Chapter 3
The Special Theory of Relativity

In this chapter we explore the special theory of relativity, in which Einstein reformulated the laws of physics in terms of the Principle of Relativity. The theory resolves the unexpected result of the Michelson-Morley experiment, but with some unexpected revelations: it turns out mass, length, and time can change, after all. We will consider Einstein's remarkable discovery of the relation between mass and energy, $E = mc^2$, and we describe some of the experimental evidence supporting the special theory. At first glance, relativity seems completely foreign, because we do not experience relativistic effects in our daily lives. But, as we shall see, relativity is crucial to understanding events at the atomic scale on the one extreme and at the scale of stars and galaxies on the other.

Possible explanations for the null result

There are a number of possible explanations for the null result of the Michelson-Morley experiment:

1. There is no ether. Light is self-propagating, and it requires no medium for its travel.
2. The Earth carries an envelope of ether with it, and this local ether is at rest with respect to the apparatus.
3. The ether exists, but its effects are too subtle to detect in this experiment.
4. Distances shorten along the direction of travel. Imagine, once again, our river ferry. If somehow the distance between A and C shortens as an effect of the current, the round trip travel time A-C-A could equal the travel time A-B-A.
5. Clocks slow in a system moving parallel to the ether current. If somehow the ferry clock ticked slower while the boat was moving parallel to the current, the clock might register only two hours on the trip A-C-A, the same as the time A-B-A.

Which of these explanations, if any, is correct?

Special Relativity

In his derivation of the special theory of relativity, Einstein had a larger purpose in mind than explaining the results of the Michelson-Morley experiment. He sought a firmer foundation for physical law. Specifically, he wanted to resolve the contradiction between Newton's absolute space and time and the Principle of Relativity.

In effect, he did so by fiat. As the crux of his argument, he made two assumptions that swept aside the dilemma. These assumptions have since been verified experimentally (the second, in fact, by the Michelson-Morley experiment), but the remarkable edifice of relativity rested initially on Einstein's insight that they must be true:

1. The laws of physics are the same for all observers in inertial frames of reference. That is, an observer moving at uniform velocity will find the same laws of Nature as any other un-accelerated observer no matter what their relative velocities. One physicist could be

traveling toward Polaris the north star) at 80% the speed of light, another away from Polaris at 50% the speed of light, and both would obtain the same results from the same experiments.
2. The speed of light is the same for any observer in an inertial frame of reference (i.e. moving at constant velocity).

Reasoning from his two premises, Einstein derived the following conclusions, and in the process explained the results of Michelson-Morley. Here is a summary:

1. Clocks slow as velocity increases. As seen by an outside observer,

$$t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$$

where *t*' is the time interval (the interval between ticks) in a moving clock as seen by an observer at relative rest outside the moving system, *t* represents time on the observer's own (stationary) clock, *v* is the velocity of the moving system relative to the stationary observer, and *c* is the speed of light (about 300,000 km/sec).

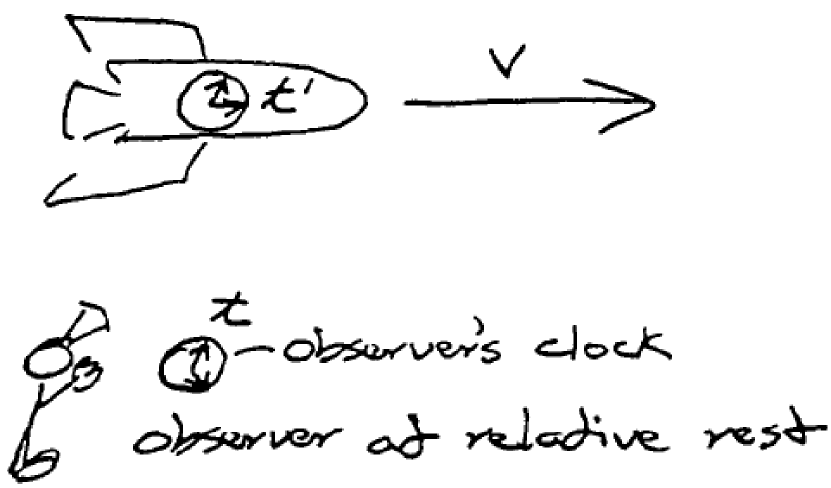

Figure 3.1. Moving clocks tick slower as seen by an observer moving at a different constant velocity (most practically considered “at rest” relative to the moving clock).

2. As seen by an outside observer, length parallel to the direction of motion decreases as velocity increases.

$$l' = l\sqrt{1-\frac{v^2}{c^2}}$$

where l' is the length of an object (e.g. meter stick) as seen by an observer at relative rest outside the moving system , l represents length of the observer's own meter stick, v is the velocity of the moving system relative to the stationary observer, and c is the speed of light.

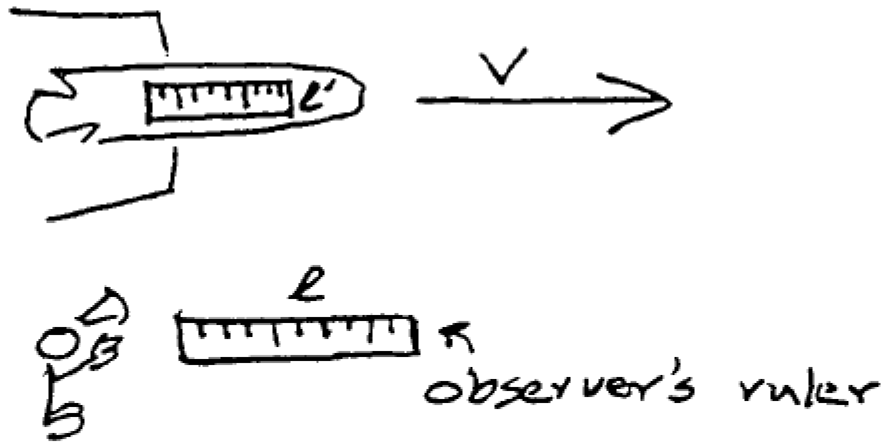

Figure 3.2. Length of a moving object shortens along the direction of motion, as seen by an outside observer at rest.

3. As seen by an outside observer, mass increases as velocity increases.

$$m' = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}$$

where m' is the moving mass as measured by the stationary outside observer, and m is the observer's fiduciary mass (i.e. the reference mass at rest with the observer).

4. Energy and mass are equivalent, and they are inter-convertible. They are related through the conversion factor c^2 (so there's an awful lot of energy in a little mass).

$$E = mc^2$$

These relationships between physical parameters of objects in motion must be true if the laws of physics and the speed of light are invariant in inertial reference frames traveling (at constant velocity, by definition) relative to each other. It seems strange that clocks slow, length decreases, and mass increases at high velocity, but that's exactly what happens. Without those compensations in time, length, and mass, the laws of physics would vary from one observer to another.

The Lorentz transformation

The basic equations of special relativity translate measurements from one frame of reference to another. *t*', *l*' and *m*' in the equations above represent the time, length, and mass in a moving system as seen by an observer at rest outside that system. The observed time, length, and mass, as seen by the outside observer, differ from her own, rest time, length, and mass, according to the equations above.

These mathematical transformations (called the Lorentz transformations) are similar to the (Galilean) transformations of Newtonian relativity, cited in the previous chapter, but the equations of special relativity demonstrate the subtle effect of velocity on our measurements. At low velocities, time, length, and mass do not differ detectably from the predictions of Newtonian physics. As velocities approach the speed of light, however, observed time, length, and mass depart radically from the Newtonian predictions.

Relative velocity ($^v/_c$)	t' **(sec)**	m' **(gm)**	l' **(cm)**
0	1	1	1
0.1	1.005	1.005	0.995
0.5	1.155	1.155	0.866
0.9	2.294	2.294	0.436
0.999	22.36	22.36	0.045

Table 3.1. Changes in clocks, meter sticks, and mass as velocity approaches the speed of light.

Now let's explore, in turn, each of these relativistic effects and show how they follow from Einstein's assumptions.

Time dilation

Following in Einstein's footsteps, let's perform a thought experiment. We build a light clock: reflect a beam of light back and forth between two mirrors. Each "bounce" of the light beam off a mirror equals one "tick" of the clock. The mirrors reflect perfectly, and the distance between them is fixed, perpendicular to the direction of motion.

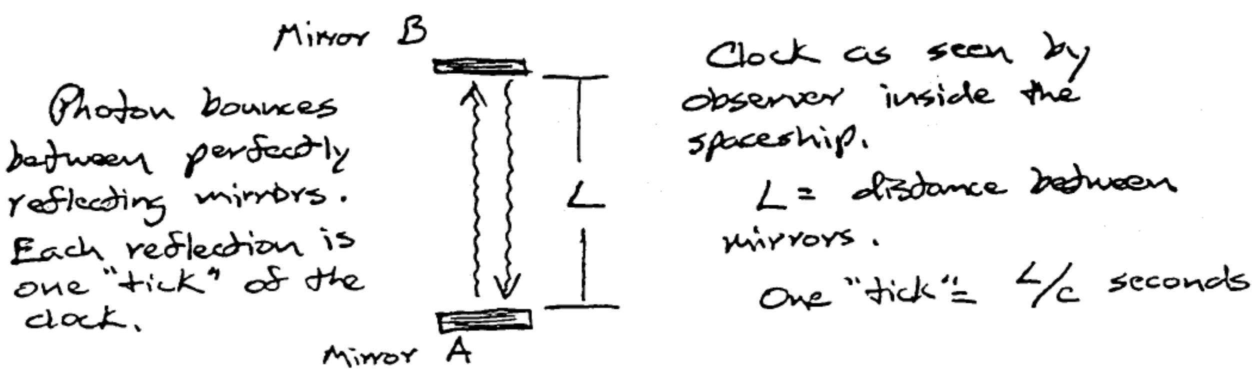

Figure 3.3. Light clock at rest. One "tick" of the clock is $L/_c$ seconds, as calculated by solving for t in the definition of velocity (in this case the velocity of light) $c = L/_t$.

We carry such a clock on a spaceship voyage between the galaxies. It records time faithfully as we rocket toward our destination, just as it did on Earth while we prepared for our trek. Same clock, same time.

Yet, to a stationary observer outside the spacecraft, the clock appears to run slower at high velocity: as seen from outside, the light beam appears to travel farther between reflections because the mirrors are moving. Since the speed of light in a vacuum is the same for any observer (as proved by the Michelson-Morley experiment), the larger the distance a light beam travels between "ticks" the longer the time interval.

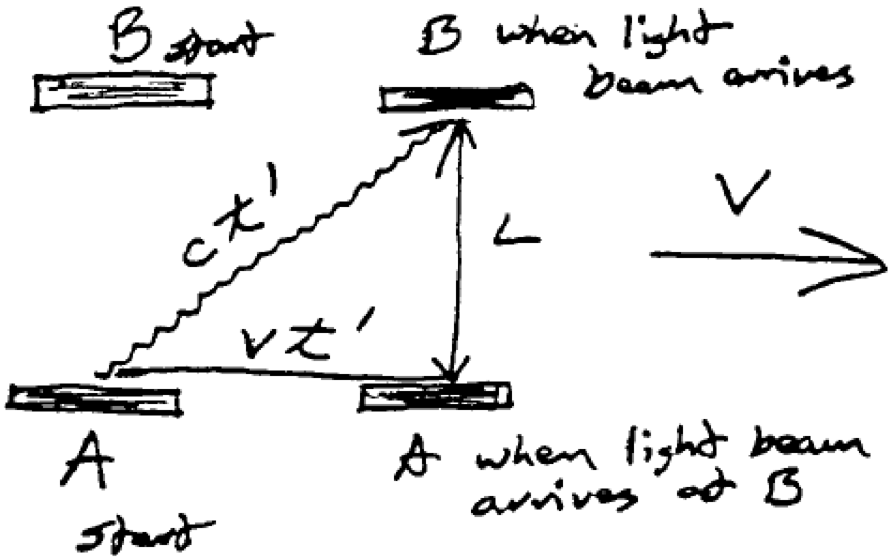

Figure 3.4. Light path in a moving clock, as seen by an outside observer, follows a longer path than light in the clock at rest.

We can find t' , the time interval on the moving clock, using the Pythagorean theorem.

$$L^2 + (vt')^2 = (ct')^2$$

$$t'^2(c^2 - v^2) = L^2$$

$$t'^2 = \frac{L^2}{(c^2 - v^2)}$$

$$t' = \frac{L}{\sqrt{c^2 - v^2}}$$

As seen by us, inside the spaceship, one "tick" of the clock takes L/c seconds. As seen by an outside observer, the light-travel time between mirrors, one tick, is

$$t' = \frac{L}{\sqrt{c^2 - v^2}} = \frac{L/c}{\sqrt{1 - v^2/c^2}} = \frac{t}{\sqrt{1 - v^2/c^2}}$$

Each "tick" recorded by an outside observer takes $\frac{1}{\sqrt{1 - v^2/c^2}}$ times as long as a "tick" seen by us, inside the spacecraft. As our velocity relative to the outside observer approaches the speed of light, the time interval between "ticks" gets longer, and longer, and longer until at the speed of light itself, the clock doesn't "tick" at all. Time stops.

Light, of course, travels at the speed of light (in a vacuum), and clocks on light beams tick not at all. Consider a beam of light produced at the instant of a supernova explosion. It travels across light-years of spacetime, and all along the way it retains information about the supernova, the record of one instant of time. When the light finally reaches our telescope, we "see" the supernova as if it is happening right "now."

Matter cannot reach the speed of light. The time constraint described above and the mass/energy constraint, to be discussed below, prohibit speed-of-light or superluminal travel.

<u>All clocks must slow, even biological clocks</u>

If a light clock slows at high velocity, as seen by an outside observer, all clocks on the spaceship must slow proportionally. Even biological processes, such as aging, must slow.

Why is this?

If the aging process or any other clock responded to velocity in a way different from the light clock, two observers could use the difference in clock rates to determine who is moving, who's at rest. This would violate the fundamental premise of special relativity, that the laws of physics appear the same to all observers in inertial reference frames and that there is no preferred (absolute) frame of reference.

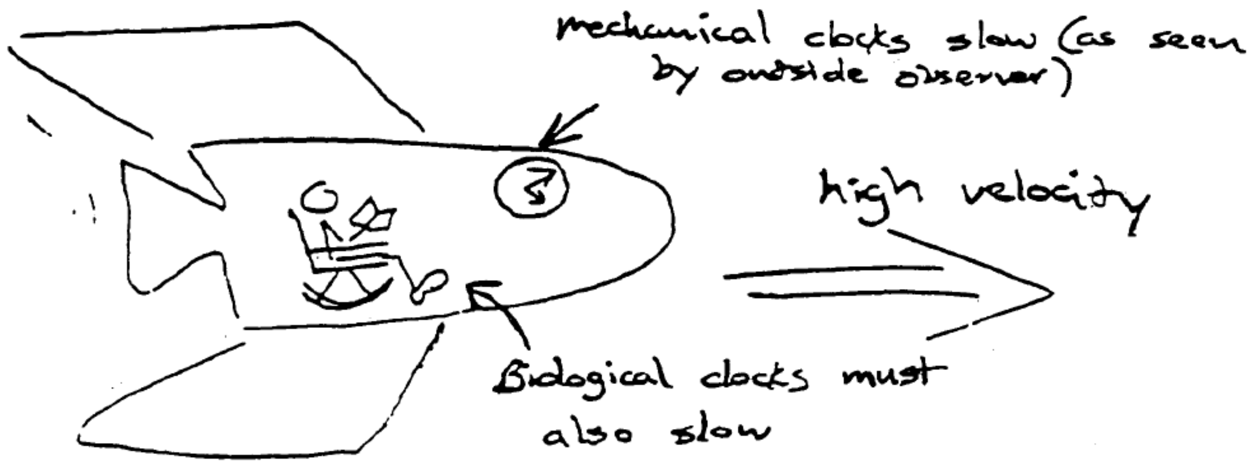

Figure 3.5. Biological clocks must slow same as other clocks, or an observer could determine who's moving, who's not.

LENGTH CONTRACTION

As we mentioned, one possible explanation for the null result of the Michelson-Morley experiment is that length contracts along the direction of travel. Einstein showed that length in a moving frame does indeed contract along the direction of travel as seen by an outside observer.

Michelson-Morley proved that the light travel time A-B-A equals the light travel time A-C-A (mirrors A and B oriented perpendicular to the direction of travel, A and C parallel to the direction of travel.)

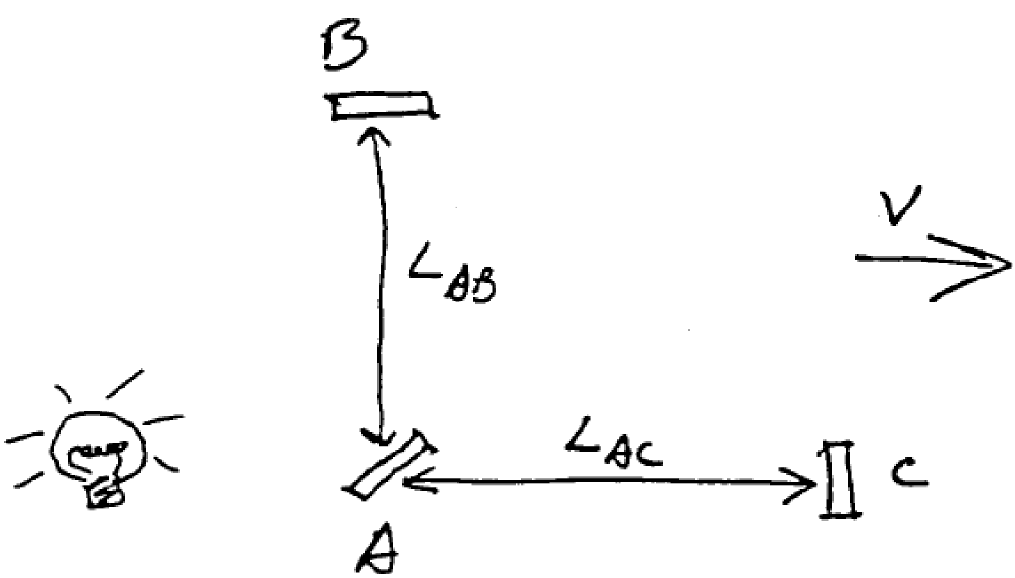

Figure 3.6. Light travel times are the same along the two different paths in the Michelson-Morley apparatus.

We've already calculated the light-travel times as seen by an outside observer:

$$t_{AB} = \frac{L_{AB}/c}{\sqrt{1 - v^2/c^2}}$$

$$t_{AC} = \frac{L_{AC}/c}{\left(1 - v^2/c^2\right)}$$

where t_{AB} and L_{AB} represent time and length measured along A-B, and t_{AC} and L_{AC} represent time and length along A-C.

We know that t_{AB} equals t_{AC} (the experiment proved this). Therefore:

$$\frac{L_{AB}/c}{\sqrt{1 - v^2/c^2}} = \frac{L_{AC}/c}{\left(1 - v^2/c^2\right)}$$

Solving for the relationship between L_{AB} (the length perpendicular to the direction of motion) and L_{AC} (the length parallel to the direction of motion):

$$\frac{L_{AB}}{L_{AC}} = \frac{\sqrt{1 - v^2/c^2}}{\left(1 - v^2/c^2\right)} = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$L_{AC} = L_{AB}\sqrt{1 - v^2/c^2}$$

That is, as seen by an outside observer, the length L_{AC} parallel to the direction of motion decreases by the factor $\sqrt{1 - v^2/c^2}$ as velocity increases. At high velocities, as seen by an outside observer at relative rest, rulers shorten along the direction of travel.

Relativistic increase in mass[1]

Funny business, this relativity! Clocks slow and rulers shorten. As we demonstrate now, even mass changes at high velocity. Our proof follows directly from the relativistic slowing of time and the law of conservation of momentum. As we have seen in Chapter 1, momentum is

[1] Following argument from Feynman, R.P. 1963. *The Feynman Lectures in Physics.* Addison-Wesley.

conserved in any collision. For example, billiard balls of equal mass exchange momentum when one strikes another.

Figure 3.7. Conservation of momentum in a collision between two billiard balls with equal mass.

Balls A and B have equal mass, *m*. Before the collision A is moving to the right with velocity *v*, and B moves to the left at velocity -*v* (the minus sign indicates direction of travel). After the collision (assuming no energy is lost to internal heating in the balls or to production of sound, etc.) A is moving left at velocity -*v*, and B is moving right at velocity *v*. The total momentum (*mv*) of the system is unchanged.

Now let's try a thought experiment tossing billiard balls from spaceships and observe the consequences. The balls were weighed before the experiments and found to be of equal mass, and the experimenters toss them out of the spaceships at a designated velocity, as measured from their own spacecraft, perpendicular to their spaceship's motion. Label the balls' perpendicular velocity *u* to distinguish it from the velocity, *v*, of the spaceships. First, with the ships at rest:

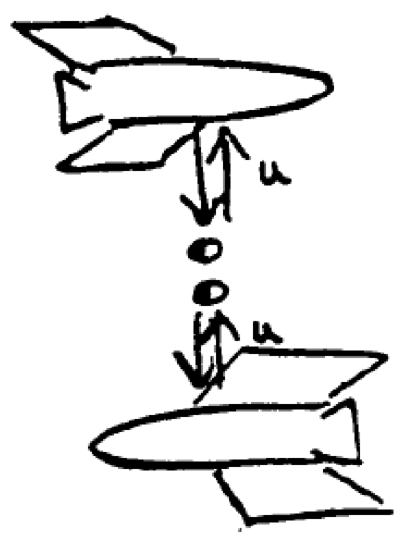

Figure 3.8. Billiard balls tossed from two spacecraft at rest.

We observe the same results as on the billiard table: the balls exchange momentum.

Next, let's watch from the perspective of an outside observer at rest as the ships fly past each other at equal but opposite velocities (*v* and -*v*):

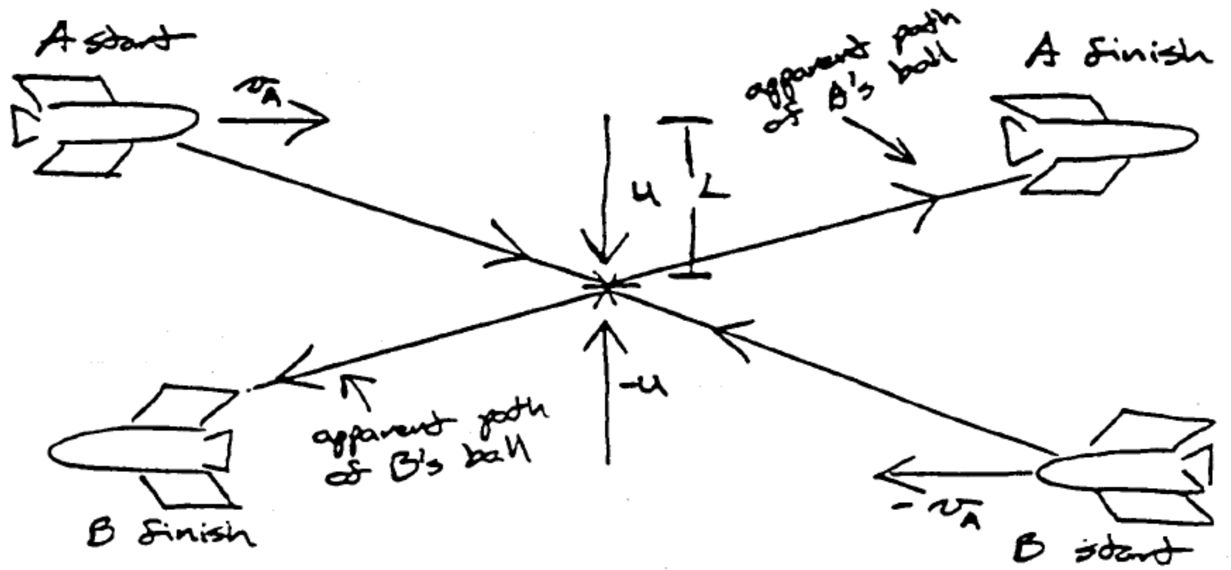

Figure 3.9. Paths of billiard balls as seen by an outside observer as two spacecraft fly past each other.

As seen by an outside observer, the balls exchange momentum at impact, and the total momentum of the two balls is conserved.

But how does the experiment appear to an observer inside one of the ships?

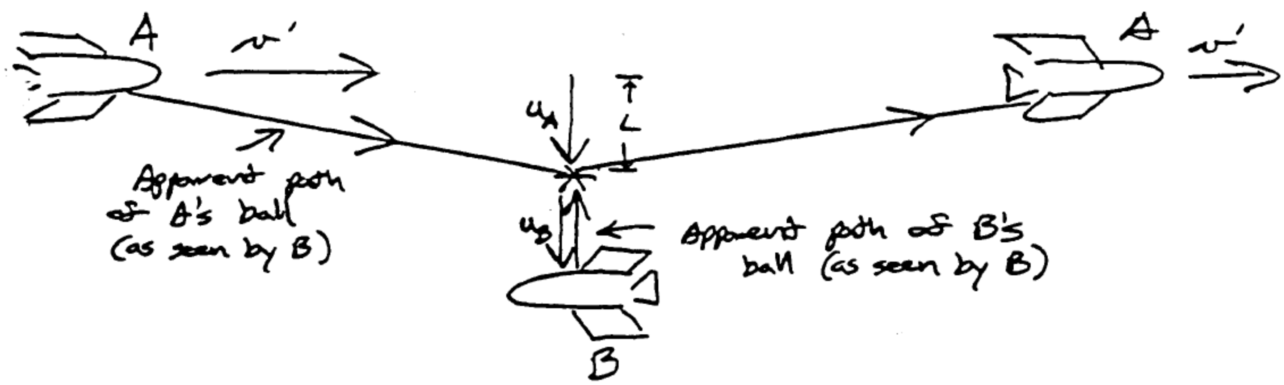

Figure 3.10. Path of the billiard balls as seen by an observer in ship B.

As seen by B, A's ball whizzes in at considerable horizontal velocity (the same horizontal velocity as the spaceship A), ricochet's off his (B's) own ball, and flies off again toward A. His (B's) own ball merely travels straight up from his spaceship to the point of impact and straight back down.

Because of relativistic time dilation, A's clock, as seen from B's perspective, is slowed according to the formula

$$t_A = \frac{t_B}{\sqrt{1 - {v'^2}/{c^2}}}$$

where t_A is the time on A's clock as seen by B, and t_B is the time on B's clock. v' is A's velocity as measured by B.

Since velocity is distance divided by time, and the distance perpendicular to the direction of motion is the same for both observers:

$$u_A = \frac{L}{t_A} = \frac{L}{t_B \Big/ \sqrt{1 - {v'^2}/{c^2}}} = \frac{L\sqrt{1 - {v'^2}/{c^2}}}{t_B} = u_B\sqrt{1 - {v'^2}/{c^2}}$$

u_A (the vertical component of the velocity of A's ball as seen by B) is slowed by the factor $\sqrt{1 - {v'^2}/{c^2}}$ compared to u_B .

We know momentum must be conserved at impact, since the outside observer saw momentum conserved and the laws of physics are the same for observers in all inertial frames. Therefore, $m_A u_A$ must equal $m_B u_B$ (where m_A is the mass of A's ball as seen by B and m_B is the mass of B's ball). Since

$$u_A = u_B\sqrt{1 - {v'^2}/{c^2}}$$

then

$$m_A u_B\sqrt{1 - {v'^2}/{c^2}} = m_B u_B$$

$$m_A\sqrt{1 - {v'^2}/{c^2}} = m_B$$

$$m_A = \frac{m_B}{\sqrt{1 - {v'^2}/{c^2}}}$$

As seen by B, the mass of A's ball appears greater than his own by the factor

$$\frac{1}{\sqrt{1-v'^2/c^2}} \quad .$$

More generally, as seen by an outside observer at relative rest, a moving mass appears greater than the mass at rest according to the formula:

$$m' = \frac{m}{\sqrt{1-v^2/c^2}}$$

where *m*' is the moving mass, as measured by an outside observer, *m* is the mass at rest, and *v* is their relative velocity.

<u>The universal speed limit</u>

We can also understand the relativistic increase of mass by analyzing the consequences of the universal "speed limit," *c*.

Suppose we accelerate our space ship closer and closer to the speed of light. Because of the "speed limit," we cannot reach *c*, no matter how much thrust we apply. So where does all that energy go, representing the work we've done to accelerate the ship? What do we get for burning the rockets? The energy is converted into extra mass: the ship, ourselves, and everything on board acquire extra mass. We don't notice the increase, but an outside observer sees our ship and everything in it gaining mass.

This is, in a way, a restatement of the law of conservation of energy. There are two components of kinetic energy, mass and velocity: $KE = \frac{mv^2}{2}$. If we pump more energy into a system, increasing the kinetic energy, but the velocity cannot increase indefinitely, that energy must appear as increased mass.

Relativity theory demands that we interpret the classical energy conservation law in a new way: total mass + energy is conserved. The total mass-plus-energy of the system remains constant.

<u>E = mc^2</u>

Our discussion of the relativistic increase of mass implies mass and kinetic energy are related. We can demonstrate the relation between mass and energy more generally. To begin our argument, we show that a beam of light has an effective mass proportional to its energy.

To simplify the discussion we introduce the photon, a "particle of light." More precisely, as we shall discuss in Ch.5, a photon is a quantum of electromagnetic radiation.

A photon has no rest mass: it is always in motion. Light does carry energy, however. It can do work, as in photosynthesis or in photovoltaic cells, devices which produce electric current using the energy from sunlight.

By definition, energy equals the work invested in an object, which is force applied over a distance: $E = Fs$, where s is the distance over which the force is applied.

Manipulating the terms:

$$Fs = (ma)s = mv^2$$

Since the velocity of a photon is always c, the speed of light, a photon's energy equals mc^2 , so the photon has an effective "mass"

$$m = \frac{E}{c^2}$$

To prove that this relationship applies to systems other than photons, imagine a mass absorbing two photons of equal but opposite momentum:

Figure 3.11. Mass absorbing two photons of equal but opposite momentum.

The mass absorbs the energy of the two photons:

$$E_f - E_i = dE = 2E_\gamma$$

where E_f is the energy of the mass after absorbing the photons, E_i is the energy before absorption, E_γ is the energy of a photon, and dE symbolizes change in energy.

Seen by an observer moving at velocity u perpendicular to the system, the incident photons appear to have a vertical component of velocity, u. By conservation of momentum:

$$M_f u - M_i u = 2mu$$
$$M_f - M_i = 2m$$

where M_f is the mass of the system after absorbing the photons, M_i is the mass before, and m is the mass of an individual photon.

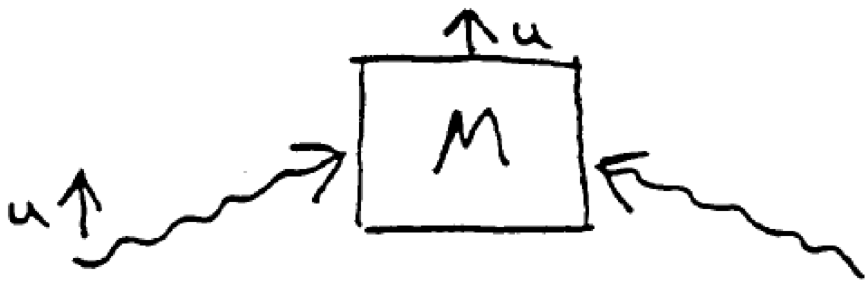

Figure 3.12. Mass and photons seen by an observer moving at velocity $-u$.

We know that the effective mass of a photon equals $\frac{E_\gamma}{c^2}$. Therefore:

$$M_f - M_i = dM = 2m = \frac{2E_\gamma}{c^2}$$

$$dMc^2 = 2E_\gamma$$

Since $2E_\gamma = dE$, the net change in energy of the system,

$$dMc^2 = dE$$

That is, the change in energy of a system is proportional to a change in mass of the system with constant of proportionality given by the square of the speed of light.

Now imagine placing a mass in a vacuum. *dM*, in this situation, is the entire mass itself (since there was no mass or energy previously), and the equation says that this rest mass carries an equivalent energy of value Mc^2.

$$E = mc^2$$

This equation does not mean you can somehow "accelerate" a mass to the speed of light squared and get energy. It means mass and energy are inter-convertible. They are flip sides of the same coin. c^2 is the constant of proportionality: a little mass represents a lot of energy.

Mass energy: fission and fusion

Mankind have harnessed nuclear fission as an energy source, and scientists are experimenting with fusion reactors, seeking an energy source for the future. Fission and fusion extract energy from atomic nuclei by exploiting the difference in mass between reactive nuclei and their products.

We can calculate the energy released in the fission of Uranium 235 (92 protons and 143 neutrons) as follows: In a typical fission event, ^{235}U disintegrates into Barium 141 and Krypton 92 plus two neutrons. The mass of the ^{235}U nucleus exceeds the sum of the masses of the products, Ba, Kr and two neutrons, by about 3.6 X 10^{-25} gm: this amount of mass is converted into energy in the fission reaction.

$$^{235}\text{U} \rightarrow {}^{191}\text{Ba} + {}^{92}\text{Kr} + 2\text{n}$$

$$M_{\text{U}} - (M_{\text{Ba}} + M_{\text{Kr}} + M_{2\text{n}}) = \Delta M = 3.6\times10^{-28}\ \text{kg}$$

$$E = \Delta Mc^2 = (3.6\times10^{-28}\ kg)(9\times10^{16}\ m^2/s^2) \approx 3.2\times10^{-11}\ J$$

$$\frac{3.2\times10^{-11}\ J}{1.6\times10^{-19}\ J/eV} \cong 200\ MeV$$

(*MeV*, "million electron volts," is the standard unit of measure for energy in subatomic processes. It represents the kinetic energy gained by an electron falling through 1×10^6 volts potential.)

Uranium fission is self-sustaining because the reaction yields high energy neutrons which destabilize nearby uranium nuclei and set off a chain reaction. In fact, uranium nuclei are unlikely to decay spontaneously: they must overcome a small energy barrier (about 6 *MeV*) to initiate fission. In-coming neutrons deliver the necessary energy.

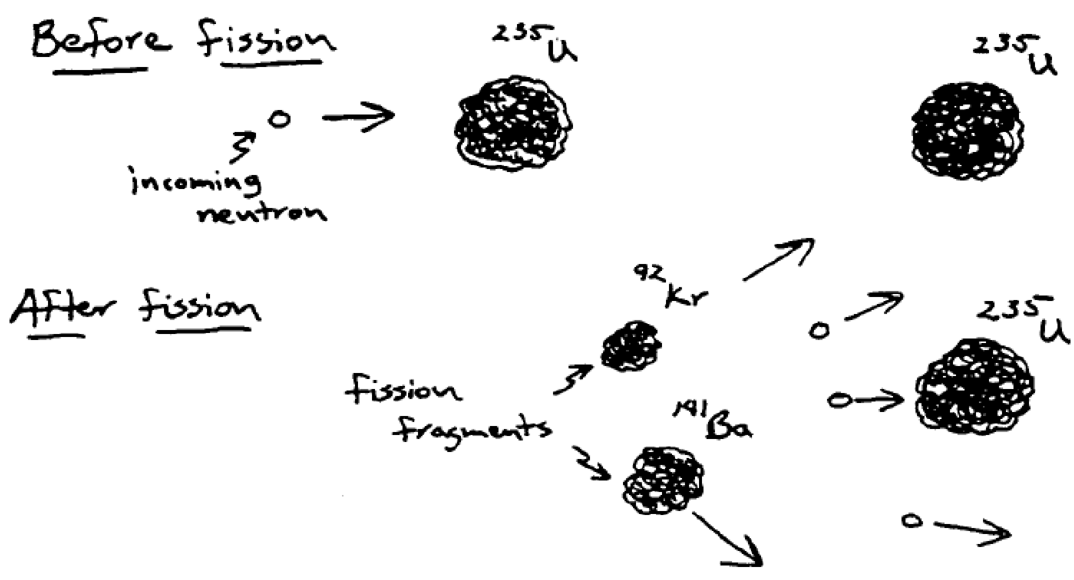

Figure 3.13. Initiation of a chain reaction. Fission of the original ^{235}U generates free neutrons which, if they hit nearby nuclei, will cause fission in those nuclei, which generate more neutrons and further fission events.

In a lump of uranium containing many trillions of nuclei, the number of fission events per unit time and the amount of energy released increases exponentially, producing an explosion, an atomic bomb. It is possible to control the rate of fission, however, by controlling the number of neutrons flitting among the nuclei.

Fission reactors intersperse neutron absorbers, usually cadmium rods, between the fuel elements (rods of uranium) in the reactor core. Engineers can cool the core (decrease the rate of fission) by dropping the cadmium rods in between the fuel elements. Conversely they can heat the core by lifting the control rods out from among the fuel elements.

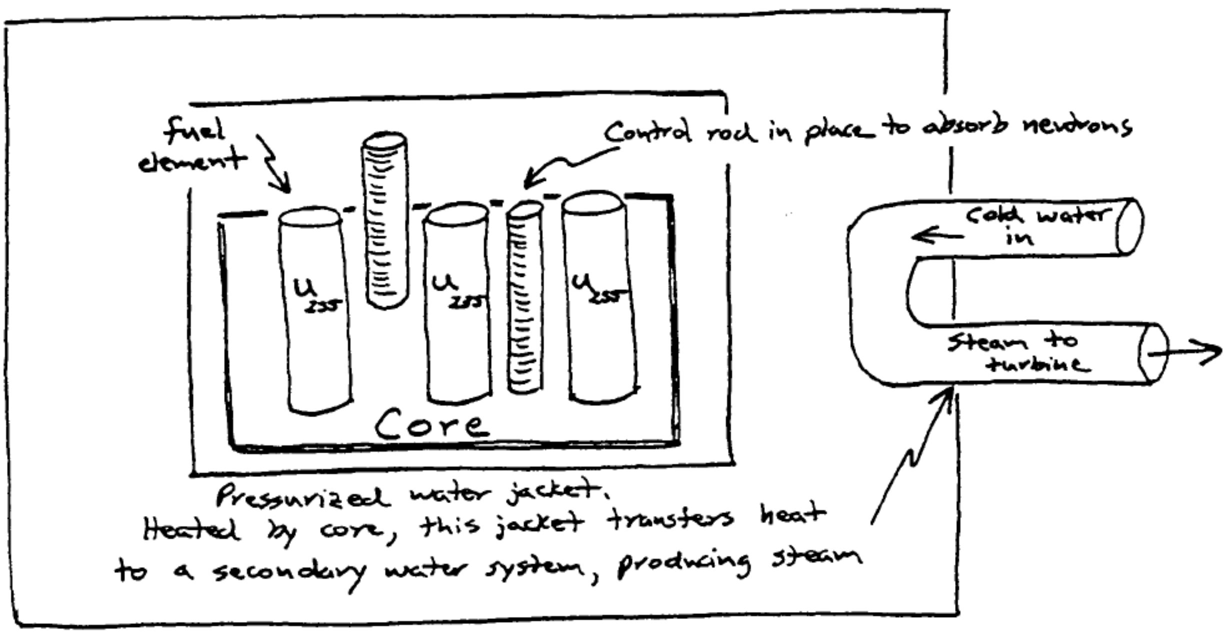

Figure 3.14. Basic design of a fission reactor. Heat from the core creates steam which drives a turbine, generating electricity.

Fusion reactions release energy because light nuclei, with large mass per nucleon, fuse to form heavier nuclei with smaller mass per nucleon. As an example, we'll tally the energy released by the fusion of a deuterium nucleus with tritium. (Deuterium (^{2}H)is an isotope of hydrogen with one extra neutron in the nucleus, tritium (^{3}H) a hydrogen isotope with two extra neutrons.)

$$^2\mathrm{H} + {}^3\mathrm{H} \rightarrow {}^4\mathrm{He} + \mathrm{n}$$

$$(M_\mathrm{D} + M_\mathrm{T}) - (M_\mathrm{He} + M_\mathrm{n}) = \Delta m \cong 3.1357\times10^{-29}\ kg$$

$$E = \Delta mc^2 = (3.1357\times10^{-29}\ kg)(9\times10^{16}\ m^2/s^2)$$
$$\cong 2.82214\ J \cong 17.6\ MeV$$

Fusion

Fusion reactions are difficult to ignite in Earth-bound reactors: ignition requires extreme temperatures (on the order of ten million degrees Centigrade) at high densities in order to overcome electromagnetic repulsion between reactant nuclei. Hence the choice of deuterium (^{2}H, hydrogen with an extra neutron in the nucleus) and tritium (^{3}H, two extra neutrons) as reactants: the extra neutrons increase the available nuclear "glue," the strong force which binds protons and neutrons in atomic nuclei and is required to overcome electromagnetic repulsion between protons.

The reaction vessel must exclude impurities (other nuclei) which would interfere with fusion, and it must tolerate not only solar-core temperatures but highly corrosive plasma. Engineers are exploring two basic reactor designs. One, a "tokomak," confines hot plasma within magnetic fields. The other uses powerful lasers to ignite reactant deuterium and tritium pellets in a vacuum chamber.

A tokomak is a toroid (a chamber like a hollow doughnut) wound with wire to produce a large magnetic field in its core. Radio-frequency oscillators heat the deuterium and tritium in the toroid. (Radio waves accelerate the ions, since electromagnetic radiation interacts with charged particles.)

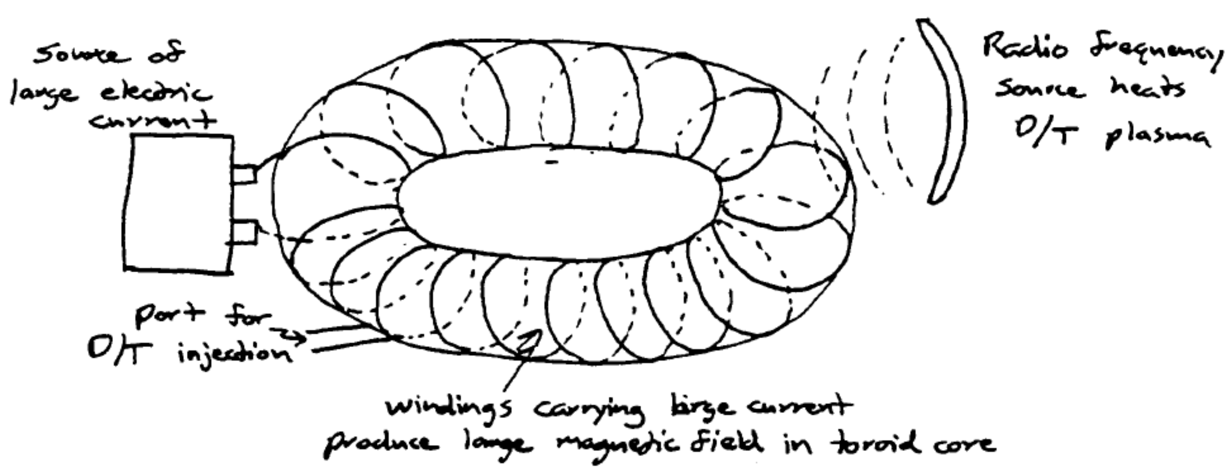

Figure 3.14. Tokomak design. The tokomak confines and compresses a circulating plasma of deuterium (D) and tritium (T).

The magnetic field confines the deuterium/tritium plasma in the toroid core since charged particles moving across the magnetic field lines will curl back toward the toroid axis.

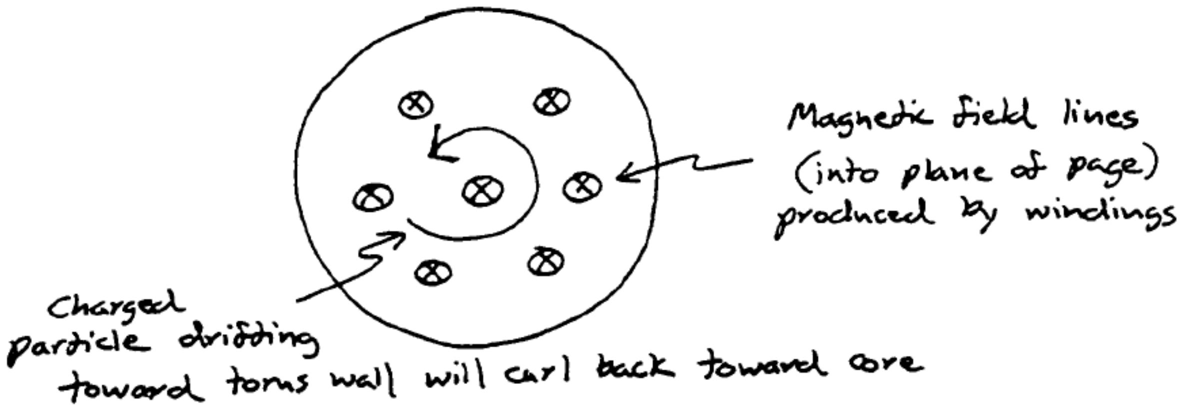

Figure 13.15. Cross section of tokomak, showing trajectory of D/T plasma relative to the toroidal axis. Strong magnetic field curls the plasma, confining it to the center of the tube as it circulates around the toroid.

A lithium blanket surrounding the toroid captures energetic neutrons released by fusion reactions, and the hot lithium vaporizes water, producing steam which drives electric generators.

Laser ignition reactors focus lasers on deuterium/tritium pellets dropped one after another into the core of the reaction vessel. High energy neutrons released in mini thermonuclear explosions transfer energy to a lithium jacket, as with the tokomak.

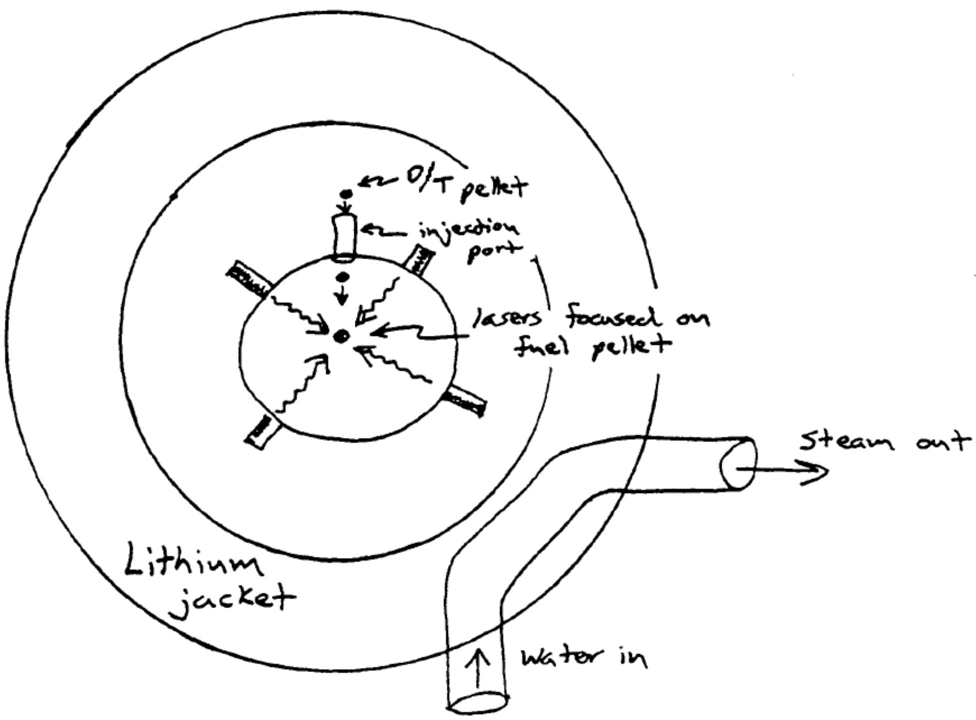

Figure 3.15. In a laser fusion device, high energy lasers heat and implode a fuel pellet, creating temperature and pressure sufficient to ignite fusion.

The spacetime interval

Is there no foundation in this Universe? We've toppled all of Newton's "constants:" time changes, length changes, mass changes. Energy and mass are inter-convertible. Is there nothing all observers, everywhere, can agree upon?

It turns out there is a measurement, the spacetime interval, on which all inertial observers can agree. The spacetime interval is the space-and-time "distance" between any two events.

An "event" is an occurrence at a particular point in space and time. The alarm ringing at 6:00 A.M. on the bedside table is one example of an event. The collision of two billiard balls is another example, and a supernova explosion is another.

Symbolically, the spacetime interval is

$$\tau = \sqrt{t^2 - x^2}$$

where τ, the spacetime interval, is the interval between two events (e.g. successive ticks on a clock) measured in the reference frame of those events, t is the time interval between the same two events as measured by an outside observer, and x is the spatial separation between the events as measured by the same outside observer.

To prove that all inertial observers measure the same spacetime interval between two events, let's return to the example of the light clock. We'll label the light leaving mirror A as Event 1, and Event 2 is the arrival of the light at mirror B.

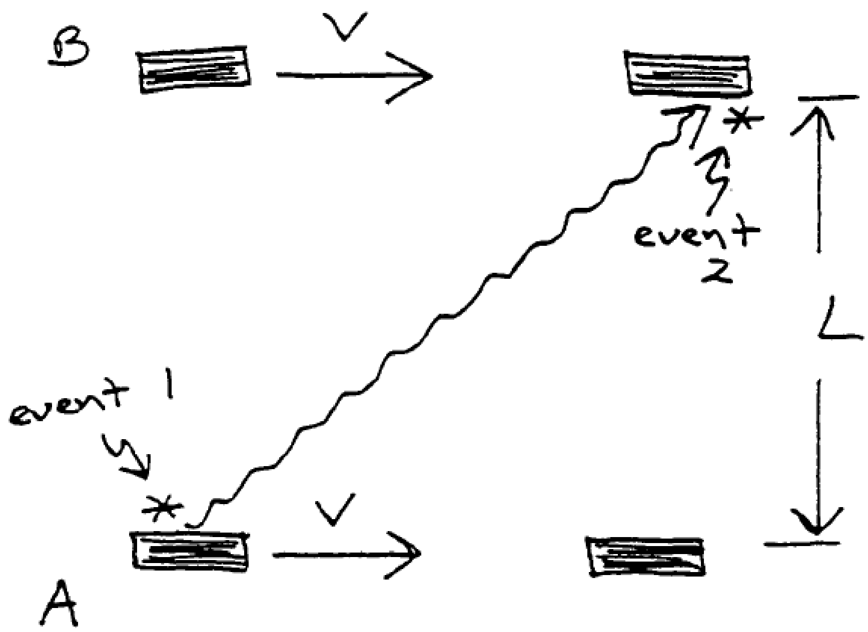

Figure 3.16. Events on a light clock. Light leaves mirror A (event 1) and, after an interval of time and across an interval of space, arrives at mirror B (event 2).

The distance between mirrors, *L*, perpendicular to the direction of travel, is the same as measured by any observer traveling in the *x* (or -*x*) direction. We prove this is true by assuming it is not true: suppose, for example, meter sticks held perpendicular to the direction of travel appear to shorten as seen by an outside observer. This assumption leads to a contradiction. To illustrate, Bill and Clarissa pass each other traveling opposite directions. Each has a meter stick held perpendicular to the direction of travel. If a moving meter stick shortens as measured by an outside inertial observer, Clarissa would measure Bill's meter stick shorter than her own. But Bill sees his own meter stick at relative rest and Clarissa's meter stick in motion, so he finds her meter stick has shortened. Now, both meter sticks cannot be shorter than the other, so our assumption is false. The meter sticks must not change in length.

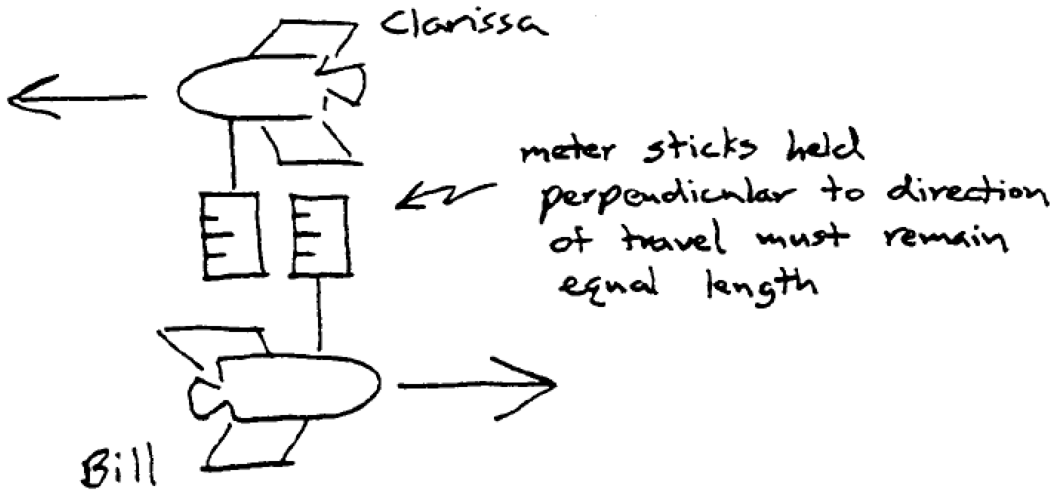

Figure 3.17. Lengths perpendicular to direction of motion remain constant.

Now we show that if perpendicular *L* is constant, the spacetime interval between events 1 and 2 on our light clock must be constant. To simplify the argument, measure distance and time in the same units, meters: one meter of time is the time it takes light to travel one meter of distance. Then

$$x^2 + L^2 = t^2$$

and

$$L^2 = t^2 - x^2 = \tau^2$$

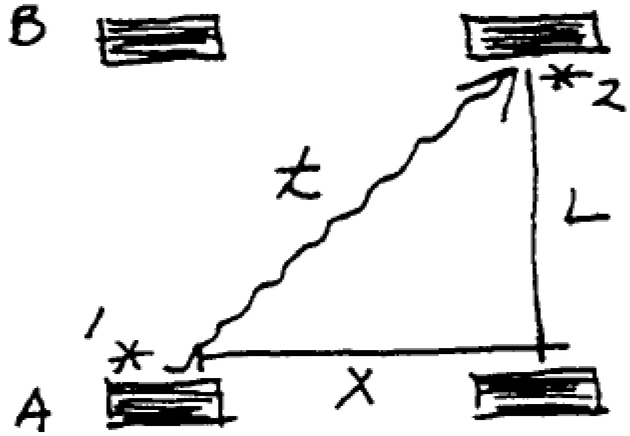

Figure 3.18. Geometry of the moving light clock. x is the distance, in meters, the clock moves in the time interval, t, also measured in meters.

where t is the time (in meters) it takes for light to travel between mirrors as seen by an outside observer, and x is the distance the rocket travels during that time. This relation holds for any observer, because L is constant: $t^2 - x^2$ is the same for all observers, even though different observers measure different t and x.

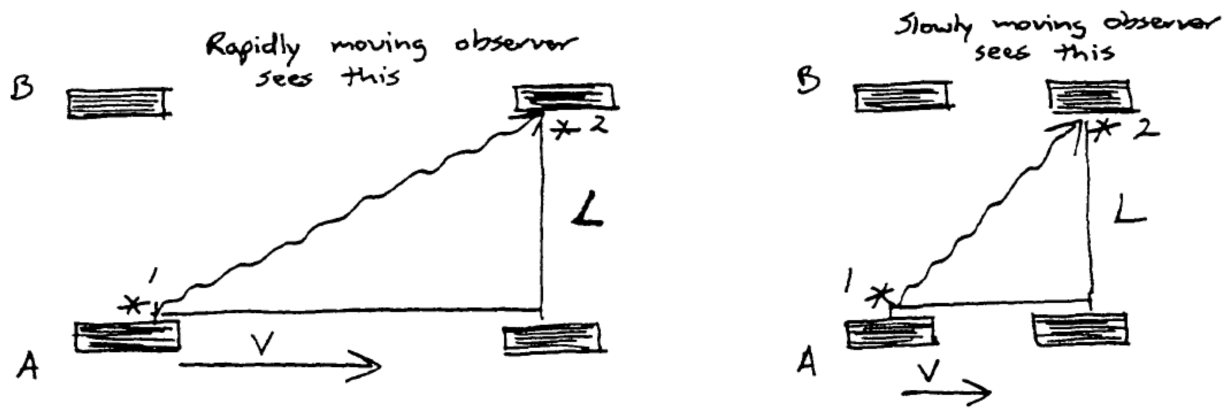

Figure 3.19. Comparison of clock configuration seen by rapidly moving vs. slow-moving observers. Events 1 and 2 occur independently of the observers, and both observers measure the same spacetime interval.

Mathematical physicists refer to the spacetime interval as a metric, one of a class of mathematical tools we use to measure "distances." A more familiar metric is the Pythagorean relation

$$c = \sqrt{a^2 + b^2}$$

where c is the hypotenuse of a right triangle, a and b are the sides. The metric allows us to compare "distances" measured by observers using different coordinate systems. For example, an observer using a Cartesian grid rotated by some angle, θ, would find *c*, the hypotenuse, the same length as another (stationary) observer using the standard Cartesian frame of reference. The spacetime interval is a metric that allows us to measure events that are separated in space and time, and, as we have shown, two observers in different frames of reference agree on the interval, even though they measure the interval in different frames of reference.

Note the connection between the Pythagorean relation and the spacetime interval in our argument above. As we'll see in the next chapter, an extension of this metric, referred to as the spacetime metric, describes the curvature of spacetime in a gravitational field and lies at the heart of the Theory of General Relativity.

A trip to Alpha Centauri

We illustrate the concept of spacetime interval with a hypothetical example. Suppose Space Command assigns Bill a mission to the planetary system at Alpha Centauri, four light years distant. Clarissa monitors the mission from Earth Base. Further suppose Bill maintains a constant velocity 0.8 *c* for the duration of the trip.

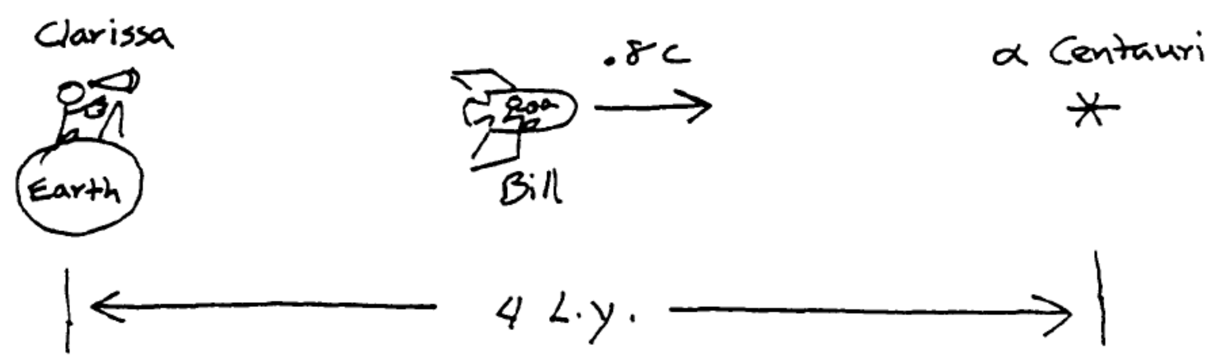

Figure 3.20. Trip to Alpha Centauri.

We use "spacetime diagrams" to represent the trip as it is seen by the two different observers, Bill and Clarissa. Event 1 is liftoff from Earth. Event 2 is arrival at Alpha Centauri.

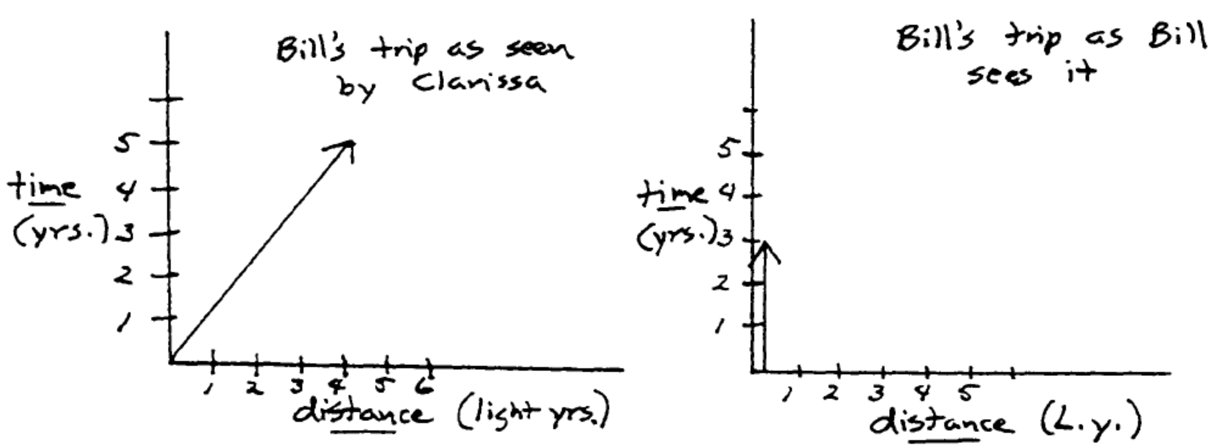

Figure 3.21. Spacetime diagrams of the same trip as seen by two different observers.

Clarissa sees Bill traveling across both space and time at 0.8 *c*. It takes 5 years, according to Clarissa's clock, for Bill to reach Alpha Centauri, 4 light years distant. The spacetime interval between Event 1 and Event 2, as measured by Clarissa, is

$$\tau^2 = (5\ yr)^2 - (4\ yr)^2 = 9\ yr^2$$
$$\tau = 3\ yr$$

Bill, on the other hand, experiences only a time interval between events. Since he is physically present at both events, liftoff and landing, he measures no spatial separation between them. He measures the spacetime interval between Event 1 and Event 2 as

$$\tau^2 = (3\ yr)^2$$
$$\tau = 3\ yr$$

Hold on, you say. How can Bill's clock read only three years between events? We've already seen why: let τ represent the time on Bill's clock, *t* the time interval as measured by Clarissa, the outside observer.

$$t = \frac{\tau}{\sqrt{1 - v^2/c^2}}$$

$$5\ yr = \frac{\tau}{\sqrt{1-(0.8)^2}} = \frac{\tau}{\sqrt{0.36}} = \frac{\tau}{0.6}$$

$$\tau = 0.6(5\ yr) = 3\ yr$$

Both Bill and Clarissa measure the same spacetime interval between these events, and so would any other inertial observer.

<u>The twin paradox</u>

Bill's trip to Alpha Centauri illustrates the "twin paradox:" if Bill and Clarissa are twins, and Bill completes a round trip to Alpha Centauri, he would return to Earth having aged only 6 years, while Clarissa ages 10 years. No one has (yet) performed such a trip, but experiments right here on Earth demonstrate the reality of the twin paradox. Atoms of Iron 57 (an isotope of Iron) emit radiation at a precise frequency, about 3.5×10^{18} cycles/sec. They provide a very accurate clock. The iron atoms' vibrations make "round trips," like Bill's round trip to Alpha Centauri: if the atoms are embedded in a crystal lattice, they vibrate back and forth in thermal motion, and the higher the temperature, the greater their thermal velocities. It has been shown by direct measurement that the higher the temperature, the lower the frequency of the emitted nuclear radiation: the "twin" atom that makes the round trip has a slower clock and returns "home" to its neutral position in the lattice younger than an atom that remained at rest. (In fact, no atom remains at absolute rest (as we'll see when we discuss the uncertainty principle in quantum mechanics), so these experiments compare the frequencies emitted by faster moving atoms, at high temperature, with slower, cooler atoms.)

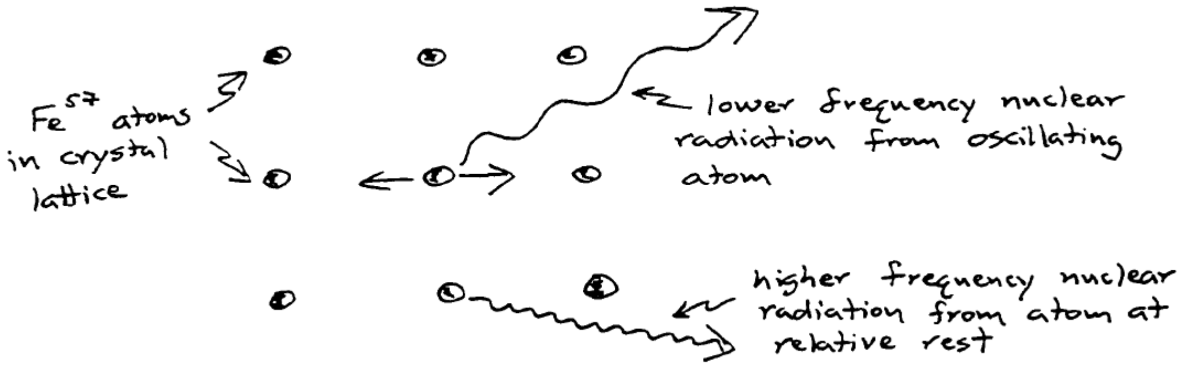

Figure 3.22. Change in clocks, as measured by emitted frequency, in iron atoms at different temperatures (and therefore different relative motion).

Other experiments also support these predictions of the special theory.

<u>Experimental verification of special relativity</u>

Special relativity describes effects that are obvious only at high velocities. Because the speed of light is so large, the term $\sqrt{1-v^2/c^2}$ departs radically from 1, and therefore mass, length and time change noticeably, only as *v*, the velocity, approaches the speed of light. We don't experience such extreme velocities in our everyday lives, so we don't see relativistic effects directly. The table on p.54 lists the changes in mass, length, and time over a range of velocities.

Even though we don't see relativistic effects in our daily lives, experiments confirm the predictions of the special theory. Following are just a few of the many experiments that have been performed:

Sensitive atomic clocks on board rapidly moving airplanes and spacecraft really do slow down compared to clocks at relative rest on the ground. (Effects of general relativity, to be discussed in the next chapter, must also be taken into account.)

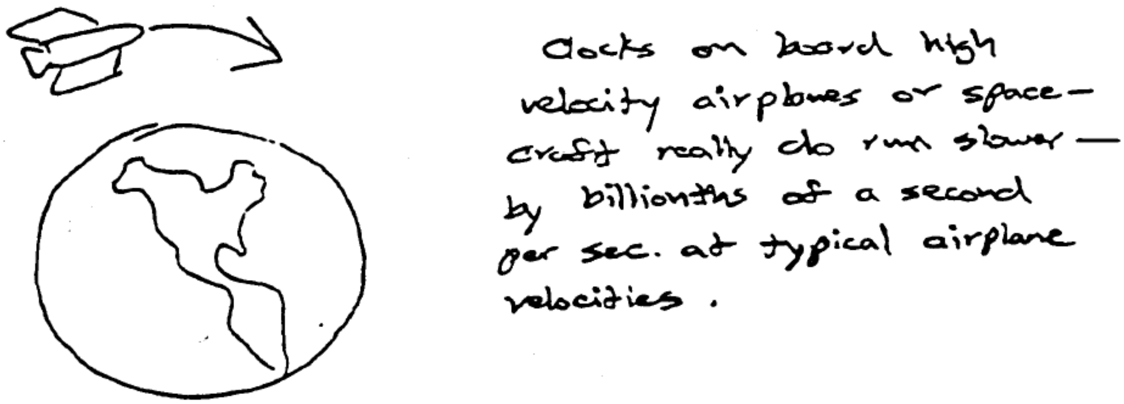

Figure 3.23. Moving clocks slow.

Muon decay also verifies relativistic time dilation. Muons are subatomic particles produced in the upper atmosphere by cosmic ray showers. At rest, they disintegrate in a very short time. At the relativistic velocities in cosmic ray showers their internal clocks slow down, as measured by ground-based observers, and they travel far enough to reach the Earth's surface before disintegrating.

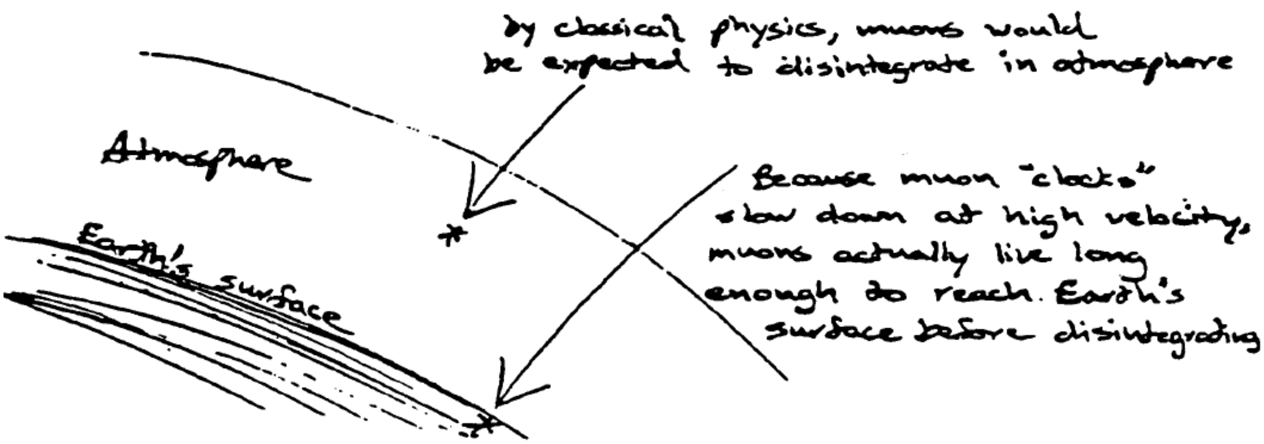

Figure 3.24. Because of relativistic effects, muon clocks slow such that they live long enough to reach Earth's surface.

Particle accelerators prove that relativistic mass increases. Just as twirling a heavy ball on a string requires more centripedal force than twirling a light ball, so holding particles in an accelerator's ring requires greater magnetic force at high velocity than expected if the particles maintained their rest mass.

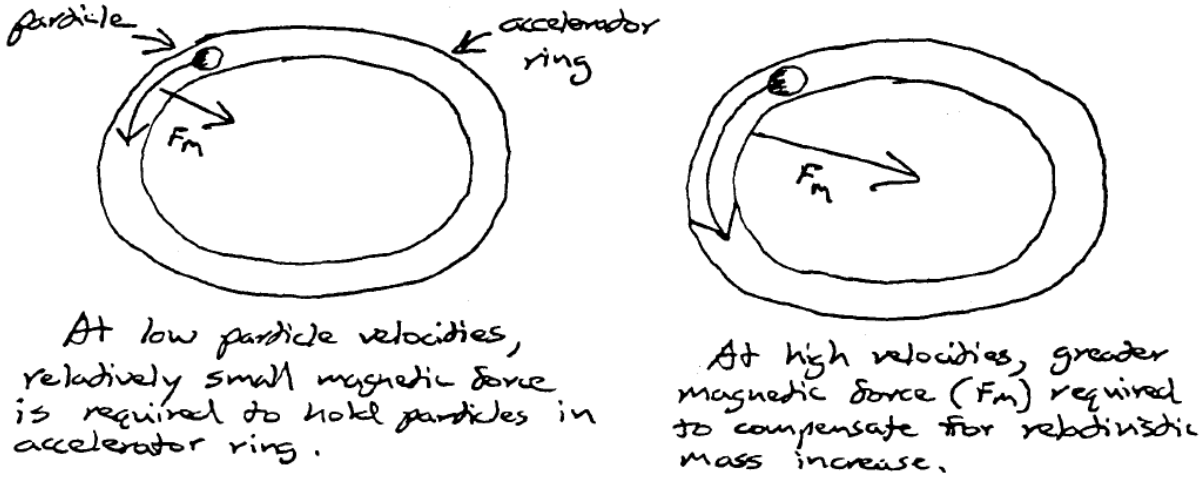

Figure 3.25. Particle accelerators must compensate for increased mass at high particle velocities.

The inter-conversion of mass and energy has been verified amply in fission and fusion reactions, where part of the mass of uranium (fission) or heavy hydrogen (fusion) converts directly into energy. And scientists operating high energy accelerators regularly witness the annihilation of particles and their antiparticles, leaving energetic gamma rays (conversion of mass to energy), and the production of particles from high energy fields (energy to mass). (We shall discuss these processes in more detail when we consider the particles and forces in a later chapter.)

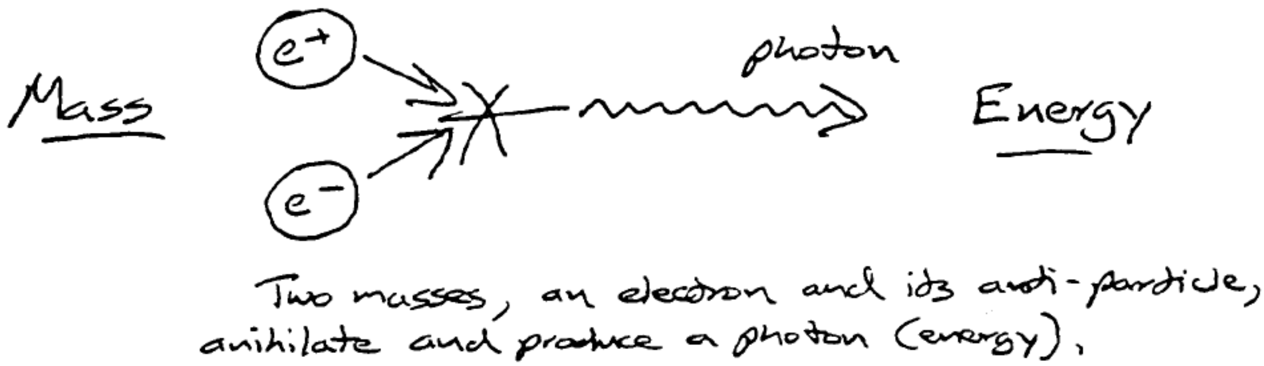

Figure 3.26. Conversion of mass into energy. A particle and antiparticle annihilate, converting mass into energy. The process can also run in reverse: photon energy can produce a particle-antiparticle pair.

Special relativity really does model how Nature behaves. It is indispensable for describing events at high energies and high velocity.

So what?

If we don't witness the effects of special relativity in our daily lives, why bother to study it? Isn't it just a shiny bauble for physicists to play with?

In fact, life on earth would not exist if Nature didn't behave as described by Einstein's special theory. Life depends on the sun for its energy: photosynthesis traps the energy from sunlight on which the entire biosphere depends. Sunlight, in turn, is generated by fusion reactions, which convert mass into energy. Every second of every day, four million tons of mass are converted to energy in the core of the sun.

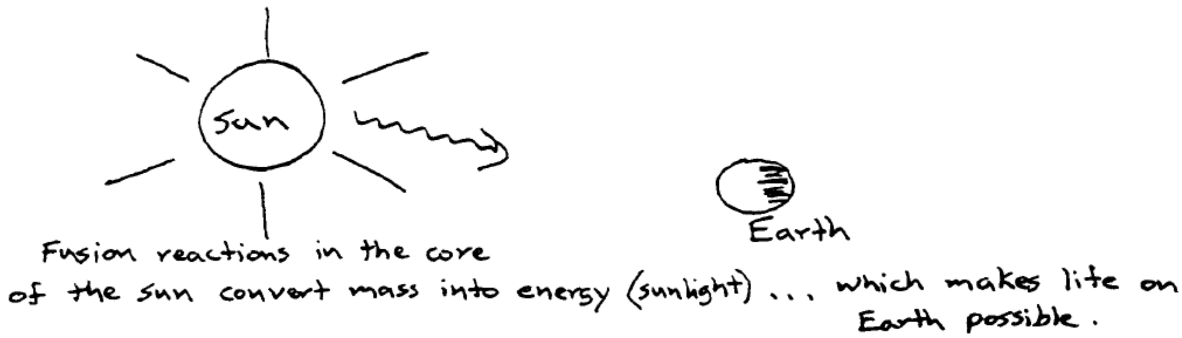

Figure 3.27. Sunlight originates from conversion of mass into energy by fusion reactions in the core of the sun.

Muons, also, affect life on Earth, including ourselves. The decay products of muons (and other cosmic ray products) can cause mutations in DNA. Mutations, in turn, provide the genetic variability that drives evolution. It is estimated that cosmic rays and their products, including muons, are responsible for about ten percent of mutations.

More practically, control of fusion and fission reactions offers an energy source for our civilization. Many nations already generate significant proportions of their electrical power in fission reactors, which convert part of the mass of the uranium nucleus (or its isotopes) into energy, and researchers are learning how to control fusion reactions similar to those in the stars for application here on Earth.

<u>Summary</u>

In his special theory of relativity, Einstein worked out the consequences of two assumptions: (1) the laws of physics and (2) the speed of light are the same for any observer in an inertial frame of reference. Reasoning from these assumptions, he deduced that, at high velocities, as seen by an outside observer,

1. Clocks slow.
2. Mass increases.
3. Length decreases.

Furthermore, mass is just a different manifestation of energy, and the two can be inter-converted. $E = mc^2$.

Even though different observers get different results when they measure moving mass, length, and time, they all agree on the spacetime interval between two events:

$$\tau = \sqrt{t^2 - x^2}$$

These theoretical conclusions have been verified experimentally, and they have direct implications for life on Earth, including human life.