Chapter 4
The Theory of General Relativity

In his theory of special relativity Einstein explored the physical consequences of the principle of relativity in inertial frames of reference. In the theory of general relativity, he extended his argument to include free-fall reference frames and, thence, to accelerated frames and the force of gravity. General relativity is Einstein's theory of gravity, and it marks a clear departure from Newton's paradigm.

In this chapter we trace how Einstein developed the theory from the principle of equivalence, using "thought experiments" to show that the path of light bends and clocks change in a gravitational field. Then we consider the implications of these phenomena on our understanding of gravity: gravity can be modeled in geometric terms as the curvature of spacetime. Finally, we discuss Einstein's unification of the (previously) disparate concepts of space, time, mass, and energy.

Free-fall

Central to Einstein's theory of general relativity is the concept of free-fall. Einstein recognized that an observer in falling in a gravitational field cannot distinguish free-fall from an inertial frame. Imagine, for example, an astronaut on board an orbiting space shuttle. The shuttle, the astronaut, and all equipment on board are falling toward Earth. (They orbit because, as the craft speeds along its orbital path, Earth curves underneath at the same rate the shuttle falls.) The shuttle is a "weightless" environment. Without looking out the cabin window, the astronaut cannot determine whether he is in free-fall orbit or drifting in empty space on course between the stars.

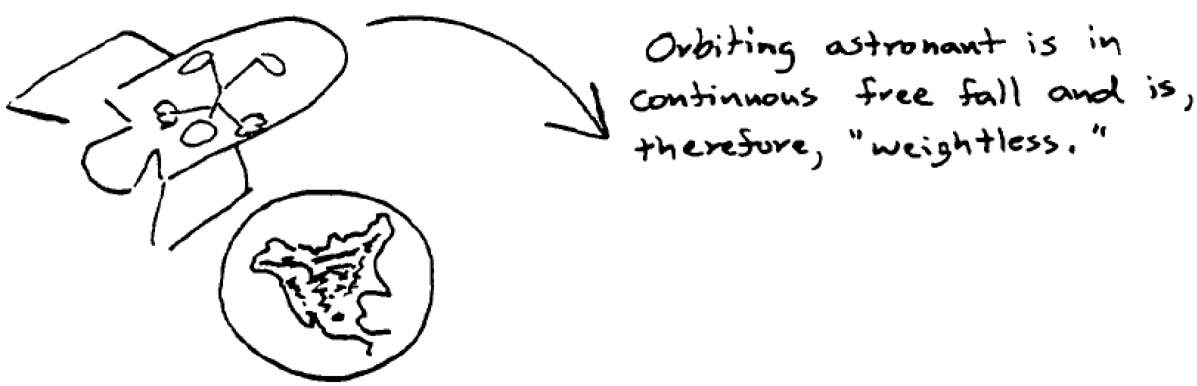

Figure 4.1. Orbiting spacecraft and all on board are always falling toward Earth. Conditions in free-fall are the same as drifting in empty space.

We can demonstrate that free-fall is equivalent to an inertial frame by extrapolating from the one to the other. Picture, first, the astronaut in near-Earth orbit, experiencing the "weightlessness" of free-fall. Now picture the astronaut in not-so-near Earth orbit. Still weightless. Imagine the astronaut in orbits farther and farther removed from Earth until he is effectively on an inertial course between the stars. Same "weightlessness." There is no boundary that the astronaut crosses between the condition of free-fall and the conditions of an inertial frame. The frames are indistinguishable. (One caveat, however: if the astronaut is in an extended frame in free fall toward a center of mass – a long shuttle craft, for example, falling toward Earth – he would see tidal effects of spacetime curvature. See below.)

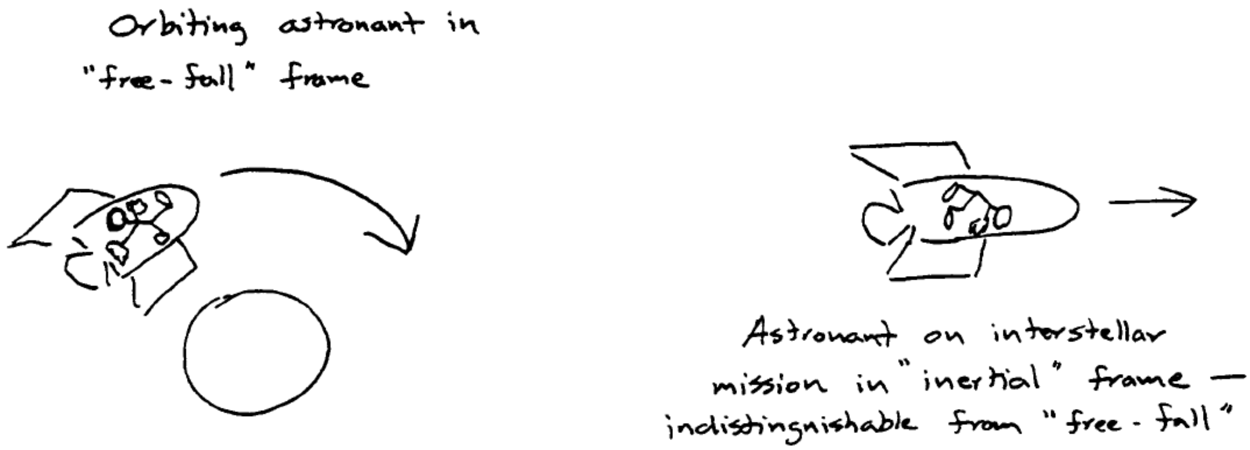

Figure 3.2. The physics in a free-fall frame are indistinguishable the physics in an inertial frame of reference.

One of Einstein's great insights was that accelerating a free-fall reference frame produced effects indistinguishable from gravity. This is the "principle of equivalence," and it serves as the basis of the theory of general relativity.

<u>The principle of equivalence</u>

Just as we used the Michelson-Morley experiment to explain special relativity, we can grasp the basics of general relativity with the help of a thought experiment. (In a "thought experiment," we imagine some hypothetical situation and explore its physical consequences. Real experiments have confirmed the findings we describe.)

Imagine a spaceship drifting in outer space, distant from any mass, with its rockets off. An object released in this "weightless" environment floats motionless (relative to the ship) unless pushed by air currents or by an observer inside the ship.

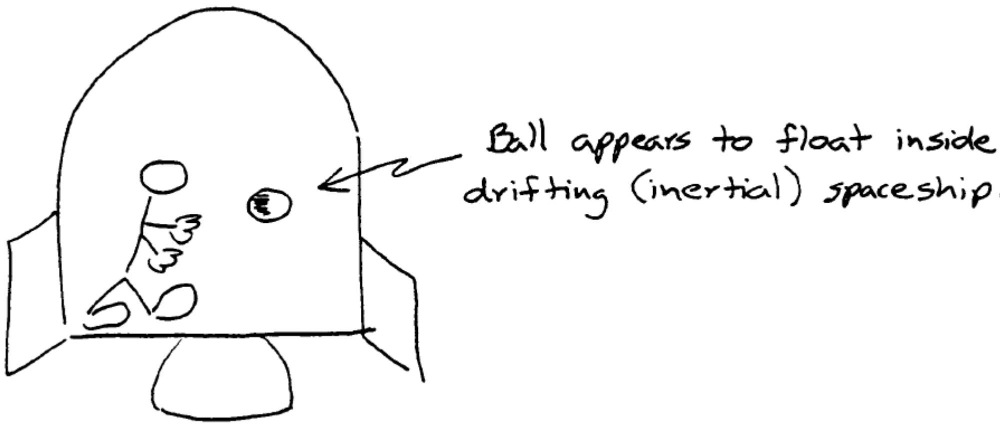

Figure 4.3. Objects in free-fall appear weightless. They drift at the same velocity as the reference frame.

Now imagine that the crew fires the rocket engine. Suppose, further, the rocket accelerates the ship 9.8 meters per second every second (the same as the gravitational acceleration at Earth's surface).

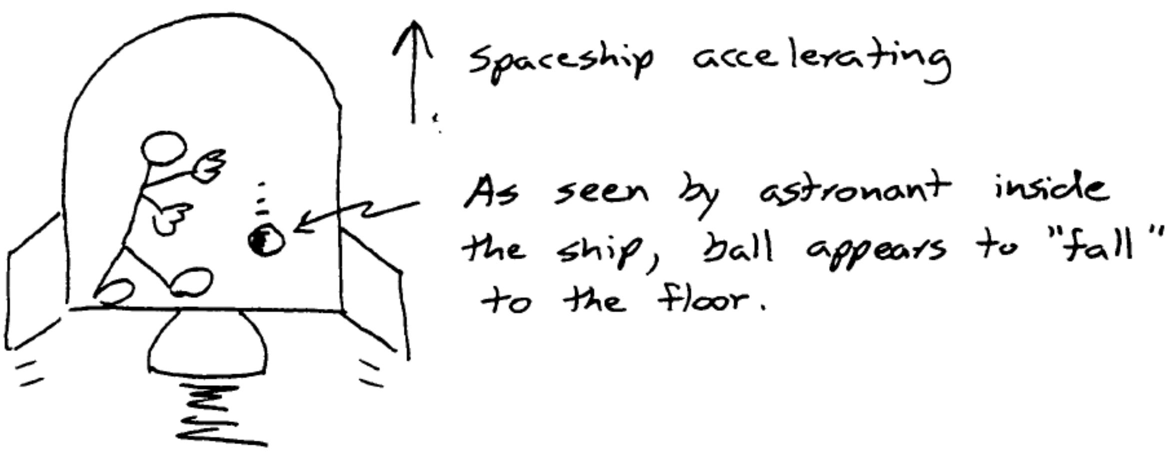

Figure 4.4. In accelerating frame of reference, objects fall just as they would in an equivalent gravitational field.

What will our observer see if she releases an object (say, a tennis ball) inside this accelerating reference frame? The system (spaceship and observer) accelerates upward in relation to the tennis ball. To our observer inside, the ball appears to "fall" to the floor of the ship. At 9.8 m/sec/sec the effects of acceleration cannot be distinguished from the effects of Earth's gravitational field. The observer cannot determine, without looking outside, if she is in an accelerating system or on Earth's surface.

This thought experiment demonstrates the principle of equivalence: an observer in an accelerating frame of reference cannot distinguish the effects of acceleration on mass from the effects of a gravitational field on mass. To put it more conventionally, in Einstein’s terms, inertial mass is equivalent to gravitational mass: the astronaut's inertia, holding the astronaut to the floor of the accelerating spaceship, is equivalent to the gravitational mass holding her to the surface of earth.

Mach's Principle

What holds the tennis ball while the spaceship accelerates past it? What is inertia? Ernst Mach, a physicist of the nineteenth century, hypothesized that momentum and angular momentum are conserved in relation to the "fixed stars." For example, a tennis ball in a spaceship maintains its momentum in relation to the distant galaxies as the ship accelerates past it.

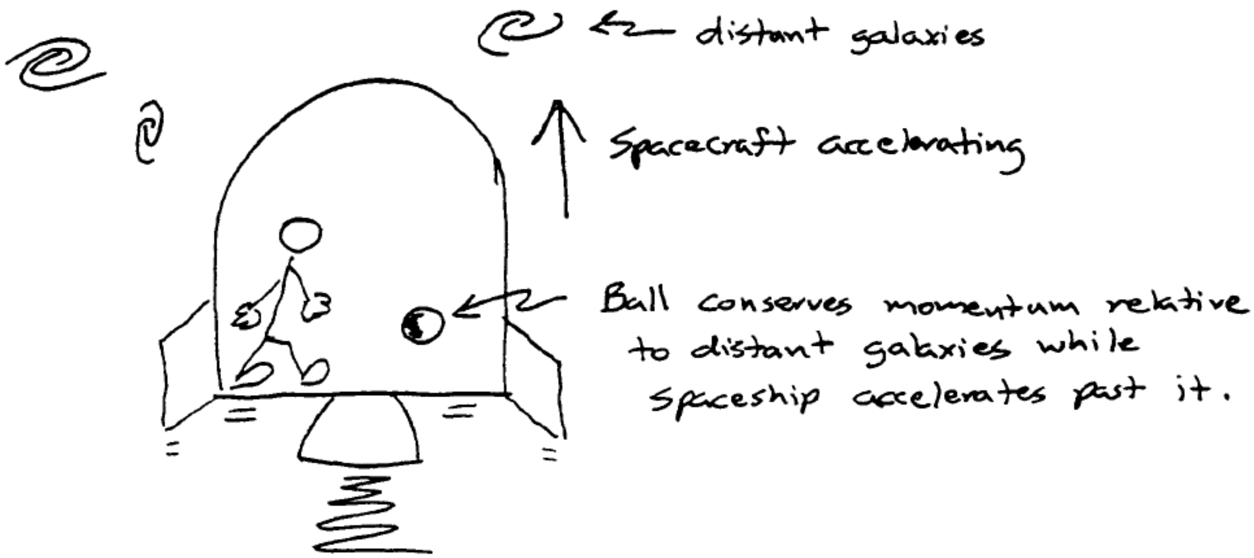

Figure 4.5. Mach's Principle. Mass is somehow caught in a web, held by effects of the distant galaxies or, in other terms, by the large-scale structure of the universe.

The Foucault pendulum provides another example of Mach's principle. The pendulum maintains its arc in relation to the distant galaxies as Earth rotates underneath.

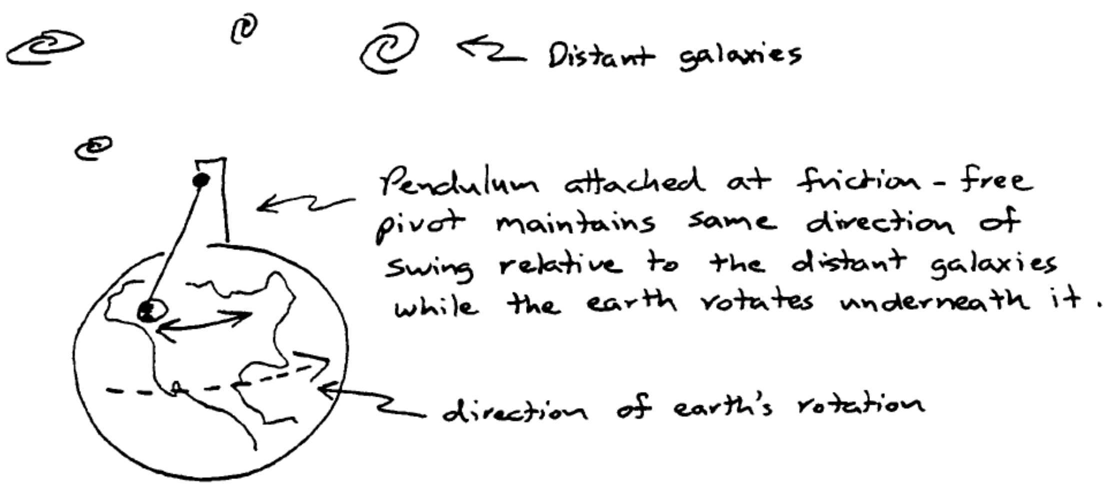

Figure 4.6. The Foucault pendulum. The path of a pendulum projected onto the surface at the North Pole, for example, would rotate around a circle like the hands of a 24-hour clock.

Just how can distant galaxies 'way out there affect mass here? By the force of gravity. One way to think about it (general relativity will provide another) is that the gravitational pull from masses in the farthest reaches of the Universe extends indefinitely in all directions, though it decreases as the inverse square of distance. Any mass feels the pull from all the distant galaxies scattered across the heavens in all directions. The mass behaves as if caught in a spider's web, anchored by strands from all points of the compass. Hence its inertia.

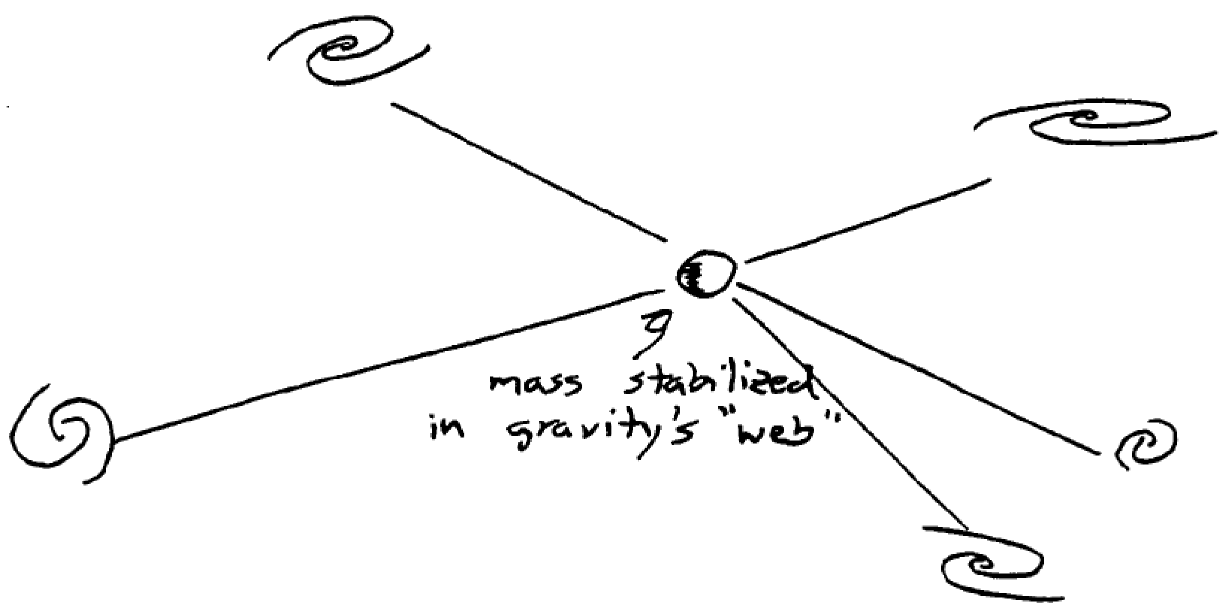

Figure 4.7. Gravitational fields from distant galaxies all pull on a mass. Since the galaxies (and the substance of spacetime in general) are uniformly distributed, the "pull" is equal in all directions.

As we shall see later in this chapter, Einstein changed the paradigm describing the large-scale structure of the Universe. In terms of general relativity, distant masses create the structure of spacetime in which test masses are embedded. It's as if there's a kind of stretchy fabric out there, the fabric of the universe, and masses here affect other masses, there, through ripples and warps in that fabric.

Gravitational effects on light

Einstein explored the consequences of the principle of equivalence and found that gravity produces some surprising effects. He discovered, for instance, that gravitational fields "bend" light.

Imagine a light source inside our imaginary spaceship, a flashlight, say, held by an observer inside the ship. Once again, imagine the spaceship drifting in empty space. The observer shines the light directly across the ship, parallel to the floor, and she observes that the light beam strikes the far wall at the same height she holds the flashlight.

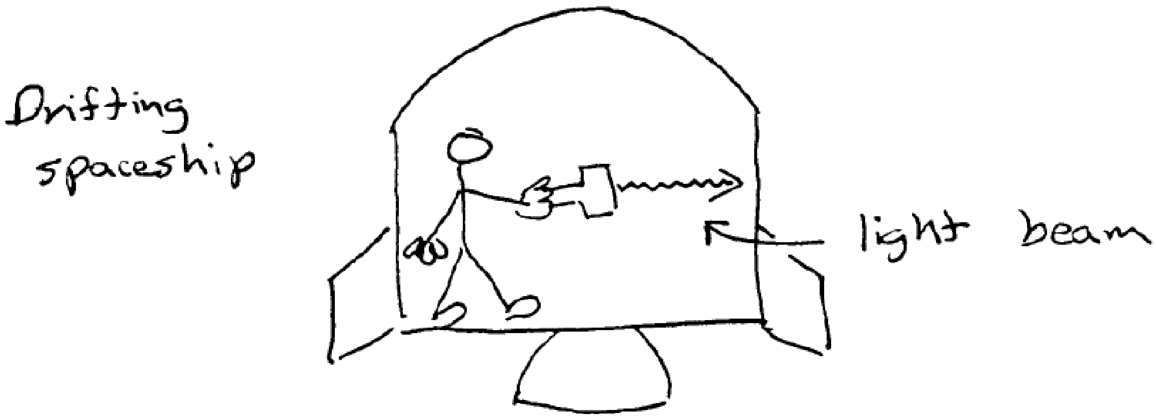

Figure 4.8. Light beam follows straight path in an inertial frame, as seen by observer in that frame.

Now the crew fires the engines. What happens to the light beam as it crosses an <u>accelerating</u> ship? An outside observer would see the light beam continue on its original course in relation to the distant stars while the spaceship accelerates past it. To our observer <u>inside,</u> the light beam appears to "fall" as it crosses the ship: the floor moves upward, toward the beam of light, in the time it takes the beam to cross the ship.

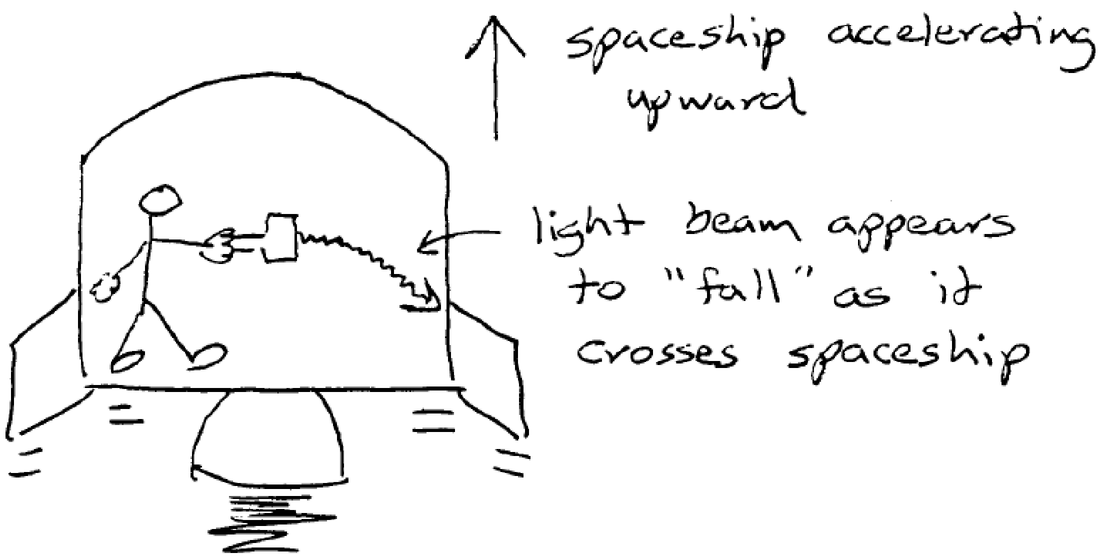

Figure 4.9. To observer inside accelerating frame of reference, light beam appears to fall.

The path of light appears to bend in an accelerating reference frame. By the principle of equivalence, since an accelerating reference frame is analogous to a gravitational field, <u>light also must bend in a gravitational field.</u>

In a real spaceship, light wouldn't fall very far: light travels so rapidly (about 300,000 km/sec) that it would cross the ship before falling noticeably. But suppose the spaceship is 300,000 km wide. It would take light one second to cross such a ship, and if the ship was accelerating 9.8 m/sec/sec the beam would fall 4.9 meters in that time.

Note our use of reference frames: an observer at relative rest outside the spaceship, in the reference frame of the "fixed stars," sees light traveling in a straight line through this (massless) sector of the Universe. Our on-board observer, using the ship as her frame of reference, sees light curving across that frame. In effect, acceleration – and, by the principle of equivalence, the gravitational field – curves space in the accelerating frame of reference.

Note, also, our argument implies that light has inertia. Just as the tennis ball "falls" – that is, follows its inertial path in an accelerating system – so light follows its inertial path relative to the accelerating frame of reference. And inertia implies light has an associated mass (or, equivalently, energy). We shall explore this idea more thoroughly when we discuss photons (particles of light).

Gravitational effects on clocks

Extending his thought experiment, Einstein showed how gravity affects time.

Imagine, once more, our observer inside her spaceship. She carries a very accurate clock, which emits a pulse of light at each "tick." She places the clock on the floor. While the ship drifts, the clock "ticks" at its regular rate, and the observer sees flashes of light at the same rate they are emitted by the clock.

Now accelerate the ship. Because the ship and the observer, pushed upward by the ship's floor, increase velocity during the time it takes light to travel from the clock to the observer's eye, the clock appears to slow down. Each light pulse has to travel farther from the clock to the observer’s eye in the accelerating frame, so the observer measures a longer interval between "ticks" than in the inertial frame. As seen by the accelerated observer, the floor clock runs slower. One might suppose that the clock would appear to run down, slower and slower, during the acceleration, but the inside observer sees the clock ticking at the same rate throughout the acceleration. It slows because the increment in the distance between clock and observer is the same from tick to tick in a uniformly accelerating frame.

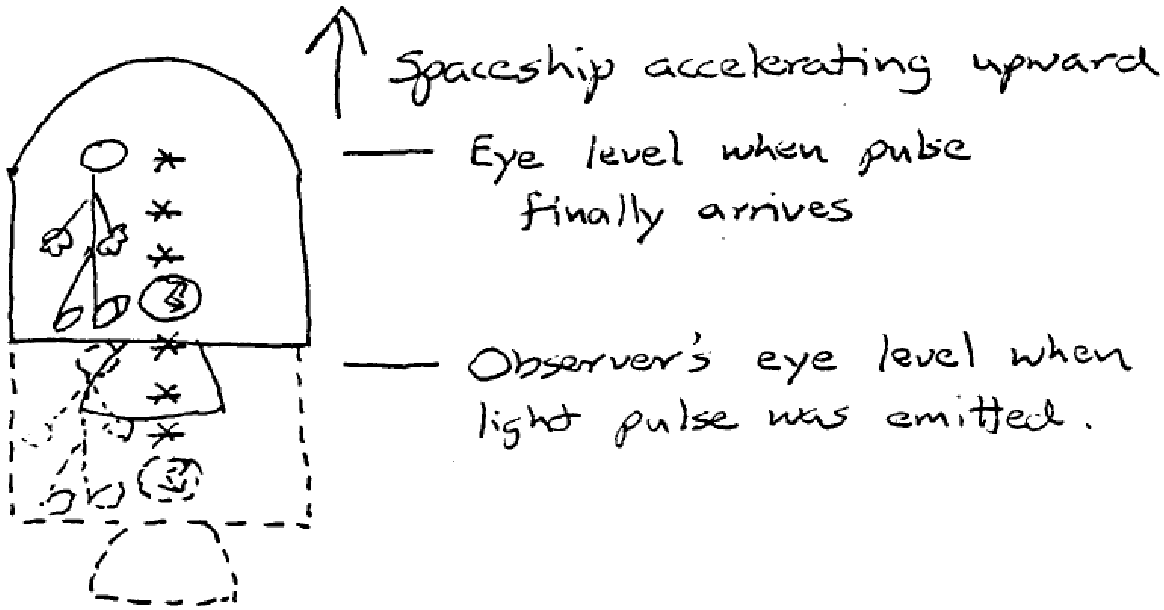

Figure 4.10. Clock on floor of accelerating spaceship ticks slower as seen by observer standing above it.

By the principle of equivalence, a gravitational field also must slow a clock located below an observer standing on a large mass (such as a planet). On the other hand, if the clock is attached to the ship's ceiling or, equivalently, above an observer on a planet surface, it would appear to run faster: the observer accelerates upward toward the light pulses, so each pulse travels less distance to reach her and arrives sooner than it otherwise would.

Experimental evidence

Einstein's model is interesting, but does it describe the real world?

A number of experiments have tested general relativity, and all of them verify Einstein's predictions to within the limits of experimental error. We shall describe only a few:

The initial verification of the theory came in 1919. Sir Arthur Eddington, a British astronomer, tested Einstein's prediction that the gravitational field of the sun should bend light from distant stars. This can only be observed (with optical telescopes) during a total eclipse of the sun.

Eddington mounted an expedition to observe the eclipse of 1919. His observations were in good agreement with the theory: the sun's mass, with its associated gravitational field, really does bend light. (More accurately, as we shall discuss below, the mass warps spacetime, and light follows a "straight" (geodesic) path through curved spacetime.)

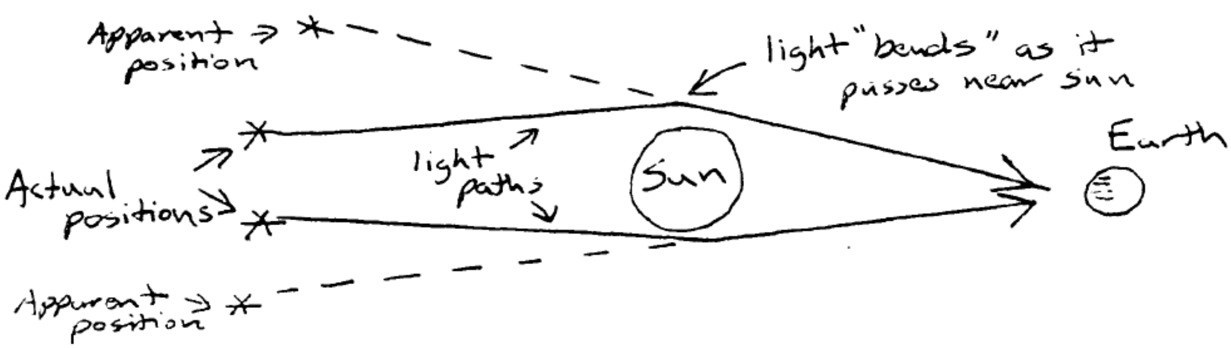

Figure 4.11. Stars that are physically behind the sun can be seen during an eclipse because gravity bends the path of light coming from the stars to observers on Earth. This phenomenon is referred to as a gravitational lens: masses bend the path of light just like lenses.

In recent years, astronomers have tested this effect of general relativity with more sophisticated equipment, including radio telescope observations of quasars occulted by the sun. These observations agree nicely with the theoretical predictions.

In a way, a massive body such as the sun acts like a lens: it focuses light from distant celestial objects. Astronomers have discovered several galaxies that lie near the path of light coming to us from quasars (exceedingly luminous and distant objects). This alignment can produce unusual optical effects: in some instances, depending on the shape of the intervening galaxy and the relative position of the galaxy to the quasar, the gravitational lens splits the image of the quasar into two or more identical images or even into a ring.

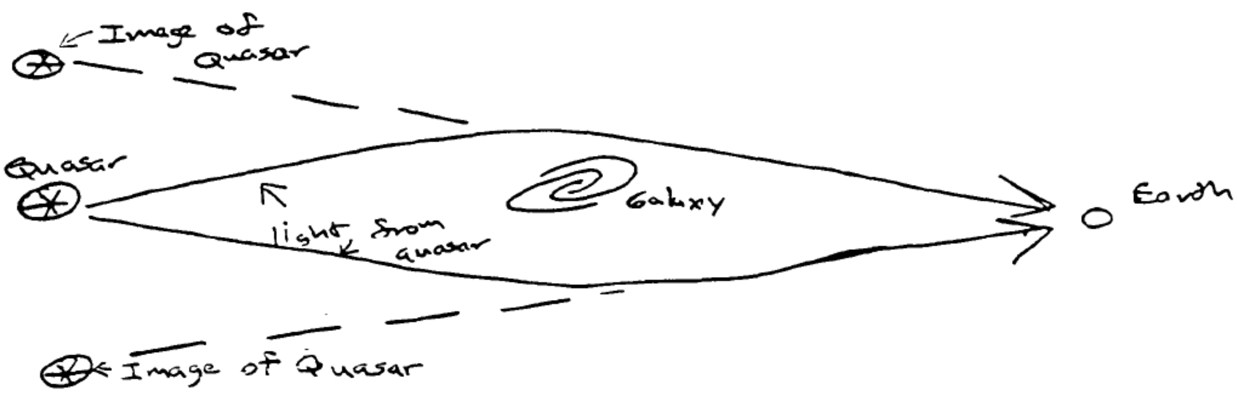

Figure 4.12. Gravitational lensing by a galaxy along the line of sight between Earth and a distant quasar.

<u>Verification of the gravitational effect on clocks.</u>

The Pound-Rebka experiment provided the premier test of gravitational effects on clocks. Two identical atomic clocks, each emitting a specific frequency of electromagnetic radiation, were synchronized at Earth's surface then separated by several tens of meters in the Harvard science tower. The higher clock ticked measurably faster than the lower clock.

The experimental method is in itself interesting. To measure the change in clock rates, the researchers used atomic clocks that emit electromagnetic radiation at a precise frequency. They moved one clock relative to the other, increasing its velocity until the frequency of the one clock matched the frequency of the other at a detector placed between the two. They found the frequencies matched if the lower clock moved at a small velocity upward, toward the upper clock, or the upper clock moved upward, away from the lower clock.

Gravity really does slow down time.

<u>Doppler shift</u>

The experimental procedure in the Harvard tower experiment depends on a phenomenon known as Doppler shift. Since we will encounter other circumstances involving Doppler shift, let's spend a moment to understand it.

When an observer is moving relative to a wave source, he measures an apparent change in frequency. There are a number of familiar examples:

- If we move a can toward a drip bottle hung from the ceiling, the drops arrive at a higher frequency. The faster we lift the can, the higher the frequency detected at the can. Alternatively, if we lower the can from the level of the drip source toward the floor, the drops arrive at a lower frequency.
- On a lake, if we paddle into the wind (toward the wave source), the waves slap the bow at a higher frequency. If we paddle down wind, in the same direction the waves are traveling, the waves slap the bow at lower frequency.

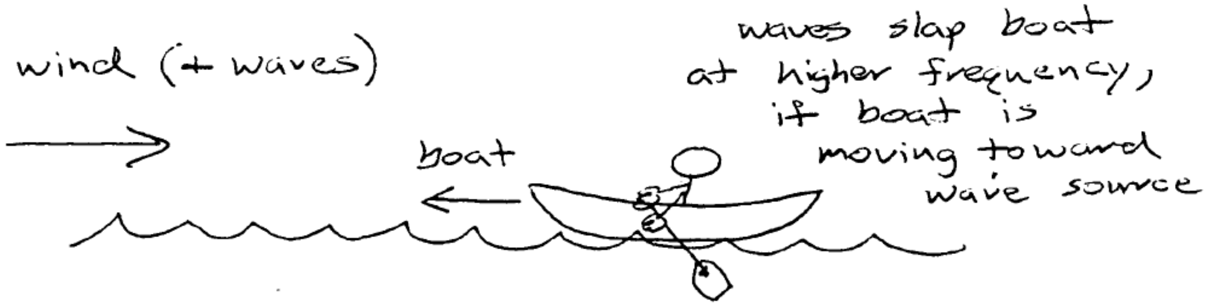

Figure 4.13. Demonstrating Doppler shift with a boat moving into the oncoming waves (increases the frequency of waves slapping the boat) or away from the oncoming waves (decreases the wave frequency).

- If we're on a street corner when an ambulance approaches, we hear the siren at a higher pitch (sound waves arriving at a higher frequency) than the pitch heard by the para-medics in the ambulance. As the ambulance passes, we hear the pitch drop to a lower frequency.

In the tower experiment, the experimenters had to move the upper clock away from the lower clock at a small velocity in order to match frequencies at the detector. Doing so decreased the frequency of the upper clock as measured at the detector. Hence, the upper clock must have been ticking at a higher frequency than the lower clock.

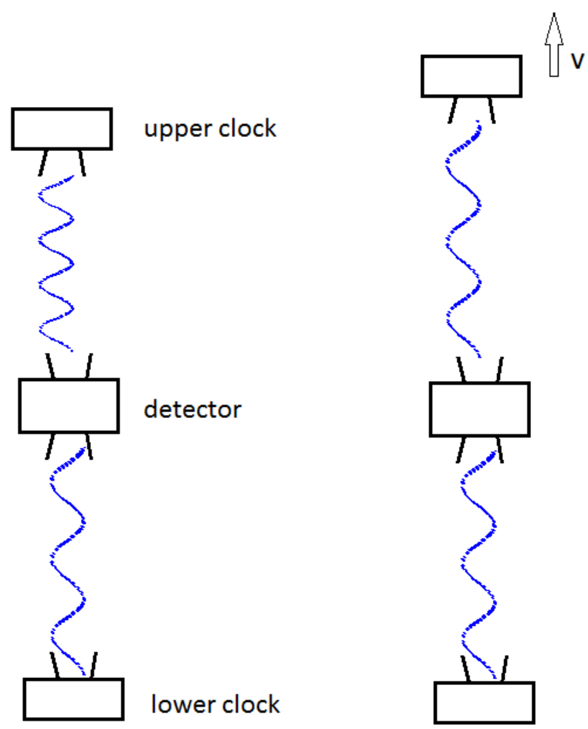

Figure 4.14. In Earth's gravitational field, frequency of upper clock, farther from Earth's center, is higher than the frequency of the lower clock as measured by detector in between. The difference in frequencies can be determined by measuring the velocity, v, required to equilibrate the two clock frequencies.

As we shall discuss in the next chapter, there's another way to interpret these experiments: light loses some of its energy, hence decreases in frequency, climbing out of a gravitational field. It gains energy, increases frequency, falling into a gravitational field.

Summary

In the theory of general relativity Einstein explores the consequences of the principle of relativity in accelerating reference frames. Particularly, general relativity is Einstein's theory of gravity. Einstein found that a gravitational field changes clock rates and bends the path of light. These predictions have been verified experimentally.

That clocks and light paths must change in a gravitational field can be deduced from the principle of equivalence: the effects of gravitational acceleration are the same as effects seen in other accelerating reference frames.

The predictions of general relativity have been verified by a number of different experiments. Among other effects that have been measured, light bends in a gravitational field, and clocks slow.