Chapter 1
Newton's Universe

Physicists attempt to find underlying order in the complexity of Nature, to explain such diverse phenomena as falling apples and the orbits of the planets in terms of simple "laws" of Nature. This endeavor is complicated by the fact that different observers witnessing the same events from different perspectives describe those events differently, like the blind men trying to describe an elephant.

In this chapter we will discuss Newton's attempts to identify Nature's invariants,those qualities on which all observers, everywhere, can agree. The chapter summarizes “classical physics," including Newton's laws of motion, the conservation laws, and the law of universal gravitation. Underlying the discussion is the tension between two different methods of measuring the Universe, absolute space and time versus the Principle of Relativity. We devote most of the chapter to Newton's chosen method, based on absolute space and time. At the end of the chapter we introduce the Principle of Relativity, which turns out to provide a more accurate description of nature and forms the basis of Einstein's Theory of Relativity.

Sir Isaac Newton lived in the seventeenth century. His laws of motion and law of gravitation dominated physics for the 200 years following his death. As we shall see in later chapters, Newton's laws are only approximately true at the extremes of scale, but even today, despite Einstein's revisions in the theory of relativity and despite revisions incorporated into quantum physics, Newton's laws provide a foundation for all physical science.

Measuring tools

Physics seeks to describe how Nature behaves and then to explain why Nature behaves as she does, i.e. to find the "laws" of Nature. It is curious, and of profound significance, that physical events can be described in mathematical terms, the "laws,” and that mathematical logic can predict the outcome of actual physical events. For example, as we shall see at the end of this chapter, Newton's mathematical law of universal gravitation can predict the orbits of the planets with great accuracy for years into the future.

In order to describe Nature, a physicist must devise a measuring system on which to base his/her descriptions. As we shall see, Newton made several assumptions in formulating his measuring system which turned out to apply accurately at the scale of human experience but not in the realms of the very small (the atomic scale) or the very large (the scale of stars, galaxies, and the Universe). Such a scenario was repeated often in the development of physics: presumed facts" which a physicist takes to be self-evident, without experimental verification, may prove false in the light of later, more accurate evidence.

Newton and physicists after him faced a related problem: if there is, in fact, some underlying order in the Universe, then all observers, everywhere, should be able to agree about certain fundamental measurements, certain "invariants." This is not so obvious as it seems. As an example, imagine that you are riding along a smooth, level road on a bicycle. You have a small stone in your hand which you repeatedly toss straight up and down as you ride. As you see it, the stone simply follows a path straight up out of your hand and straight back down. But people standing on the side of the road watching you go by see something quite different: to them the stone appears to be travelling in long arcs through the air. You and the other observers all witness the same stone in flight, but you "see" different phenomena. How can you ever agree on the true "flight of the stone?"

Newton realized this dilemma. In formulating his physics he assumed that our measuring tools (i.e. rulers and clocks) were invariant. Relativity, on the other hand allows variable measuring tools so that certain invariant quantities, such as the speed of light, remain constant for all observers. We will return to this idea later.

Newton's assumptions

In devising a method to describe Nature Newton made several important assumptions that go to the very foundation of how we perceive the world around us. The most important of these is the assumption of absolute space and time. By this he meant that a point in space or a moment in time has an independent existence. Even if all material objects were removed from the Universe, Newton claimed that a location in space and time would retain its identity.

By this assumption of absolute space and time one can imagine a huge three-dimensional gridwork, a three dimensional Cartesian coordinate system, extending throughout the Universe. The position of any object could be measured relative to the principal axes of the grid, and all objects simply move through the grid, which itself remains fixed. The grid could exist even in an otherwise empty Universe.

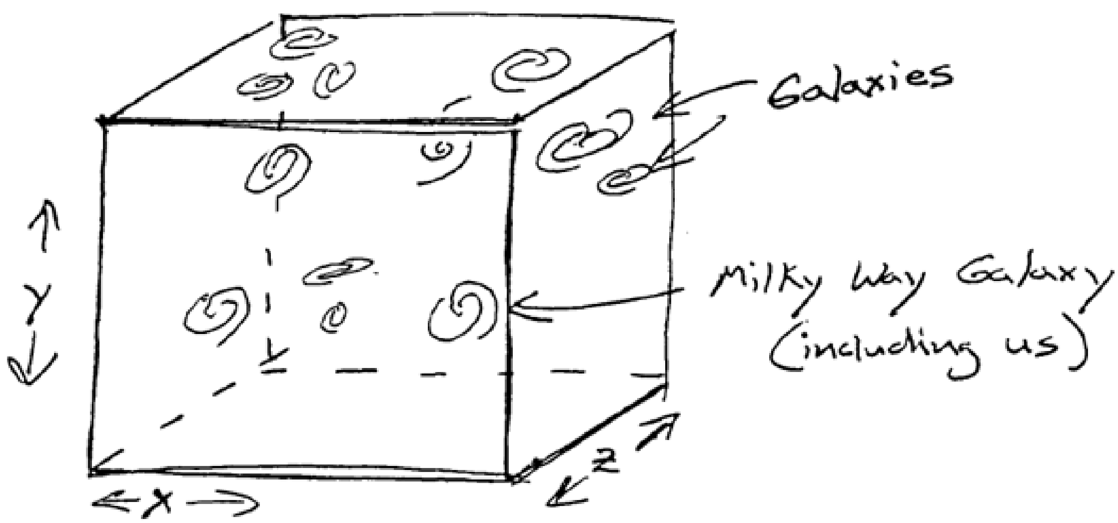


Figure 1.1. Universe as a giant grid. Galaxies can be located by position in x, y, z coordinates.

On a more familiar scale, as an example, think of locating a fish in an aquarium. Arbitrarily, we call the lengthwise direction the "x" axis, the width of the tank the “y" axis, and the depth the "z" axis, and we choose one corner of the aquarium at water level as the origin (the zero point for each axis). We can then locate the fish (or any other object in the water) according to how far along the x-axis it is from the origin, how far along y, and how far along z : i.e. we determine its x, y, and z coordinates (x,y,z). In the diagram, the fish is located at approximately (10 centimeters, 5 cm., -5 cm.). The minus sign in the z coordinate refers to a direction: the fish is 5 cm below the surface.

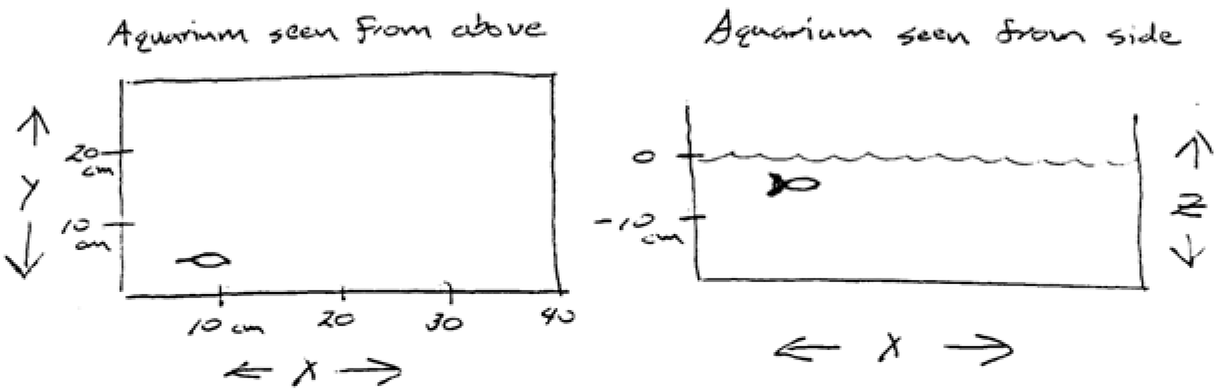


Figure 1.2. Fish in a Cartesian grid.

In Newton's Universe, the grid is Euclidean: parallel lines never meet, and the grid lines do not curve. Astrophysicists use the term "flat" to describe such a Euclidean Universe. Here "flat" refers to a three dimensional geometry that does not bend in any direction.

Since everything in the Universe moves, that is changes position with time, scientists must measure position with reference to time. The earth, for example moves in its orbit around the sun, the sun orbits the nucleus our Galaxy, and our Galaxy moves in relation to other galaxies. Newton assumed there was an absolute clock which could specify for everyone in the Universe exactly when an object occupied a particular position on his absolute grid. No matter where a sequence of events occurred, any observer would measure the same time intervals between them. According to Newton, there is no difference in the flow of time in the various nooks and crannies of the Universe.

Besides the capacity to measure space and time, Newton required some means to quantify the amount of matter in an object. Hence the notion of mass. An object's mass is a measure of the amount of matter it contains. For example, a tennis ball has a relatively small mass compared to a bowling ball.

One measure of mass, and an important concept in itself, is inertia, the resistance to any change in motion. The greater an object's mass, the greater its inertia. To illustrate inertia, shake a tennis ball side to side, then shake a bowling ball similarly. It is much more difficult to accelerate the bowling ball than the tennis ball; the bowling ball has greater inertia, proportional to its greater mass.

It is important to distinguish mass from weight, which is a measure of the force of gravity pulling a mass toward earth. A bowling ball in orbit around earth is weightless, but it has the same mass as before: we would find the same inertia if we tried to accelerate it side to side.

The notions of absolute space and time imply that length and time are unchanging quantities. A meter stick measures a meter anywhere in the Universe, no matter how it is oriented, no matter how it is moving, no matter what time. And clocks tick off seconds at the same rate everywhere in the Universe.

Newton also included mass as a fundamental quantity. A kilogram mass, according to Newton, is a kilogram in every nook and cranny of the Universe, at all times, and no matter how fast it is moving.

One must be aware of such underlying assumptions related to any theory of Nature. Newton's assumptions of absolute space and time lead to significantly different predictions about events at the extremes of scale than are evidenced by experiment and predicted by Einstein's theory. When studying physics we must be prepared to accept the fact that quantities which are immediately accessible to our senses may not be the ones which best describe the workings of Nature, just as a person's outward demeanor may not reveal his or her true character (ask poor Othello). As theories are refined by experimental test, we may have to give up the intuitively obvious in favor of the more abstract. This is not done arbitrarily but by testing the theory under different circumstances and at different scales. For example, using measuring tools much more accurate than those available to Newton, twentieth century physicists find that the theory of special relativity and the rules of quantum mechanics describe events at the atomic scale much more accurately than does Newtonian mechanics. As it turns out, length and time intervals, taken by themselves, are not really fundamental, fixed quantities, but that's our story in a later chapter.

Derived quantities

Newton based his entire mechanics (the study of matter in motion) on the three "fundamental" quantities – mass, time, and length. From these he fashioned the other tools he needed to describe Nature.

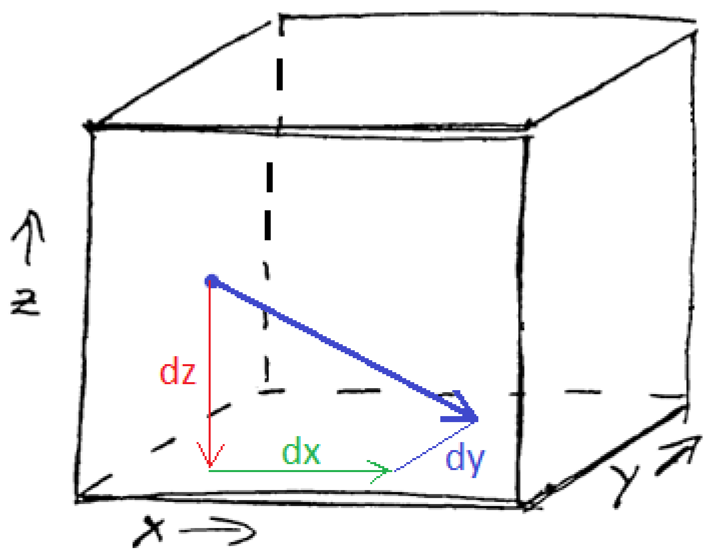
The speed tells us the rate of motion of an object. In simplest terms, it is the ratio of the distance and object travels to the time it is in motion. For example if you drive sixty miles in one hour's time, your average speed for the trip was sixty miles per hour.

There are situations, however, where the average speed proves inadequate in describing motion: if an object's speed is changing while we study it, we must refine our definition of speed in order to describe its motion more accurately. For example, on our hypothetical car trip, the fact that we averaged sixty mph doesn't reveal that we stopped at a stop light along the way, then zipped along the freeway at 70 m.p.h. for a while, and it doesn't reveal which direction we traveled.

We can refine the concept of speed by considering the distance traveled over very tiny time intervals. This allows us to describe the speed of an object anywhere along its path of motion. Analyzing motion over tiny time intervals gives us what is called "instantaneous speed, the speed at some particular instant of time. For example, if we glance at the speedometer while driving down the highway we see the measure of the car's instantaneous speed. In this text the generic word "speed" will refer to instantaneous speed. And we can further improve our measuring system by introducing a quantity called "velocity," which includes in its definition both the speed of an object and its <u>direction</u> of travel. This allows us to analyze motion more precisely, telling us not only how fast an object is moving but also where it is going: traveling 60 mph north on the interstate takes us to a completely different destination than traveling 60 mph. south.

We can make the concept of velocity more concise by using the simple equation $v = {}^{ds}/_{dt}$ where v is the velocity of an object, and ds is the small distance in a particular direction in space over which the object moves in the small time dt. Directions are indicated with $+$ (e.g. "moving away from") or $-$ (e.g. "moving toward") signs, or by compass direction.

As a final refinement of this idea we can define the "components" of velocity along three mutually perpendicular directions, call them x, y and z. If an object moves a small amount ds in the time dt in some particular direction, this results in small displacements, which we label dx, dy and dz, along the three axes.



<u>Figure 1.3</u>. Components of motion of an object moving downward, to the right, and away from us.

We define the velocity in the direction of each axis as the "component" of the velocity along that axis:

$$v_x = {}^{dx}/_{dt}$$

$v_y = {dy}/{dt}$

$v_z = {dz}/{dt}$

where v_x is the component of velocity along the *x* axis, etc. If we choose an extremely small time interval, dt, we have a fairly complete description of the motion at a particular instant.

To illustrate the idea of the components of velocity we can return to the example of a fish in an aquarium. Suppose the fish is resting in the near bottom left corner of the tank at a location (0, 0 , -20), i.e. 0 cm. along the x axis, 0 cm. along y, and 20 cm in the negative direction, i.e. below the water's surface, along z.

We drop food in the far upper right corner, and the fish swims toward its meal. Observing from the side of the tank we can measure the x and z components of the fish's velocity.

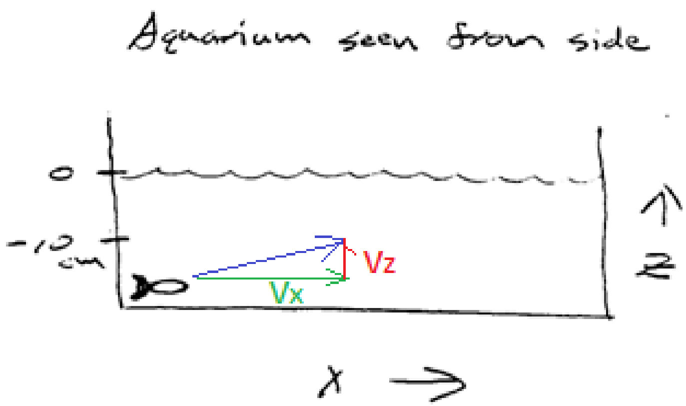


Figure 1.4. Components of fish's velocity as seen from the side of the aquarium.

Observing from above the tank we can measure the x and y components of the fish's velocity.

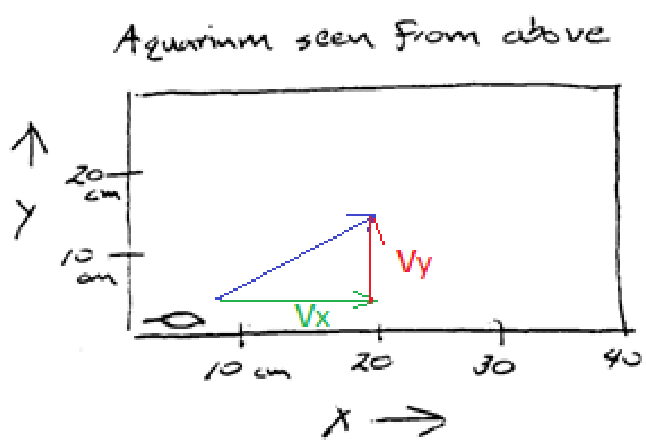


Figure 1.5. Components of fish's velocity seen from above.

Knowing the x, y, and z components of the velocity we can calculate the speed and direction of the fish through the water. (Can you do this? Hint: use the Pythagorean relationship between the sides of a right triangle and its hypotenuse.)

If the velocity of an object changes from one instant to another we say it undergoes an "acceleration." Acceleration can result from either a change in the speed of the object or from a change in its direction of motion. The concept of acceleration resulting from change in direction may seem surprising until one considers the fact that by changing direction you have increased the speed at an angle to the original direction of motion. At first, all the speed was along one direction in space. Now there is a speed along a direction where there was none previously, hence an acceleration.

As a simple example, suppose you are driving a car down a long straight road. If you step on the gas or brake pedals you accelerate simply because you are changing the speed of the car. You can see this on your speedometer. If the road takes a 90 degree turn to the right gently enough so you can keep the speedometer reading constant, you still accelerate, because your speed along the old direction diminishes to zero, and you gain speed along the new direction.

Using the concept of components of velocity, if the speeds v_x , v_y and v_z change by the amounts dv_x, dv_y and dv_z in the small time interval dt, then we define the components of the acceleration to be:

$$a_x = {dv_x}/{dt}$$

$$a_y = {dv_y}/{dt}$$

$$a_z = {dv_z}/{dt}$$

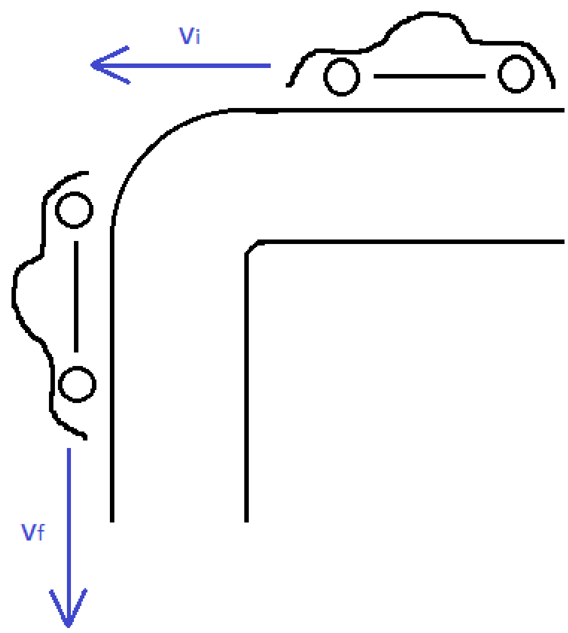


Figure 1.5. Acceleration around a 90° corner. Components of car's velocity change incrementally (a little bit at a time around the corner) from all $(v_x, 0, 0)$ to all $(0, v_y, 0)$.

Those quantities, velocity and its congeners, which include magnitude and direction are called "vectors." Symbolically, we represent a vector as an arrow. The length of the arrow signifies the magnitude of the vector quantity, and the direction of the arrow in relation to a Cartesian grid represents the direction of the vector. For example, the following arrows represent the velocities of two airplanes approaching O'Hare International Airport. One plane is approaching from the west at 500 mph, and the other approaches from the south at 250 mph. (We assume they are flying at uniform altitude, neither climbing nor descending.)

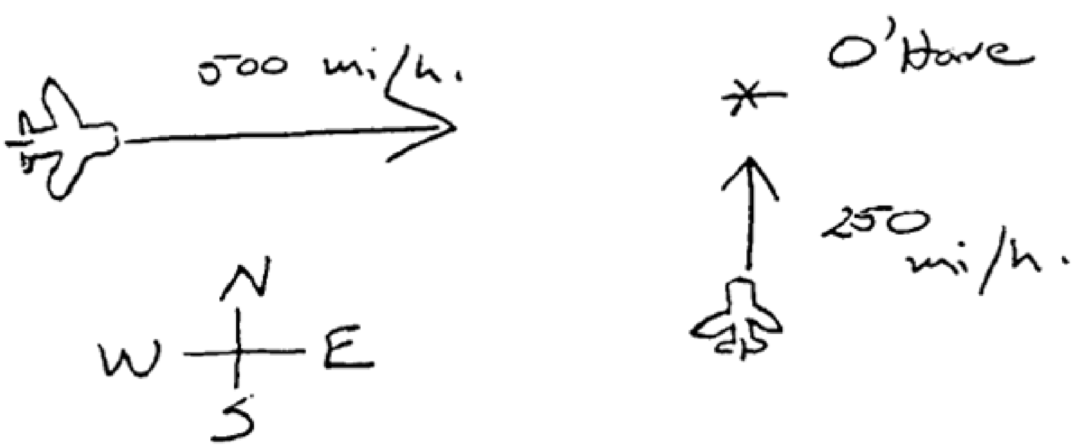


Figure 1.6. Vector representation of planes approaching Chicago O'Hare airport.

An important attribute of vectors is that if an object is subjected to two vector quantities simultaneously, the combined effect is the sum of the two vectors. Suppose, for example, an airplane is flying west at 400 mph relative to the surrounding air, and the air mass itself is moving south (i.e. the wind is blowing) at 100 mph relative to the Earth's surface. We can find the velocity of the airplane relative to the Earth's surface by adding the wind vector to the plane's airspeed vector. Simply draw the vector representing the airplane's velocity as an arrow, say 4 cm long and pointing to the left edge of the page. Then beginning at the tip of this arrow draw a second one, say 1 cm long and pointing to the bottom of the page, to represent the wind. Now draw an arrow from the beginning of the first vector to the end of the second. A person on the ground would see the airplane moving with this speed and direction (calculation gives 412 mph and 14° South of West).

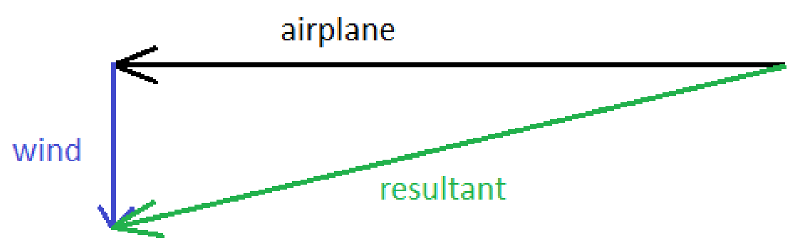


Figure 1.6. Vector representation of airplane flying west at 400 mph in wind carrying the plane to the south at 100 mph. Resultant vector is the actual path of the plane relative to the ground.

Vectors and vector addition aid analysis in a number of physical situations, such as calculating the total force on an object when several forces are applied at once, calculating the change in momentum of one object when it collides with another, calculating the change in angular momentum when spinning objects interact, etc.

Frames of reference

Now, how can we apply these measuring tools, Newton's "fundamental" (e.g. mass, time, distance) and "derived" (e.g. velocity, acceleration) quantities, to study Nature? In order to discern underlying patterns in all Nature's complexity, physicists study natural phenomena from different perspectives, different "frames of reference," seeking what is invariant among those several frames.

A frame of reference is the setting in which an event occurs or the setting from which an event is observed. For example, consider the following situation: a conductor is walking (say at two miles per hour eastward) through his train as the train itself moves to the east through a station (say at twenty miles per hour). Three separate frames of reference may be considered: the "fixed" frame of reference of the station; the moving frame of reference of the train; and the conductor's frame of reference which moves relative to the train. From the conductor's perspective, the seats on the train move west (backward) at two miles per hour and, looking out the window, the station appears to move west at twenty-two miles per hour (the conductor's speed relative to the train plus the train's speed relative to the station). Observers in the other reference frames obtain different numerical values for the same events: a passenger waiting at the station sees the train moving east at twenty miles per hour and the conductor moving east with a velocity of twenty-two mph. As seen by a passenger seated on the train, the station moves west at twenty mph and the conductor walks east at two mph.

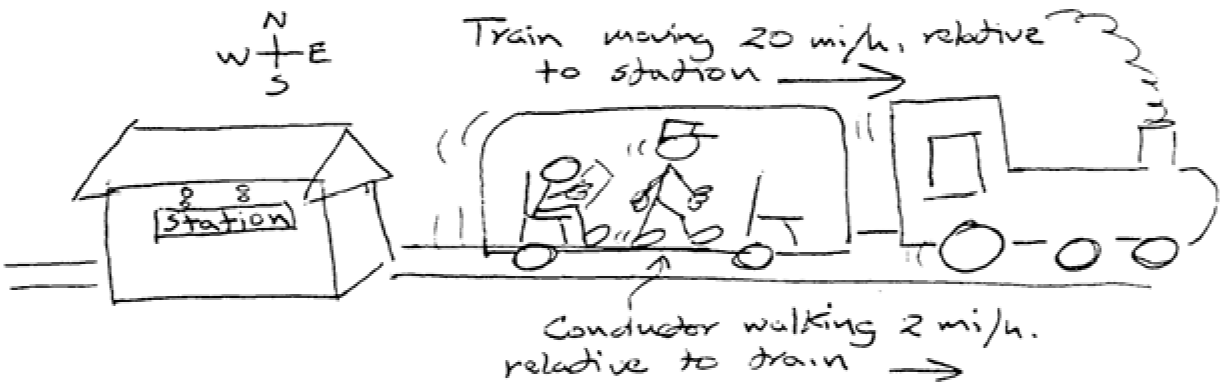


Figure 1.7. Frames of reference.

From this example, we see that the same quantities can have different numerical values when measured from different frames of reference. It turns out that many quantities which depend on velocity (including quantities we have not yet defined, such as momentum, energy, electric and magnetic fields, etc.) behave in this fashion. However, as Einstein proved in the theory of Special Relativity, when certain quantities are combined in pairs, e.g. space and time, momentum and energy, electric and magnetic fields, in just the right way, the numerical result turns out to have the same value even in different frames of reference. These combinations represent entities, invariants, which are fundamental to the structure and workings of Nature.

Galilean invariance

In the arguments presented above we assumed that the speeds of objects as viewed from different frames of reference could be related in very simple ways. For instance the speed of the conductor relative to the train was simply added to the speed of the train to obtain the speed of the conductor as seen from someone at the station. This seems intuitively obvious to us: it is based on ordinary every-day experiences. If someone throws a ball to us, we are prepared to catch it in a different manner if we are running toward that person rather than away from him. The sensation of catching the ball will be quite different in the two situations because relative to us the ball is moving faster in the first case and slower in the second.

If we cling to these assumptions as to how speeds are related then we must devise a means of transferring numerical values for position and time from one frame of reference to another so that different observers in different frames can communicate what they see. It turns out that a set of equations called the "Galilean transformations" will do this for us.

Consider the simple case of two frames of reference in which we set up Cartesian coordinate systems. Let the x axes of these systems overlap, and let the y and z axes of one frame of reference parallel those in the other. Assume that the first frame of reference is stationary in absolute space and call the axes drawn in it the x, y and z axes.

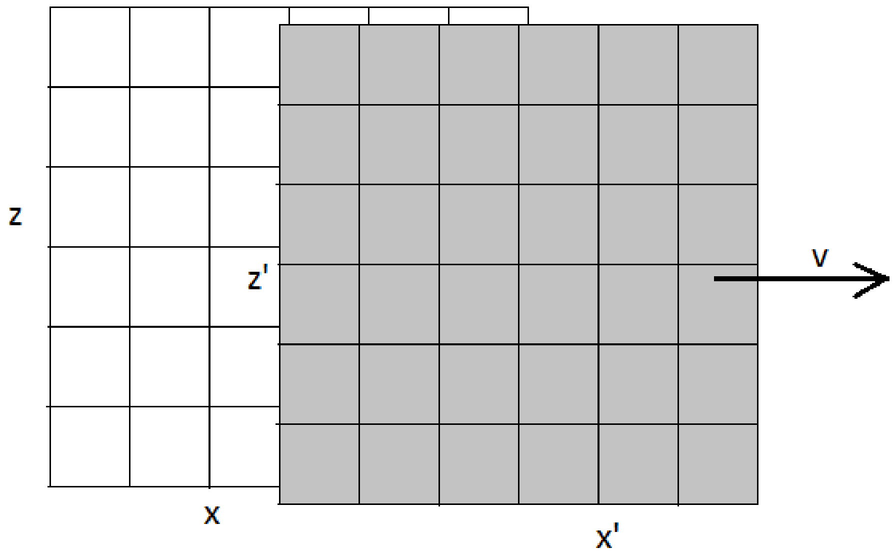


Figure 1.8. x', z' frame of reference moving along x axis with velocity v relative to x, z frame.

Assume that the second frame of reference is moving past the first with velocity v_x along their mutually parallel *x* axes. Call the axes in this second frame $x', y',$ and z' . For example, the *x*, *y* and *z* axes could refer to the train station mentioned above and the $x', y',$ and z' axes could be moving with the train. The position of the conductor, then, would have different numerical values as measured in these two coordinate systems.

In order to have the simplest possible equations through which we can relate quantities in the two frames of reference we can assume that everyone agrees to measure time from when the rear of the train passes through the station. If x is the position of the conductor relative to the station and x' is the conductor's position as measured from the rear of the train, then we can write at any elapsed time t' in the train x = x' + vt', (or alternatively x' = x - vt), where v is the train's velocity relative to the station, 20 mph in our example, and by the assumptions stated above t = t' .

These remarkably simple equations (called the Galilean transformations after Galileo Galilei, who formulated the idea a century before Newton) allow us to take time and position measurements in one frame of reference (x and t) and use them to calculate the values of these same measurements in another frame of reference (x' and t') which is moving relative to the first.

As another illustration, imagine two cars start at the same time in the same direction along a straight road. One car (the Chevy) travels at 60 miles per hour and the other (the Honda) travels at 50 mph. After one hour, the Chevy will be 60 miles (distance x) from the starting point. However, as measured by someone riding in the Honda (a moving frame of reference), the Chevy is only ten miles away (distance x'). We can visualize this diagrammatically (see illustration) or calculate x' according to the formula x' = x - vt, where x is the Chevy's distance as measured from the starting point, v is the Honda's velocity = 50 mph, and t is the elapsed time = 1 hour.

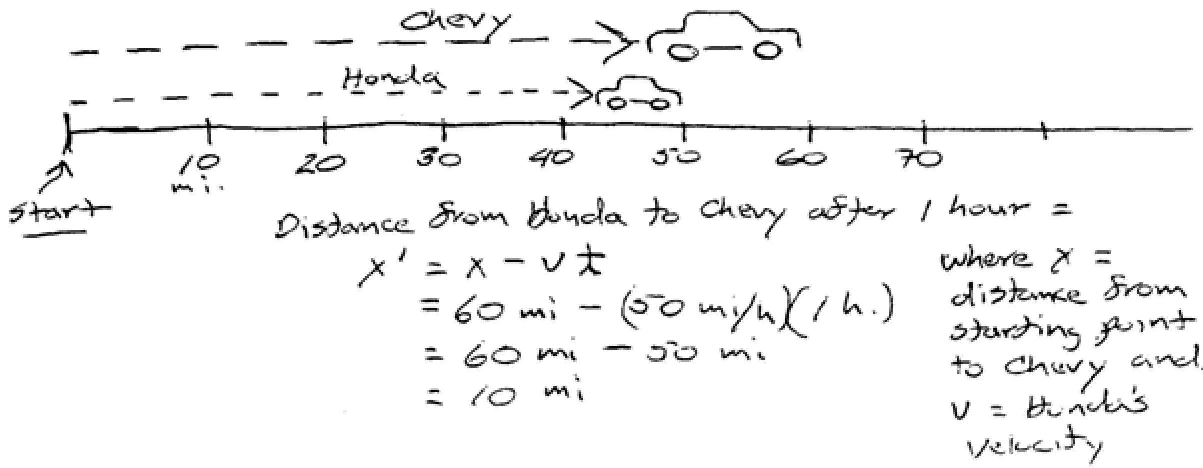


Figure 1.9. Relative distance between two cars traveling at different speeds in same direction.

We can also show that these equations are compatible with our concept of addition of velocities. Let the velocity of the conductor as seen by a passenger on the train be $v_{cond\ in\ train} = {dx_{cond\ in\ train}}/{dt}$. What is the conductor’s velocity relative to the station? From the Galilean Transforms we obtain $x_{cond\ from\ station} = x_{train\ from\ station} + v_{cond\ in\ train}t$.

Since time is the same everywhere, by Newton’s assumption,

$$ {dx_{cond\ from\ station}}/{dt} = {dx_{train\ from\ station}}/{dt} + v_{cond\ in\ train} \cdot $$

That is, $v_{cond\ from\ station} = v_{train\ from\ station} + v_{cond\ in\ train}$

where $v_{cond\ from\ station}$ is the conductor's velocity seen from the station (22 mph), $v_{cond\ in\ train}$ is the conductor's velocity relative to the train (2 mph), and $v_{train\ from\ station}$ is the train's velocity relative to the station (20 mph). In other words add the speed of the train to the speed of the conductor within the train to find the conductor's speed relative to the station.

The Galilean transformation provides a consistent system relating measurements made by different observers. No two observers agree about what they see, because they are in different states of motion, but they can relate their observations using the transformation.

<u>Motion</u>

Based on Galileo's ideas and on observations of his own, Newton proposed that Nature behaved according to certain universal laws governing the motions of all physical objects, In formulating these laws, Newton considered a fairly abstract but apparently fundamental quantity which he called "motion" and which we now call momentum, the product of the mass of an object and its speed along its direction of motion. If we use the symbol p for momentum, m for mass, and v for speed, we can write the defining equation

$$p = mv$$

Just as we analyzed speed in terms of its components in a Cartesian coordinate system, so we can represent momentum in terms of it components:

$$p_x = mv_x$$

$$p_y = mv_y$$

$$p_z = mv_z$$

We encounter momentum when we exert a force (in simplest terms a push or a pull) to change the motion of an object. For example, if you wish to stop a slowly rolling automobile in a given amount of time using your own muscle power, you know that the force necessary is proportional to the mass of the car and how fast it is rolling, i.e. the necessary force is proportional to the car's momentum. Both quantities, mass and speed, must be considered. For example, it is more difficult to stop a slowly rolling Cadillac than a slowly rolling Honda (mass effect on momentum). It's more difficult to stop a rapidly moving Honda than a slowly rolling Honda (velocity effect on momentum).

Another consideration in stopping a rolling car is how quickly we want to do it. Intuitively, the sooner we wish to stop the car, the more force we must apply. If a car strikes a rigid object like a telephone pole, it undergoes an extremely rapid change in motion. In fact, the force necessary to stop a car in a very short time is usually so great that the pole is not able to supply the needed force, and it breaks.

From experience, we are also aware that in stopping a car, not only do we exert a force on the car but the car exerts a force back on us. Only when the forces involved are small enough so we can "dig in" to the ground with our feet can we actually stop the car: sometimes the force back on us is so strong that our feet simply slide along the ground. Again it is the car's momentum which determines how much force we must exert on the car and how much force the car exerts on us.

Newton combined all these ideas into his Laws of Motion. They may be paraphrased as follows:

1. Law of inertia: An object will continue to move along a straight line with constant velocity unless it is acted upon by an unbalanced external force.

Figure 1.10. Inertia. In the absence of force, an object maintains its initial state of motion.

2. Force law: The rate of change of the motion (momentum) of an object is directly proportional to the net force acting on it.

$$F = {}^{dp}/_{dt}$$

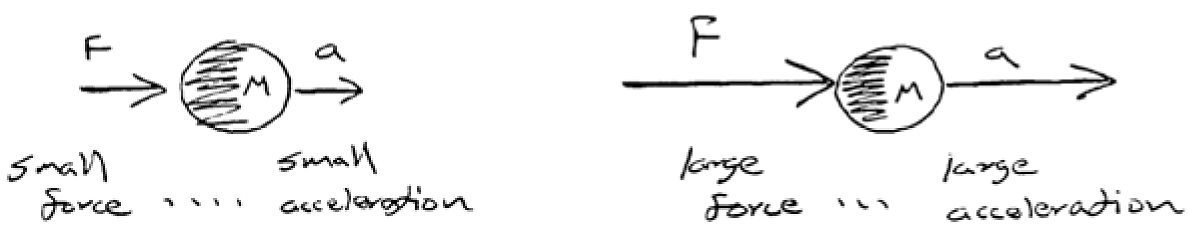


Figure 1.11. Acceleration (rate of change of momentum) is proportional to the force acting on an object.

3. Action – reaction: Whenever one object exerts a force on a second object, then the first object will simultaneously experience a force of equal magnitude but in the opposite direction.

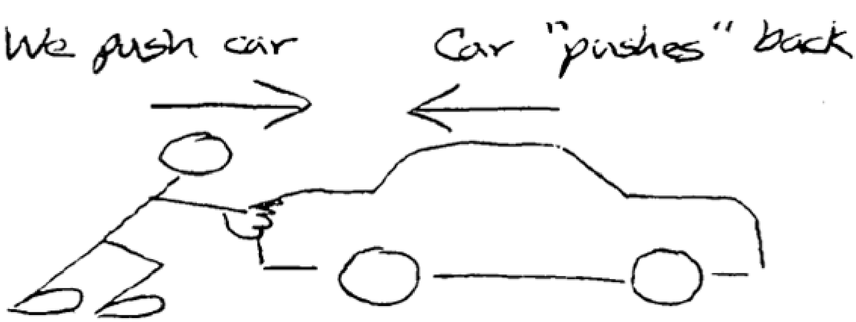


Figure 1.12. Reaction force is equal and opposite (in direction) to the applied force.

Notice the qualifiers placed on these laws. To effect a change in motion, more is needed than just the application of force: if two people push equally hard on opposite ends of a car, there certainly is force present, but we don't get any effect from it because the forces cancel each other. Only the net force, that which remains after the individual forces have been combined, can influence the motion. Also, sitting inside an automobile and pushing on the interior obviously does not move the car. We can get temporary motions by jumping to and fro inside, but the motion ceases when we stop. Only continuous forces can move a car steadily. This is the reason why Newton stipulated external forces in the Law of Inertia.

<u>The conservation laws</u>

Newton's Laws of Motion are invaluable because they allow us to predict the motion of an object when the forces are defined clearly enough to state in mathematical form. Sometimes, however, the forces in a certain process are not clearly known or are so complicated that it is almost impossible to write them concisely.

However, certain implications of Newton's laws lay the foundation for predicting the motion of complicated systems of objects. To see this, consider the Force Law, which states that the force on an object is equal in magnitude to the time rate of change of momentum. We can manipulate the mathematical form of the law by multiplying both sides of the defining equation by the small time interval dt :

$$Fdt = {}^{dp}/_{dt}\,dt\ ,$$

so

$$Fdt = dp$$

This tells us that the product of the numerical value of the force and the time over which it acts gives the corresponding change of momentum change of the object. For example, if we push our Honda by ourselves (small F) for a short period of time (small dt), its momentum changes only slightly (small dp). If our friends help us push (big F) for an extended time (large dt), we get a big increase in momentum (large dp), and someone better get in the driver's seat to control it!

Now suppose that the net external force acting on the object is zero. The implication is that the momentum change, dp, is also zero. In other words, the momentum is a constant, simply a restatement of the Inertia Law.

Next consider the case where the force operates on an object more complex than just a simple, single mass. The object may have many parts to it or may include several unconnected parts. If the net external force acting on this system of masses is zero, then the momentum of the system remains constant.

As a simple example, consider a collision of two billiard balls. If the rolling friction of the balls is small enough to disregard, there are no forces acting in the direction of motion of the balls as they roll toward the collision point. During the collision itself, the only forces acting are the ones from each ball pushing against the other. Newton's third law says that these forces are equal and opposite in direction, so the net force is zero. Since no net force is operating, the total momentum after the collision must be the same as it was before the collision. Momentum is conserved in the collision: the total momentum of the two billiard balls after the collision is the same as the total momentum before the collision.

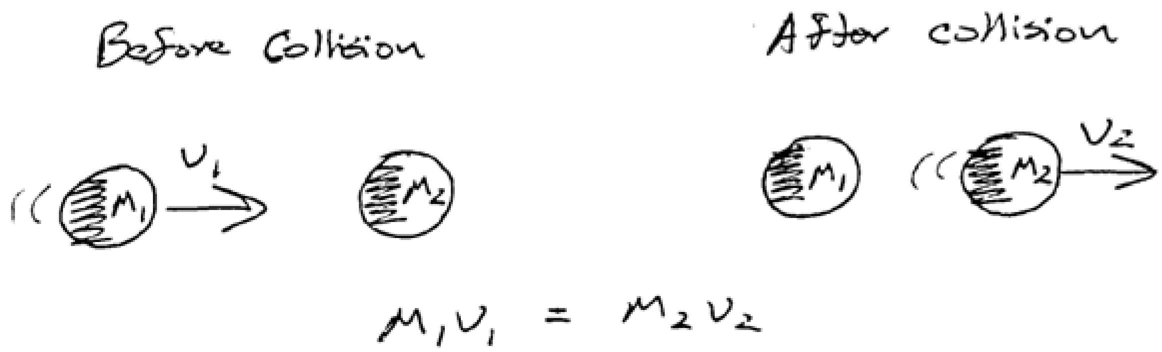


Figure 1.12. Conservation of momentum in collision of two billiard balls.

This conservation law gives us a powerful tool to determine the motion of objects after they collide or interact in some fashion. For example, consider two identical railroad boxcars, one of which is stationary and the other slowly rolling toward the first. They collide and their couplers connect them so they become a single entity with twice the mass of each boxcar individually. We can calculate the speed of the connected pair by equating the total momentum before the collision to the momentum after the collision:

$$mv = 2mV$$

where m is the mass of each car, v is the speed of the moving car before the collision, and V is the final speed of the coupled pair. Simple algebra tells us that

$$V = v/2$$

That is, the velocity of the coupled pair after the collision is half the original velocity of the single moving boxcar.

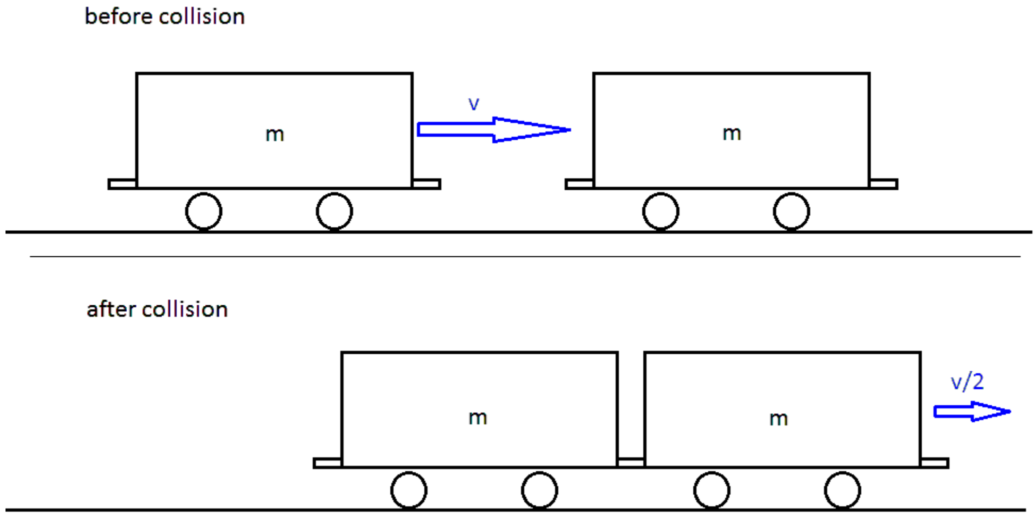


Figure 1.13. Conservation of momentum when boxcars couple. Initial momentum is mv, and final momentum of the coupled boxcars (with combined mass 2m) is still mv.

Notice that we have calculated all this without knowing the magnitude of the force in the couplers between the cars during the collision. This illustrates the power of the conservation laws: we do not need to know the details of the interactions between objects to predict their behavior after an interaction. We simply need to know the values of the appropriate conserved quantities before the interaction.

Another powerful analytical tool is provided by the Law of Conservation of Energy. In the same way that momentum is generated by the application of a force over an interval of time, a quantity called energy is transferred when a force acts on an object as it moves over a distance (an interval of space).

Energy appears in various forms. For example, if the force is exerted on an object which is free to move, such as when a hockey player slaps a puck on smooth ice, then the energy shows up as kinetic energy, i.e. energy of motion: the puck zips across the ice. Kinetic energy is described by the equation

$$K = \frac{1}{2}mv^2$$

where K is the kinetic energy of a mass, m, moving at a velocity, v.

Another form in which energy may appear is potential energy, or energy of position. If you exert a force to lift an object, such as lifting a bag of flour up onto a shelf, then the resulting potential energy is

$$U = mgh$$

where U is the potential energy of an object with mass m raised to height h above its original position. g is the value of the free fall acceleration of any object dropped near the Earth's surface (9.8 meters/sec/sec).

Many other forms of energy are possible -- such as chemical energy, nuclear energy, heat, and electrical energy -- but most are variations of kinetic or potential energy.

The concept of conservation of energy is useful when we consider motions within a system of objects isolated from external forces. If all the forces are internal to the system, Newton's third law tells us that the forces form opposing pairs, and therefore the sum of all the products of force and displacement within the system must be zero. Hence no new energy is generated, and the total energy is conserved. The amazing part of all this is that Nature guarantees this conservation, and we can use that guarantee to predict motion after an interaction even though we cannot fully describe all the internal forces involved.

For example a swinging pendulum conserves energy: the pendulum converts energy from potential energy (energy of position) to kinetic energy (energy of motion) back to potential energy, and so on. (Ideally, we would suspend the pendulum in a vacuum in an insulated box on a friction-free pivot, to avoid any transfer of energy.) The total energy of the pendulum, potential plus kinetic, remains constant.

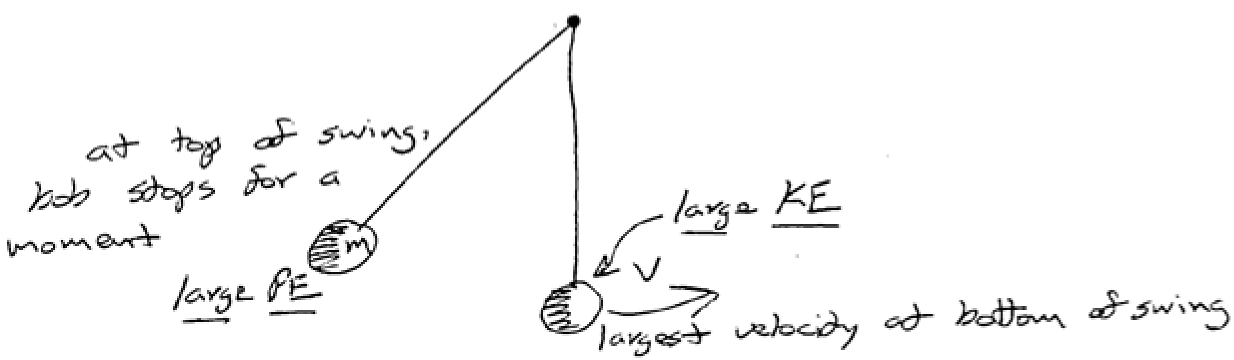


Figure 1.14. Pendulum transforms energy back and forth between kinetic energy and potential energy.

By extrapolation from the law of conservation of energy, the total energy of the Universe is constant, since the Universe is a "closed box" with nothing outside.

Keeping track of energy can be difficult, and the conservation of energy in processes like chemical reactions or during the interaction between two subatomic particles may not be obvious. For example, energy flows in various forms from the sun through the biosphere: nuclear reactions in the core of the sun convert mass (which, as we shall see, is itself a form of energy) into radiant energy (light). The light escapes into space, and some of it reaches Earth. Some of the light strikes water molecules in the atmosphere and sets them vibrating (mechanical energy), while some of the light traverses the atmosphere and is absorbed by chlorophyll in green plants, initiating a series of chemical reactions that ultimately produce glucose (stored chemical energy). A cow eats the grass and converts the chemical energy of glucose into energy of motion (mechanical energy), using the energy from glucose for muscle contraction to walk to another patch of grass. Throughout this pathway, and not forgetting the energy lost as heat (as when, e.g. heat from the cow's muscles is transferred to random molecular motion in the air), the total amount of energy distributed in the system remains constant.

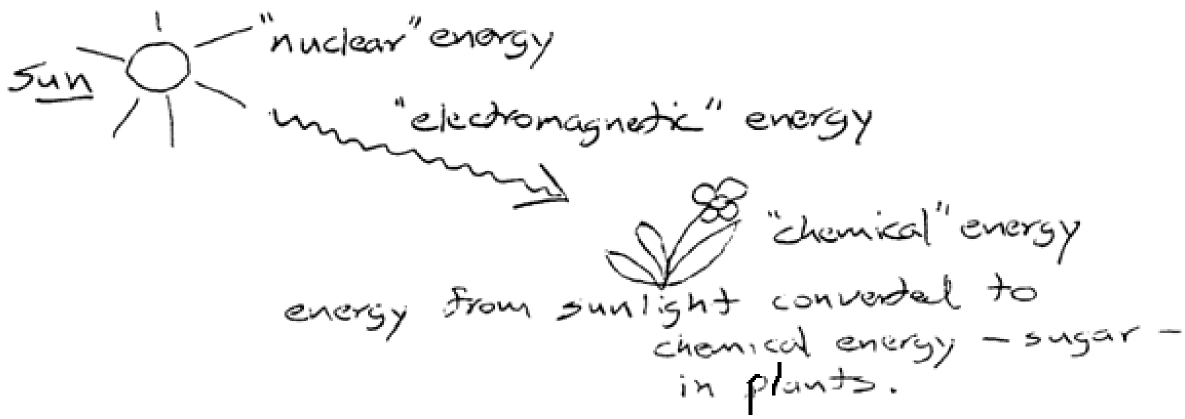


Figure 1.15. Energy transformations from the sun into the biosphere.

A third conservation law (which will prove quite useful, especially when we consider quantum mechanics) is the law of Conservation of Angular Momentum. This law obtains when one considers forces applied in such a way as to produce a "twisting" or spinning motion in an object. An important quantity which appears in such systems is angular momentum. In its simplest form, angular momentum can be defined as

$$L = mvr$$

where L is the angular momentum of a mass m moving with instantaneous velocity v on a circular path of radius r. This equation would apply directly to the case of the steady circular motion of a ball being whirled around on a string.

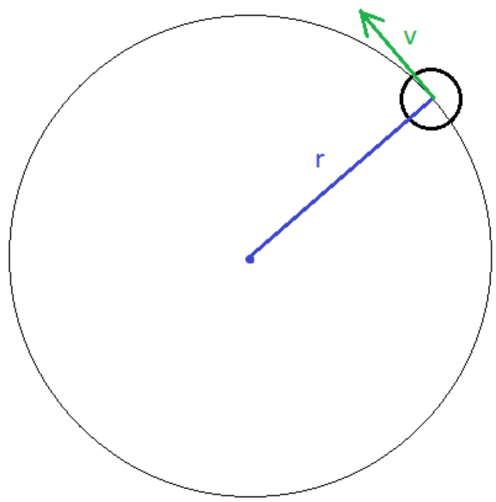


Figure 1.16. Ball with mass m on a string twirled in a circle of radius r with instantaneous velocity v has angular momentum $L = mvr$.

When one considers an object, or system of objects, in which no external "twisting" forces are applied, it turns out that the total angular momentum, or "spin," of the system is conserved. For example, if two spinning tops collide, the angular momentum of the two tops after the collision equals their combined angular momentum before the collision. Or if an ice skater starts spinning with extended arms, then pulls her arms in toward her torso, her rate of spin increases such that angular momentum is conserved.

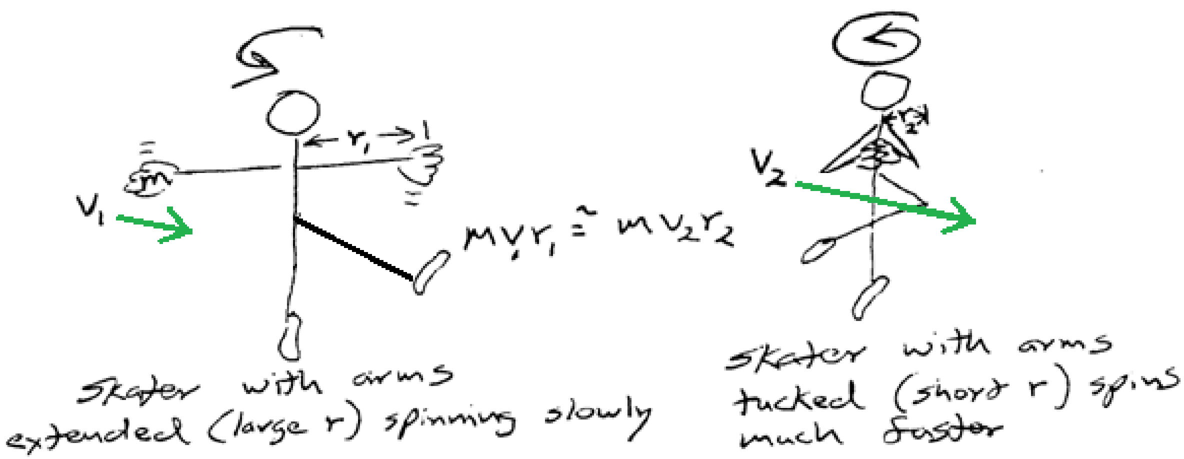


Figure 1.17. Rate of spin increases when spin radius decreases (instantaneous velocity v_2 greater than v_1).

These conservation laws, conservation of momentum, conservation of energy, and conservation of angular momentum, are among the most powerful ideas of physics. They apply at all scales, from the scale of subatomic particles to the scale of galaxies, and physicists use them to predict how Nature, in all her guises and all her realms, will behave.

Friction

"Wait just a minute," you say. "From my own experience, I can see that momentum is not conserved. If I roll a ball along the floor, it eventually slows down and stops: its momentum decreases. And if I spin a top, it, too, eventually stops: angular momentum is not conserved either, evidently."

So it might seem at first glance, but interactions occur at scales we cannot see, and we must consider all scales while keeping track of quantities such as momentum and angular momentum.

As the ball rolls across the floor, atoms on the surface of the ball bump into atoms in the floor and set the floor atoms jiggling: the ball's atoms transfer momentum to the floor atoms, hence to the Earth itself. The ball also bumps into air molecules and transfers momentum to them. If we add the momenta of all the jiggling atoms in the floor, the change in the Earth's momentum, and the momentum of the air molecules after the ball stops, the sum equals the original momentum of the ball.

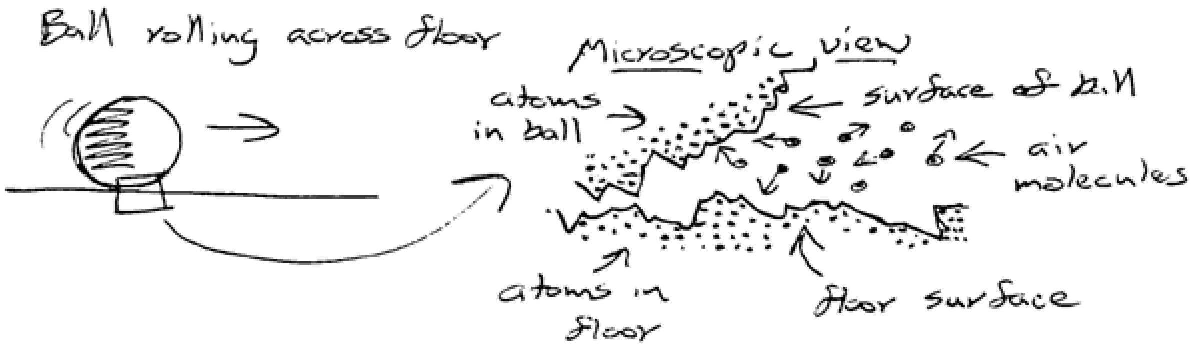


Figure 1.18. Microscopic view of the microscopic momentum exchanges we call friction.

Similarly, a spinning top transfers angular momentum to atoms in the floor (and by extension, to the Earth) and to atoms in the air. The book-keeping in such complex interactions, involving untold millions of individual atoms, is the realm of a particular discipline in physics called thermodynamics.

Newton's law of gravitation

One of Newton's most impressive achievements was his discovery of the mathematical form of the law of gravity. He based his discovery on the remarkable insight that gravity was a universal force and that it operated the same on objects near the earth as it did on celestial objects. From observations on the moon's orbit compared to (as legend has it) falling apples, he deduced the Law of Universal Gravitation.

Newton proposed that the force which made an apple fall to the ground and the force which drew the moon into a circular orbit around the earth were one and the same. For the apple, the acceleration is easily observed and measured, its value being about 9.8 meters per second per second. That is, the apple's speed increases by 9.8 meters/second for each second it falls.

That the moon must be continually accelerating toward earth is evident when one realizes that it would simply move off in a straight line otherwise. To effect the observed circular path of its orbit, the moon must be given a component of velocity at right angles to its instantaneous momentum, i.e. it must accelerate toward earth, pulled by some force. In this regard, we can liken the moon's orbit to a ball on a string: the spinning ball is held in its circular path by the string. Let go the string, and the ball flies off along the straight line path given by the instantaneous velocity at the instant it was released.

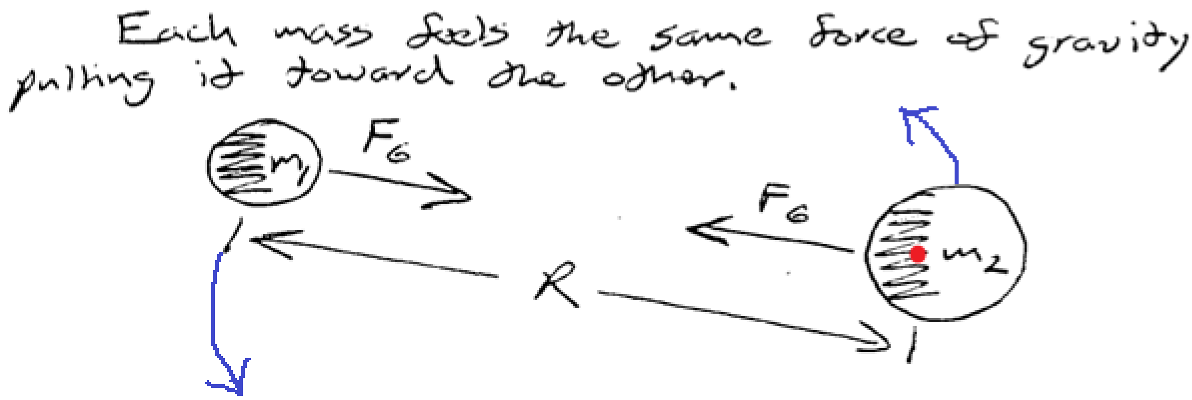


Figure 1.19. Earth and moon orbit a common center of mass (red dot under earth's surface), held by their mutual gravitational attraction.

The moon's earthward acceleration can be computed from its orbital distance and speed. Each second, the moon moves about one kilometer along its inertial path, and it must fall toward earth about 2.7 millimeters to maintain its (nearly) circular orbit. This results in an acceleration of 0.0027 meters/second/second.

The ratio of the acceleration of the apple to that of the moon is about 3600:1. Newton observed that if one compares the distance from the moon to the center of the earth to the distance from the apple to the center of the earth, one finds these values to be in a 60:1 ratio. Since the square of 60 is 3600, the implication is that the strength of gravity falls off with the square of the distance from the center of the earth.

Newton could also explain why apples of all sizes fall at the same rate. He assumed that the force of gravity on an apple is simply proportional to its mass. But <u>inertia</u>, the resistance to change in momentum, is also proportional to mass. So earth's mass pulls on a large apple with greater gravitational force than on a small apple, but the large apple has a greater inertia which counteracts the greater gravitational force, so the two apples fall at the same acceleration.

From these considerations about the effects of distance and mass on the force of gravity, Newton concluded that the gravitational force law must be of the form

$$F = G\, mM/r^2$$

where F is the force of gravity, M is the mass of the earth, m is the mass of the object being attracted, r is the distance from the object to the center of the earth and G is a constant of proportionality which produces the correct numerical value of F in a given system of units (e.g. meters, seconds, and kilograms). This mathematical form may be generalized to any two masses. Newton considered gravity to be universal, i.e. he assumed there is a gravitational force between any two objects, not just between an object and the earth. One could let M and m represent the masses of a bowling ball and a baseball, or two neutron stars orbiting each other, or a water molecule and an oxygen molecule in the air, or any two masses, and the formula would give the gravitational force between them. Note that the force of gravity is always attractive: each object experiences a force in the direction of the other.

It is evident that the gravitational law is consistent with Newton's Third Law of motion. The force on each object has the same numerical value but they act in opposite directions. The dependence on distance is also interesting. If one doubles the distance between the objects, the force drops to one quarter of its original value. Gravity weakens very quickly as objects are separated. Since the planet Jupiter is five times further from the Sun than the planet Earth is, it falls toward the Sun only 1/25 as much as the earth in a given time interval.

The value of the gravitational constant, G, was found experimentally by the English scientist Henry Cavendish and was published in 1798. In the system of units used most frequently by modern physicists (length in meters, mass in kilograms and time in seconds) its value is approximately

$$G = 6.67 \times 10^{-11}\ \text{N-m}^2/\text{kg}^2$$

The resulting force is expressed in terms of a unit called the "newtons"; one newton is equivalent to about 3 ounces, which appropriately enough is the weight of a small apple!

The small value of G reflects the fact that the gravitational force between ordinary objects is extremely small. For example, the attractive force between two people standing one meter apart is about one millionth of a newton. Scientists believe the value of G is the same throughout the universe, and it can be used to solve many practical problems. For instance, by knowing the size of the earth's yearly orbit around the sun, one can use the gravitational formula to find the mass of the sun (about 2×10^{30} kg). And, by measuring the acceleration of a falling apple and the radius of the earth, we can find the mass of the earth (about 6×10^{24} kg). We can weigh the earth by measuring a falling apple!

Newton's law of gravitation, applied to celestial navigation, enabled man to set foot on the moon and to probe the distant planets. It is remarkably accurate on the familiar scale of our daily existence – gravity holds us to the surface of Earth – and the scale of the solar system. Only near exceptionally massive objects, on the order of the Sun's mass and greater, does the law falter. Around white dwarfs, neutron stars, black holes, and other immense masses, the equations of General Relativity (Einstein's theory of gravity) prove more accurate.

Newton’s Universe

Note the nature of the Universe assumed in these laws: if something changes, it changes because a force was applied. If something moves, it must have been pushed or pulled.

Newton's is a cause-and-effect Universe. Events are caused to happen, and nothing happens without cause. There is no randomness or uncertainty. Hypothetically, if we were clever enough to determine the motion of all particles everywhere in the Universe, we could deduce the past and predict the future completely. (In practice, it is not possible to measure the motion of even a few objects to arbitrary accuracy. Predicting the positions of the planets precisely over a few millions of years is an intractable problem.)

Furthermore, Newton's Universe exists whether or not we are here to observe it. It ticks along, effect following cause producing new effect, millennium to millennium. No matter Who or What designed the Universe originally, it runs on its own (according to Newton), following Newton's laws.

Newton's laws were a triumph for physics. For the first time, physicists had a logical system that explained phenomena at all the scales then accessible to measurement. With Newton's laws physicists could plot the trajectories of falling apples and the paths of the planets.

One remarkable demonstration of the power of Newton's laws was the discovery of the planet Neptune. Astronomers knew Uranus deviated in its orbit from the path of a perfect ellipse. They could account for part of the deviation due to gravitational effects of Jupiter and Saturn, but there remained an unexplained wandering. Astronomers guessed that the discrepancy resulted from the gravitational tug of a more distant planet, and Newton's laws enabled them to calculate the mass and position of the new planet. With their calculations (and a bit of luck), they pointed their telescopes to the predicted sector of the sky and, sure enough, there was Neptune!

Faraday’s fields

Newton realized his laws were incomplete. He could not quite bring himself to accept the “action-at-a-distance” implied by his law of gravitation: how could one mass pull on another across apparently empty space?

Physicists studying electricity and magnetism found a tentative answer. They noticed (and navigators among the Vikings and Chinese had probably noticed long before) that one magnet can accelerate another without physically touching it. There is some property about a magnet that can reach across space and affect another. That property is called the magnetic field.

As children, many of us discovered that magnets can push and pull at each other even while separated by several inches. Some magnets can repel each other with surprisingly strong force and are extremely difficult to push together.

We can visualize the magnetic field with the help of iron filings. Place a magnet on a table and cover it with a sheet of paper. Then sprinkle iron filings onto the paper. Gently tap the paper to distribute the filings evenly. The filings trace the lines of the magnetic field permeating space around the magnet. Note that the magnetic field requires two poles for its existence, as if it was some sort of fluid emanating from the North pole and returning to the magnet at the South pole. No isolated monopoles (one pole without the other) has (yet) been detected.

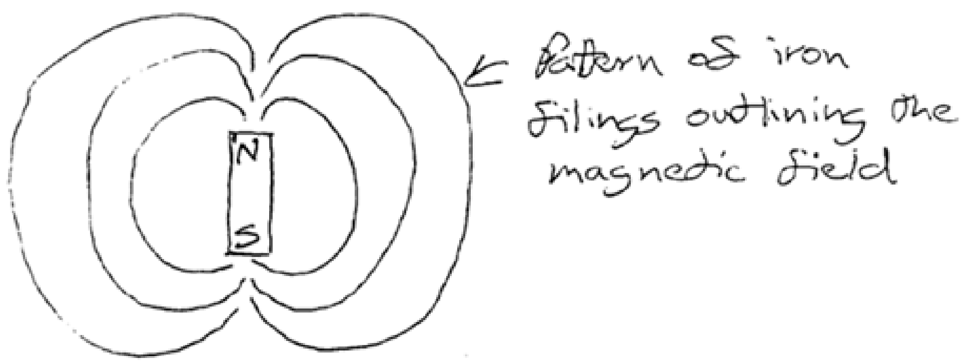


Figure 1.20. Pattern of iron filings sprinkled around a bar magnet.

The field itself transfers momentum and energy: when we push the north pole of one magnet toward the north pole of another, the one magnet accelerates the other without touching it directly. Force is transmitted through the fields, one field “pushing” against the other.

Similarly, one electric charge can accelerate another charge without physically touching it. The force between electric charges is mediated by the electric field.

We can demonstrate the existence of the electric field simply: Rub a plastic pen vigorously with a cotton cloth. (The cloth strips electrons from some of the atoms in the pen, leaving the pen with a net positive charge.) Bring the pen near to small bits of torn paper. The bits of paper will be attracted to the pen, some so strongly that they jump to the pen and stick to it. As with magnetism, you see evidence of a force transmitted across space even though there is no direct contact between the pen and paper.

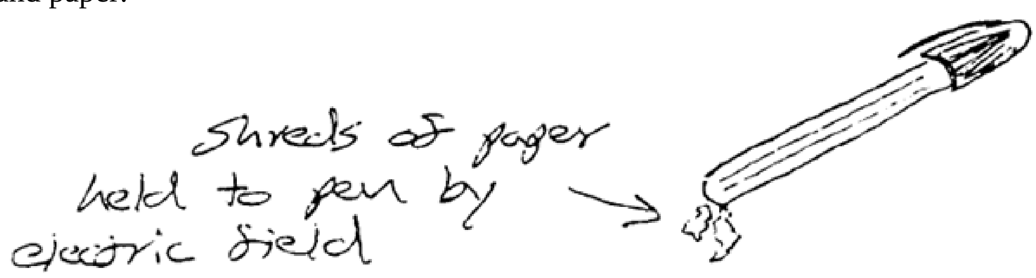


Figure 1.21. Static electricity attracts bits of paper to pen.

It is possible to outline the electric field with more sophisticated apparatus. It differs from the magnetic field in that electric field lines radiate away from isolated positive charges, as if the positive charge was the source of the field (like a water sprinkler). The electric field converges toward an isolated negative charge, as if the negative charge was a drain for the field.

The field itself transfers momentum and energy: when we push the north pole of one magnet toward the north pole of another, the one magnet accelerates the other without touching it directly. Force is transmitted through the fields, one field "pushing" against the other.

Extrapolating from their knowledge of electric and magnetic fields, physicists hypothesize the existence of a gravitational field and also fields associated with the strong and weak nuclear forces. The forces of nature are transmitted across "empty" space by the fields. Planets pull one another through the mediation of the gravitational field. Electrons (each with a negative electric charge) push one another through their electric fields.

We sense that "empty" space really is not so empty. It is permeated by the fields.

To summarize, field theory provides a model which resolves the problem of "action-at-a-distance." Masses have an associated gravitational field, which permeates surrounding space and affects other masses. Electric charges have an associated electric field which affects other charges. The fields mediate the forces of nature: they transmit energy and momentum from one object to another.

<u>Self-propagating fields</u>

The next great discovery was that the fields could be self-propagating and, therefore, might have an independent existence distinct from their sources. This realization followed from experiments performed by Andre Ampere and Michael Faraday and was proved by James Clerk Maxwell. Ampere found that an electric current (which we know today is the flow of electrons through a metal wire) produces a magnetic field. Faraday, on the other hand, discovered that a magnet could induce an electric current in a wire.

We can demonstrate these phenomena with the following apparatus: To show how a magnetic field generates an electric current, coil a copper wire around a cylinder, such as the cardboard tube in a toilet paper roll. The more wraps of wire, the more obvious will be the effect. Attach the two ends of the wire to the current indicator on a galvanometer. Now move a bar magnet back and forth through the core of the coil. Moving the magnet through the coil creates an electric current, as indicated on the galvanometer. Apparently, the magnetic field reaches out a pushes electrons through the wire.

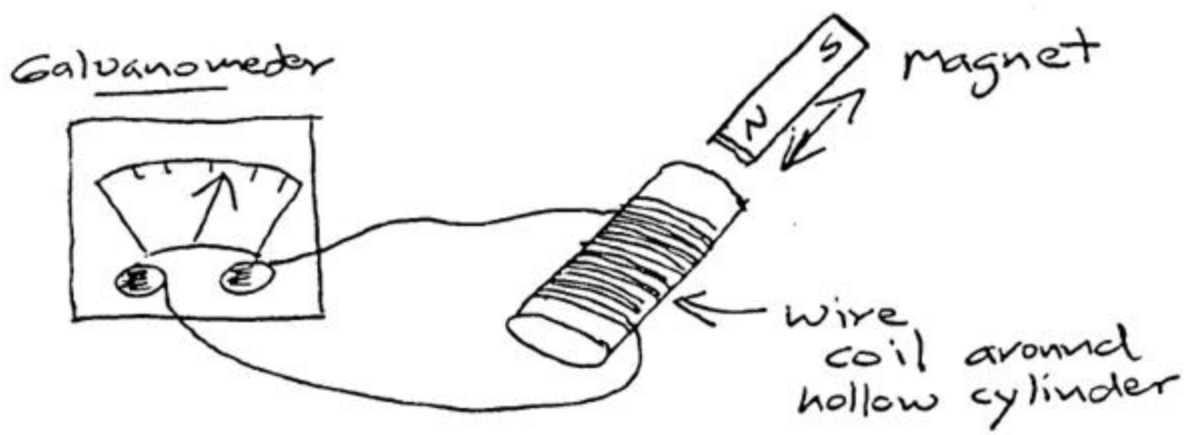


Figure 1.22. Magnetic induction: a moving magnet generates an electric current in the coil.

To demonstrate the creation a magnetic field by an electric current, wrap copper wire around a nail or an otherwise non-magnetic iron rod. Again, the more wraps of wire, the more obvious will be the effect. Attach the two ends of the wire to a nine-volt battery. The current round and round through the coil of wire creates a magnetic field, and the nail will pick up metal objects just like an ordinary magnet. (In fact, same thing is happening in ordinary magnets but at a much smaller scale: the magnetic field results from millions of electrons orbiting round and round their nuclei.)

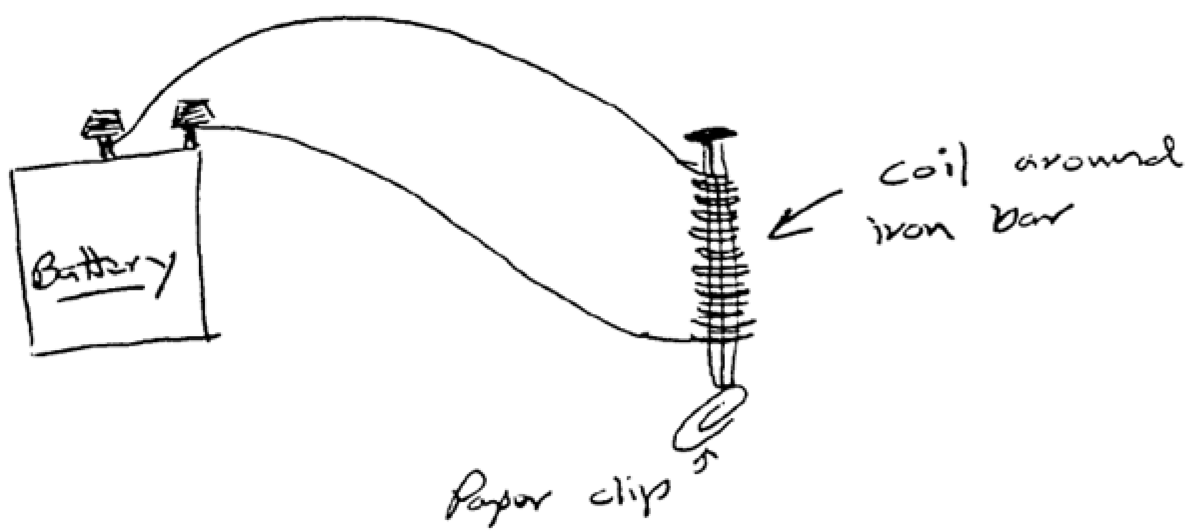


Figure 1.23. A simple electromagnet demonstrates the origin of the magnetic field from electrons in circular motion around the coils of wire.

As these two demonstrations intimate, moving a magnet (changing the local magnetic field) creates an electric field (which accelerates electrons down the wire in the first of the demonstrations above), and moving electrons (changing the local electric field) creates a magnetic field(as in the second demonstration). Maxwell proved this must be so: a changing magnetic field creates an electric field, which induces a magnetic field, which creates an electric field, and so on.

The result is a self-propagating wave – an electromagnetic wave. This is the mechanism that produces visible light, radio waves, X-rays, and the rest of the electromagnetic spectrum of radiation. The fields have an independent existence, and they can propagate from place to place.

<u>Contradictions in Newton's paradigm</u>

Such was the extraordinary contribution of Sir Isaac Newton: a self-contained model of the Universe, a model which successfully accounted for scientific observations available at his time, provided the basis for our thinking for the next two hundred years, and which even today plots trajectories of spacecraft to the moon, the planets, and beyond.

Yet the model breaks down. In the realms of the very fast, the very massive, and the very small, Newton's laws make predictions at odds with observation.

In fact, Newtonian physics contradicts itself on the theoretical level. On the one hand, Newton assumed that the laws of physics were the same for any observer in uniform relative motion (his Principle of Relativity) and, hence, that an observer traveling at uniform velocity cannot determine he is looking outside his frame of reference. On the other hand Newton's concept of absolute space and time implies a fixed frame of reference against which absolute motion <u>can</u> be measured, hence that an observer can determine his state of motion <u>without</u> looking outside his frame of reference.

To see why this is so, recall our discussion of the Galilean transformation. Classical physics assumes that light travels at uniform velocity in absolute space and time. So a moving observer should be able to detect a change in the speed of light dependent on her own state of motion. Detecting a change in the speed of light would allow her to determine her absolute motion, in violation of the Principle of Relativity.

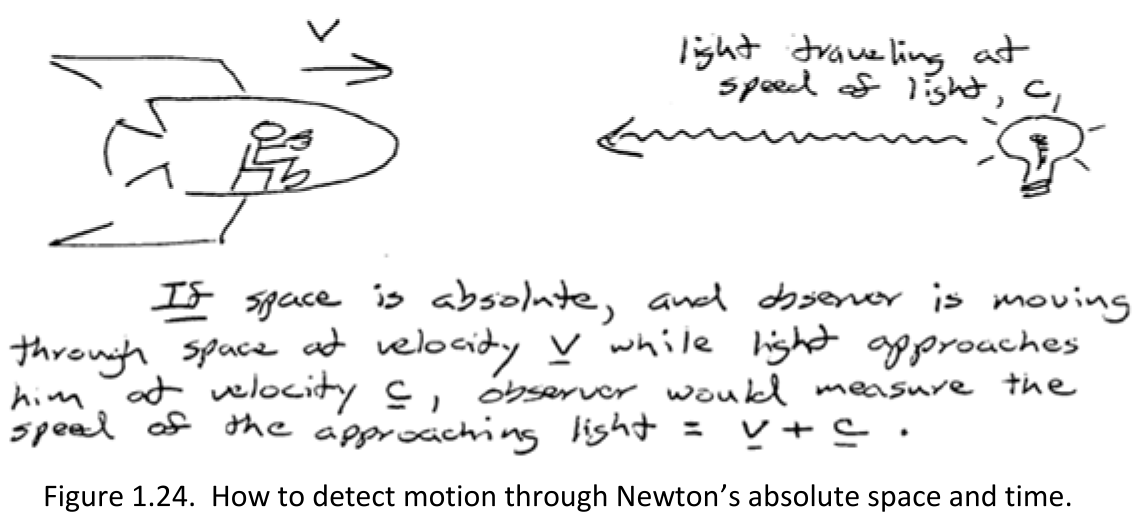


Figure 1.24. How to detect motion through Newton's absolute space and time.

Cracks in the mortar

By the end of the nineteenth century experimentalists had discovered other serious discrepancies between the predictions of Newton's laws and experimental fact. Most importantly, two American physicists, Albert Michelson and Edward Morley, designed an experiment to detect changes in the speed of light due to Earth's motion through absolute space. As we shall discuss in detail in the next chapter, their results shook the foundations of classical physics: they could detect no variation in the velocity of light due to Earth's motion.

Another blow to Newton's model came when astronomers discovered clocks in different parts of the solar system keep different times. Jupiter's moon, Io, orbits the planet like hands around a clock dial. Viewed at opposition (i.e. Jupiter "opposite" the sun in Earth's sky) Io disappears behind Jupiter with clockwork precision every 1.77 days. Viewed six months later, with Jupiter in conjunction (i.e. near to the sun in the sky) Io revolves with the same periodicity but is occulted by Jupiter seventeen minutes later than predicted by the observations at opposition. The "Io clock" changes according to the relative position of Jupiter and earth. (As we'll see, the variation results because of the finite speed of light: it takes roughly seventeen minutes for light to cross the diameter of Earth's orbit.)

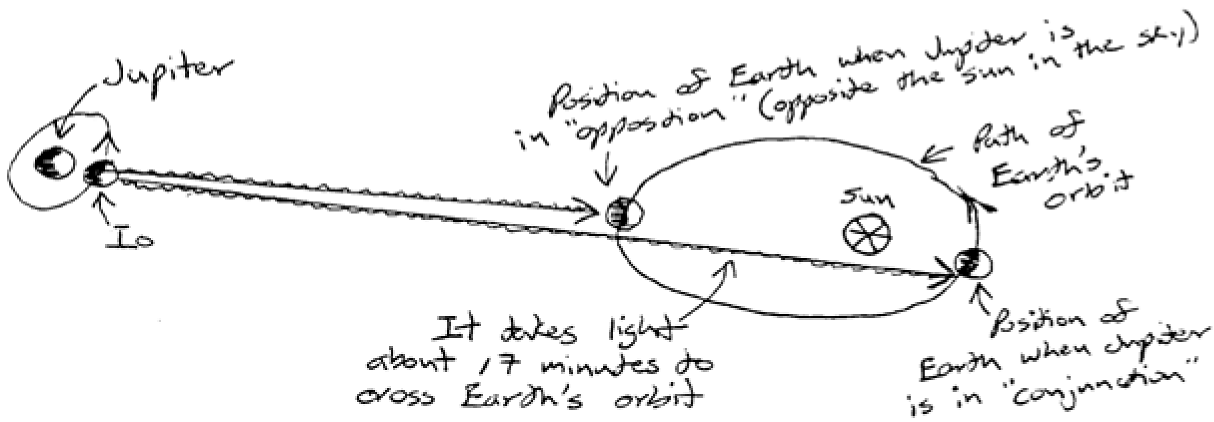


Figure 1.25. Relative positions of Earth, sun, and the Jupiter system as Earth orbits the sun.

No absolute space, no absolute time: physicists at the end of the nineteenth century began to suspect the Universe was a very slippery place, indeed. The accumulation of data contradicting Newton's theories demanded a reassessment of physical law. The man primarily responsible for that reassessment was Albert Einstein, and his work is the subject of the next two chapters.

Summary

Newton built a measurable, determinate model of the Universe. According to classical (Newtonian) physics:

- We can measure events precisely in time and space.
- The Galilean transformation accurately translates observations from one uniformly moving reference frame to another.

- Event follows cause. We can deduce the past and predict the future completely.
- The conservation laws allow us to predict the behavior of complicated physical systems.
- The force of gravity, mass pulling mass, applies at all scales in the Universe. It determines the fall of apples and the paths of planets.

Field theory shored up Newton's paradigm, explaining action at a distance. In fact, field theory provides a powerful tool even today for describing the forces of Nature.

Newton's is a wonderful model, and it works well at the scale of our direct observation, even sending spacecraft to the moon and planets. But it fails to account for the uniform speed of light, measured by all moving observers (as proved in the Michelson-Morley experiment), and it contradicts itself by predicting uniform laws of physics in an absolute frame of reference.

In the following chapter, we shall see how Einstein extended our understanding of physics at the scales of the extremely large and how quantum mechanics describes the indeterminate world of the very small.