\gamma A}{4G}$. Image from Ryu-Takayanagi, 2006.

What's being measured here? What is entanglement entropy? What is entangled with what?

The idea is that information on the boundary is entangled with information in the bulk. If we think of AdS/CFT as a universe with a boundary at infinity, we can use terms same as with the black hole. An observer "outside" the boundary, if she measures the information on a patch of the boundary, is also measuring the information in the bulk region contained between the boundary and the bulk geodesic. In higher dimensions, that boundary information represents information in the "causal wedge" between the boundary and the geodesic (which is now a surface of minimal area).

More properly, in the language of AdS/CFT, QFT (quantum field theory) operators in the connected region on the boundary are entangled with bulk operators in the cosmic wedge. There are two mathematical systems (sets of operators) at work, one in D dimensions in the bulk and the other in D-1 dimensions on the boundary, that mimic each other in their (operator) calculations / measurements / observations.

Ryu and Takayanagi derived their theory in AdS/CFT. That area formula has since been generalized. It turns out it applies to any volume and its boundary. Select a galaxy cluster, the great Virgo cluster say. Encompass it with a suitable boundary. That boundary area encodes all the information in the Virgo cluster. How? By all the threads of entanglement connecting inside with the holographic boundary.

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Discoveries in AdS/CFT: Tensor networks

Among its wonders, mathematics avails the skilled user a whole bunch of toolkits that might solve any particular problem. For example, you can find the approximate value of pi by comparing the diameter of a circle to the summed lengths of the edges of higher and higher order (more edges) polygons fit into that circle. Or you can find pi with the tools of calculus. Or you can approximate pi with Monte Carlo algorithms on a computer. And so on. A lot of the work in problem solving involves trying to find the right toolkit, especially a toolkit that makes the problem simple to solve.

In 2009 Brian Swingle (and others) proposed a new mathematical toolkit to describe the bulk-boundary relation. Tensor networks.

Think computer circuits. Wires connect processors. A single processor may have several inputs and several outputs. Build a tree graph, leaves on the boundary, trunk in the center. That provides an architecture for AdS/CFT.

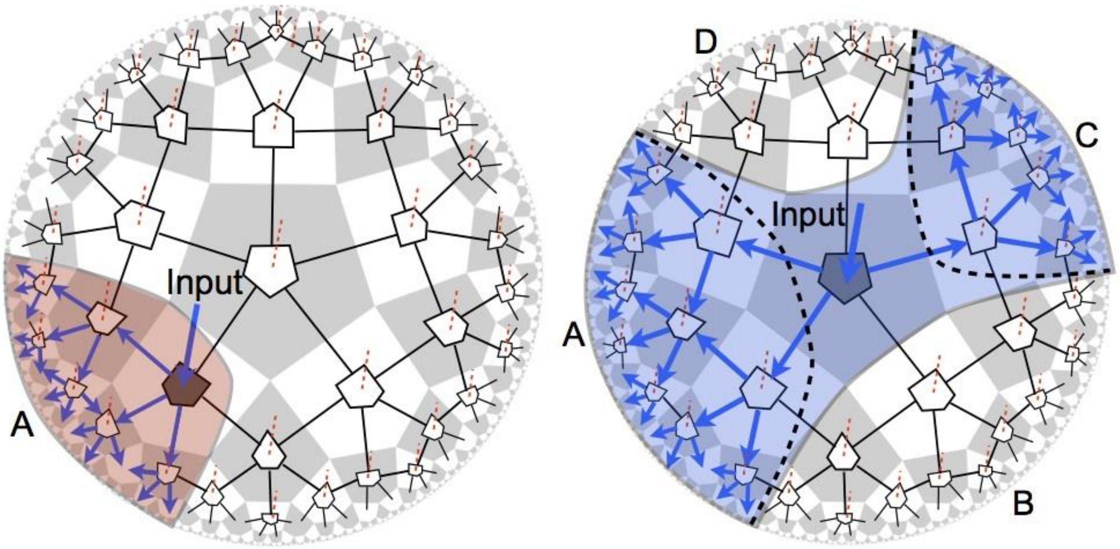

Figure 15. Tensor network. Pentagons represent tensors (think quantum circuit gates) each processing inputs coming from center of the bulk and distributing three output qubits to the periphery. Shaded regions show entanglement resulting from this particular tensor network. Other networks can have different tensor composition. Note that in order to access the core of the bulk you have to have access to information / operators on more than half of the boundary. Image credit Beni Yoshida.

Tensor networks provide a physical picture how the boundary is entangled with the bulk and how information processing is distributed. This puts a new perspective on the Ryu-Takayanagi (RT) model, too. Minimal surface area in the RT model of the AdS bulk is the same as counting the minimal number of cuts through edges (the links between nodes) in the tensor network, from one edge of the boundary region to the other. Edges represent entanglement in the physical system and network connections in the model. That makes sense.

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Discoveries in AdS/CFT: Spacetime is entanglement.

If entanglement links the D-1 dimensional boundary to the D dimensional bulk, then it seems reasonable that maybe the extra dimension emerges from entanglement and that spacetime is entanglement. Mark van Raamsdonk proved that this is so, at least in the most familiar AdS/CFT models (van Raamsdonk, 2010).

van Raamsdonk reasoned backward from RT entanglement entropy and tensor networks. What happens to the bulk if you sever entanglement between boundary regions? We have a description of a universe out there on the CFT boundary. Inside, in the bulk, are all the gravitational operators – all the maths of spacetime curvature – in the dual description of that same universe. van Raamsdonk showed that if you dis-entangle regions on the boundary then their corresponding entanglement wedges pinch off. Spacetime in the bulk dissolves. See van Raamsdonk 2016 for detailed background and derivation.

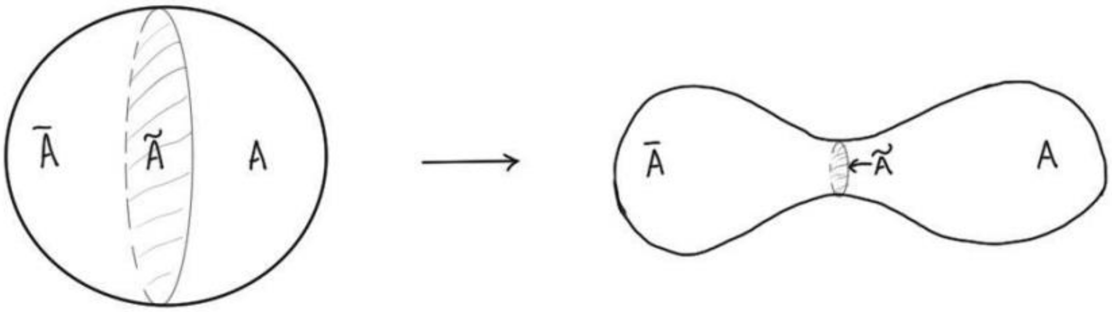

Figure 16: If you cut entanglement between the two CFT hemispheres of an AdS/CFT, then the bulk pinches off. You've split the bulk spacetime in two. Spacetime, by van Raamsdonk's calculations, is entanglement. Image from van Raamsdonk, 2016.

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Discoveries in AdS/CFT: The AdS/CFT duality is a quantum error correcting code

We have a mathematical correspondence, the quantum gravity / quantum field theory (AdS/CFT) duality, in which entanglement links bulk to boundary. We have a conjecture that spacetime itself is built from entanglement. That entanglement can be modeled by a tensor network. And that tensor network sure looks like quantum computer circuitry.

What stabilizes the entanglement / tensor network circuitry? van Raamsdonk showed that spacetime might dissolve if you cut the entanglement. What keeps it from disintegrating?

John Preskill, Daniel Harlow, Xi Dong, Beni Yohsida and others familiar with ideas from quantum computation found that AdS/CFT behaves like a quantum error correcting code. Ditto black holes – they behave like quantum error correcting devices. The idea of quantum error correction itself does not necessarily model the actual physics of the universe. The AdS/CFT may (or may not) be an actual computer, some HAL invented long long ago in a galaxy far away by some super-intelligent race of space aliens. But research into quantum error correction has already provided robust theorems that help to understand entangled systems, and the maths of quantum error correction (QEC) may lead the way to new discoveries.

What is quantum error correction? Some background:

Errors due to interaction with the environment are the bane of quantum computing. The heart of quantum computing is entanglement. If a qubit interacts with its environment, that messes up whatever calculation was in progress. If a stray photon, for example, hits an ion in an ion trap, then the state of the ion changes and we've lost the coherence necessary to factor a large prime product or whatever problem was in process. That's why quantum qubits have to be isolated from the environment, in vacuum chambers protected from electromagnetic radiation and kept at extremely low temperature.

One way to correct random errors in regular old digital communications systems is to translate bits into triplet code. If Alice sends a 0 , the computer converts that to 000 and sends the triplet to Bob. If there's a glitch in the transmission and Bob receives 010 he still knows Alice probably sent a 0 , unless the transmission error rates are very high.

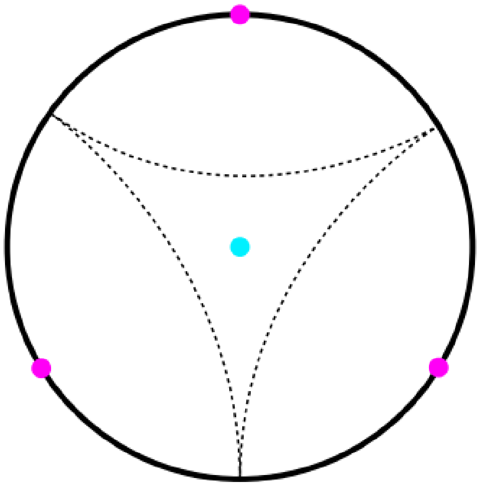

Figure 17: A qutrit error correcting code in holography. An operator (i.e. something that can be measured) at the center of the bulk (blue dot) can be encoded in any two of the operators on the boundary (red dots), but not by just a single boundary operator alone. The boundary operators are entangled, e.g. $|\phi\rangle = \frac{1}{\sqrt{3}}(|000\rangle + |111\rangle + |222\rangle)$, such that knowing any two qutrits determines the state of the third. Image from Harlow TASI Lectures 2018.

Quantum error correction works similarly, but the key is entanglement. The more qubits you entangle in your quantum computer the more robust is the information. Entanglement stabilizes information storage. It provides a kind of information inertia. A highly entangled system, as in AdS/CFT or the horizon and interior of a black hole, is stabilized by the QEC of entanglement. Built in. (More on that in a bit – or qubit.)

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The benefits of entanglement: ER = EPR

Entanglement is key. All these models are built on entanglement – what connects bulk to boundary, what stitches spacetime together, how spacetime is stabilized. What is the physical basis of this entanglement?

Enter ER = EPR. The acronym derives from the authors' initials on two of Einstein's papers, both published in 1935 a few months apart (Einstein et al, 1935). (There's interesting history here. The two papers, when first published, had no obvious relation, and Einstein himself never accepted the underlying quantum mechanics, "spooky action at a distance," for which the papers have come to play a central role!)

Einstein and Nathan Rosen (ER) discovered wormholes. In an effort to avoid singularities in the Schwarszchild solutions to general relativity (which predict black holes), Einstein and Rosen proposed tunnels through spacetime to bypass them. (Singularities are conditions, e.g. at the center of a black hole, where the equations of GR give infinities as solutions, as in dividing by zero. Infinities cannot be actual physical conditions.)

Einstein, Rosen, and Boris Podolsky four months later formulated the "EPR paradox." They devised a thought-experiment to prove that quantum mechanics could not be a complete theory of nature. Quantum mechanics predicts that if Alice and Bob share an entangled spin pair and Alice measures her particle's spin, then she immediately knows the spin of Bob's particle far far away. And when Bob measures the spin of his particle, that's what he finds – what Alice already knows. EPR argued that it violates the principle of relativity for a measurement performed on one particle instantaneously to determine the state of the other particle. No information can travel faster than the speed of light.

But it turns out that argument doesn't obtain. Bob doesn't know his particle's spin until he measures it himself or Alice sends him a message. That message can't travel faster than light. On the other hand, reading between the lines, it suggests that two entangled particles are physically connected – by a wormhole.

That's ER=EPR. Juan Maldacena and Leonard Susskind conjectured that entangled particles are connected through wormholes (Maldacena and Susskind, 2013). If all the universe is entangled – all the particles here there and everywhere – then the physical structure of spacetime is a cobweb of wormholes.

And there may be different varieties of wormholes. Douglas Stanford lists three different circumstances in which wormholes play a role – in particle entanglement, wormholes connecting black holes, and wormholes connecting information in the black hole interior to information outside (Stanford, 2020). We'll look into that next.

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The benefits of entanglement: Resolution of the black hole information paradox (?)

Two groups generated considerable excitement recently reporting progress toward resolution of the black hole information paradox. Entanglement-wormholes, holography, AdS/CFT, information theory, path integral QM, all have contributed. What's especially interesting is that the (apparent) resolution uses some classical tools with which Newton would have been familiar, maybe even Euclid.

After years of collaboration back and forth, two groups on opposite sides of the (North American) continent published their results at about the same time (Almheiri et al, 2020; Penington et al, 2020). Information dropped into a black hole is not lost. It becomes entangled with Hawking radiation and returns to the outside universe (highly diffuse and thermalized, but still potentially accessible) in that radiation. The details are complicated, but the basic idea is simple. If the black hole horizon records all the information inside the black hole (that's holography) and if the horizon is entangled with the inside (that's AdS/CFT and tensor network models), then it's not surprising that evaporation off the block hole horizon should carry information about the interior.

Nice and tidy, but complications result from the "no-cloning" theorem. QM doesn't allow you to copy information. You can't copy a qubit. So how can the same information be both inside the black hole and also (entangled) outside in the Hawking radiation? The East Coast and West Coast groups found a way to resolve the problem with good ol' calculus using a sum over histories approach invented by Richard Feynman. In this instance, the breakthrough was to sum over all possible spacetime geometries connecting black hole to universe. Those geometries are built from interconnected wormholes.

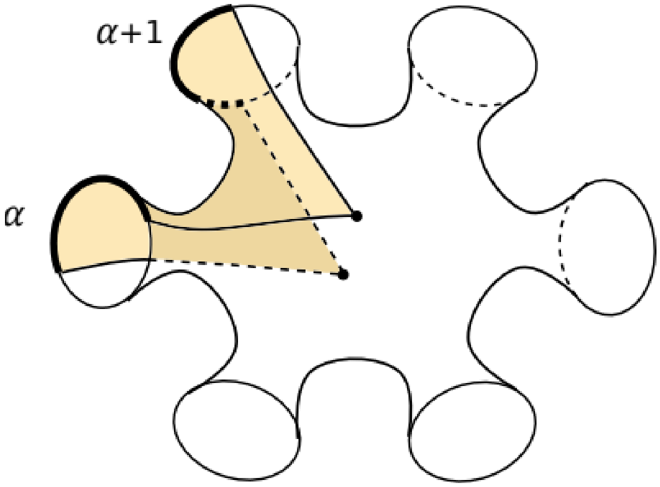

Figure 18: Wormhole geometry for a sum over spacetime topologies with six interconnected wormholes, six entangled spacetimes, as an example. The connected wormholes (as described by path integrals) share information. That information can be extracted by calculating the limit of mathematical cuts that reduce the multiply-connected spacetime to a single bounded region. As shown here, gluing regions α to $\alpha + 1$ along the cut edges captures the wormholes' interior, therefore information, inside the cut boundary. Figure credit Penington et al, 2020.

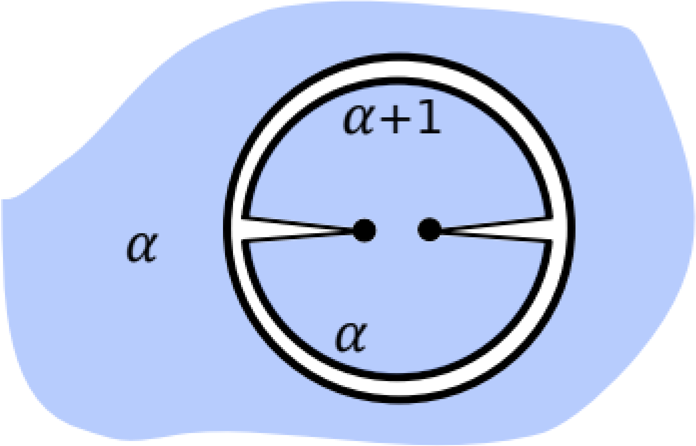

Figure 19: The result of the gluing described in Figure 18. The multiply-connected wormhole topology has been reduced to a familiar bulk and boundary. In doing so, information in the bulk is also shared outside (blue). Bulk and environment share information. Figure credit Penington et al, 2020.

There's lots going on in these arguments. Read the papers carefully, and figure them out – I'm still trying to understand them fully myself! The East Coast review article, especially, offers a beautiful summary of the logic (Almheiri et al, 2020).

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Complexification

So far we've built a spacetime structure from entanglement. But how can we model the dynamics? Objects move around through that spacetime. Fields oscillate. On the largest scales the whole universe is expanding. How do we get dynamics in an entangled spacetime?

The thermofield double state provides a useful model. The thermofield double comprises two entangled black holes, in fact the original Schwarszchild solution to Einstein's field equations. The wormhole of entanglement stretches rapidly over time, growing longer and longer at a rate faster than light.

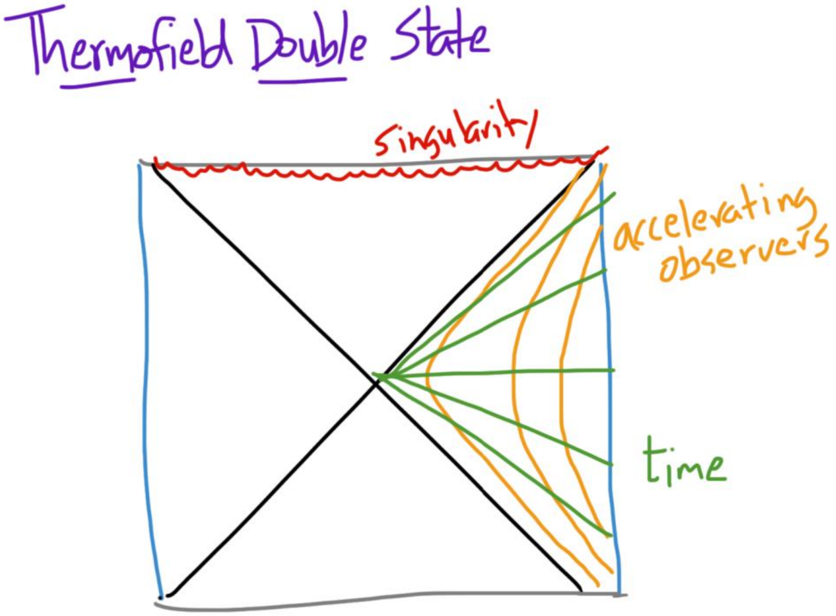

Figure 20: Penrose diagram of the thermofield double state, two entangled black holes connected by a wormhole. Coordinates are shown for the black hole on the right; mirror reflection would show coordinates in the black hole on the left. Time runs upward. Three spatial dimensions are reduced to a single radial coordinate with $r = 0,\ t = 0$ where the black lines cross. Coordinates of equal radial distances outside the black hole shown as orange curves, equal times coordinates shown as green lines.

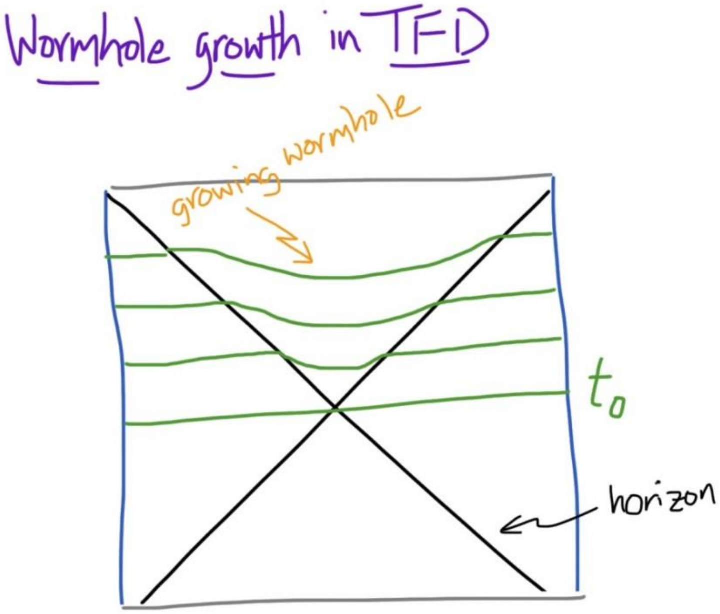

Figure 21: Evolution of a wormhole in the thermofield double state. The wormhole between two entangled black holes, represented by the sequence of green curves, stretches over time. It is evident in this Penrose diagram that the stretching, starting from the "now," $t = 0$, must be faster than light. $45°$ lines on the Penrose diagram are light rays.

What drives that stretching? Entropy and the second law can't suffice. Entropy equilibrates very quickly. Something falls into a black hole, the black hole oscillates for a split second then settles back into perfectly smooth thermal equilibrium. (The LIGO gravity wave detectors have measured that equilibration, and so have laboratory experiments on a quantum computer model and in strange metal materials. More on that to come.) There has to be something else at work here.

Leonard Susskind and Adam Brown have proposed that something else is quantum complexity. As we've already seen, one measure of complexity counts gates in a quantum circuit. Another measure counts states in a quantum system. Entropy is measured by Shannon's rules. It is a power of N where N is the number of bits in a system. The number of possible states in a system of N bits is 2^N . Complexity on the other hand is doubly exponential, increasing as 2^{2^N} . It takes an awfully long time for the complexity in a large system, such as a black hole, to equilibrate. And if it is complexity driving an expansion of spacetime then that spacetime can grow really really big.

Susskind and Brown showed that calculations for wormhole growth using the maths of quantum complexity match the results based on GR. Not a proof, but compelling evidence that complexity may drive the dynamics of spacetime.

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Quantum gravity in the lab: black holes in strange metals and superconductors

How on earth can we test these notions? We will not likely measure the insides of a thermofield double wormhole. But lately the experimentalists, bless them, have created models for quantum gravity physics in the lab. I'll discuss just a couple examples. Condensed matter physicists have discovered "strange metals" and superconductors that behave like black holes. Their work, based especially on the SYK model, provides new insights into quantum gravity.

SYK, after authors Subir Sachdev, Jinwu Ye, and Alexei Kitaev, models electron behavior in so-called "strange metals." Conduction in those metals depends on pair-wise, randomly exchanged entanglement between electrons in a two-dimensional system. It turns out that SYK, a QFT model for electrons in 2D, is dual to a model including gravity in 3D. It's an AdS/CFT! And lab measurements of the materials' behavior agree with predictions in the QFT/gravity duality. Among other correspondences, relaxation time for electrons in the strange metal is the same as the relaxation time for a black hole. Disturb the electrons in the metal and they relax to thermal equilibrium in a time $t = {}^{\hbar}/_{kT}$ (Sachdev, 2020). Disturb a black hole and it relaxes to thermal equilibrium in that same time (Abbott et al, 2016).

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Quantum gravity in the lab: Black holes in quantum computers

If gravity really is, at heart, a quantum phenomenon, and if quantum computers really do express quantum reality as per the Church-Turing Thesis, then you should be able to model black holes and other natural systems on a quantum computer. Well, it's been done! Even relatively primitive quantum computers now in operation (2021) have been able to simulate black hole phenomena (Landsman et al, 2019).

The main players in these investigations so far have been trapped ion devices (see Schleier-Smith, 2021), but many other quantum computer platforms are in the works. Among the most exciting are efforts to model the thermofield-double state and its wormhole (Schleier-Smith, 2021; and see TIQI.) For a nice presentation of quantum circuit design to produce the lab models, see Brown et al, 2021.)

Most recently (as of March 2023) a Harvard-Caltech-Google team claims to have created a wormhole on the Google Sycamore machine. See Ch.15 for some of the details. The authors' conclusions, whether or not they actually created a wormhole, are still contentious, but their experiment represents a significant step on the trail of quantum gravity. (Quanta Magazine, 2022.)

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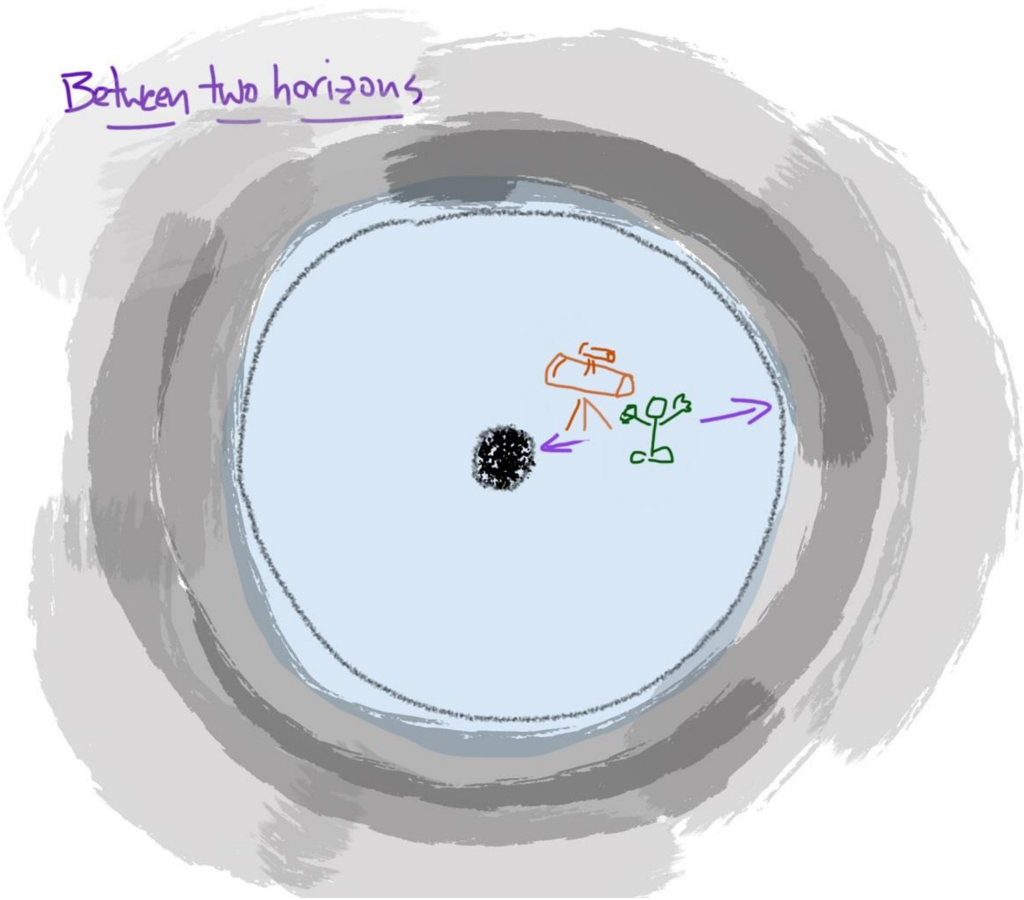
Onward to de-Sitter space?

Mind-boggling and fascinating. The people in this exciting field have made enormous progress. We may be close to, if not already arrived at, a good grasp of quantum gravity. Only problem is that the work on the gravity side has been mostly in anti-de-Sitter space. That's not our universe. By observation, we live in a near-flat, positive energy density de-Sitter universe. We need a theory in de-Sitter gravity to complete the program.

And we may be on the way. Lenny Susskind – again – has leaped ahead with new ideas. He has recently (2021) published arguments showing that, under certain assumptions, quantum calculations for the "entropy deficit" between two horizons in de-Sitter spacetime are dual to the GR calculations. That implies a QFT/GR duality in de-Sitter space. Whether that duality applies more broadly, to other parameters of de-Sitter space, is yet to be determined.

Here's the gist of Susskind's argument. We live between two information horizons: black hole horizons and the cosmological horizon. The cosmological horizon is 'way out there at the edge of the visible universe. It's the boundary beyond which we cannot see, where the recession velocity of our expanding spacetime reaches the speed of light. Just like at the event horizon of a black hole, galaxies approaching the cosmological horizon are redshifted, their clocks slow, their images are distorted – all familiar phenomena, same as an object falling into a black hole.

Presumably, by the holographic principle, all the information about all the universe is encoded there on the cosmological horizon. That information includes what's inside the black holes scattered around our spacetime. We can't access that information (not until after the black holes have evaporated). So we live in a residue of information about our universe. Cosmological horizon tells all, but black holes hide stuff from us. The residual includes all the information here in between the horizons. Susskind shows that we can calculate that residual with either toolkit – a matrix theory analog of QFT or the maths of GR – and the results come out the same.



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Looking ahead

We’ve covered a lot, and this review only traces the logical threads of quantum gravity as information. We haven’t considered other possible models such as loop quantum gravity or emergent gravity or twistor theory or bootstrapping from quantum theory. There’s a whole lot more to consider, but the program we’ve outlined seems to be making the most progress, by far.

Problems solved? Well, we have tantalizing most-likely-seems-consistent-with-actual-experiments explanatory models. Black hole information paradox? The verdict is in: black holes don’t destroy information. Quantum mechanics survives. And the structure of spacetime? It’s entanglement. In retrospect of 13.8 billion years, that doesn’t seem so awfully strange, a universe of entanglement. All observations support a big bang origin from a singularity. So the whole shebang must have been entangled from the get-go, all fields jam-packed entangled at the Origin. Entanglement, a.k.a. wormholes, weave a tapestry in the voids between the galaxies, and knots of entanglement form the galaxies themselves. Moreover, the universe behaves like an information processor, a computer. We can already model some aspects of the universe on our primitive quantum computing devices. Some day we might be able to build a universe on a machine.

Well, at least all those speculations seem to be on a consistent track. I hope some of you might want to push forward these ideas, or seek out new and better!

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Appendix

Glossary of abbreviations (that I include in the text and forget to define):

AdS	anti-de-Sitter space
CFT	a quantum field theory that is invariant under conformal transformations, i.e. angles between state vectors are preserved
GR	general relativity
QC	quantum computing
QEC	quantum error correction
QFT	quantum field theory
QG	quantum gravity
QM	quantum mechanics
RT	Ryu-Takayanagi

The standard universal set of quantum gates

Note that these gates are equivalent to vector operators – matrices – that rotate state vectors in three-dimensional vector space. For example, the action of the X gate is to rotate a vector around the X axis. We can choose X to represent direction of a physical parameter such as spin or "direction" in some other state space, such as color charge. For example, an X gate, in matrix form, operating on spin down is represented as

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

where $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is the vector representation of spin up and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is the vector spin down. With this set of gates, we can rotate state vectors to any orientation in space, i.e. we can represent any of the infinitude of states on 2-D or 3-D coordinate systems.

<table>
<tr><th>Gate</th><th>Truth table or matrix form</th></tr>
<tr><td>CNOT
flips target qubit in the second wire if qubit in the input wire is 1</td><td><table>
<tr><td>Input AB</td><td>Output AB</td></tr>
<tr><td>$|00\rangle$</td><td>$|00\rangle$</td></tr>
<tr><td>$|01\rangle$</td><td>$|01\rangle$</td></tr>
<tr><td>$|10\rangle$</td><td>$|11\rangle$</td></tr>
<tr><td>$|11\rangle$</td><td>$|10\rangle$</td></tr>
</table></td></tr>
<tr><td>Hadamard
creates mixed states</td><td><table>
<tr><td>Input</td><td>Output</td></tr>
<tr><td>$|0\rangle$</td><td>$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$</td></tr>
<tr><td>$|1\rangle$</td><td>$\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$</td></tr>
</table></td></tr>
<tr><td>X
rotates state vector around the x-axis</td><td>$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$</td></tr>
<tr><td>Y
rotates state vector around the y-axis</td><td>$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$</td></tr>
<tr><td>Z
rotates state vector around the z-axis</td><td>$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$</td></tr>
<tr><td>Phase shift</td><td>$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$</td></tr>
<tr><td>$\frac{\pi}{8}$ Phase</td><td>$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{bmatrix}$</td></tr>
</table>