General Relativity
Study Questions

1. What is the principle of equivalence? Give an example illustrating the principle.

2. A news reporter covering a manned space launch reports that the astronaut is subjected to a "force" of "3g's" at liftoff. What is the astronaut's acceleration?

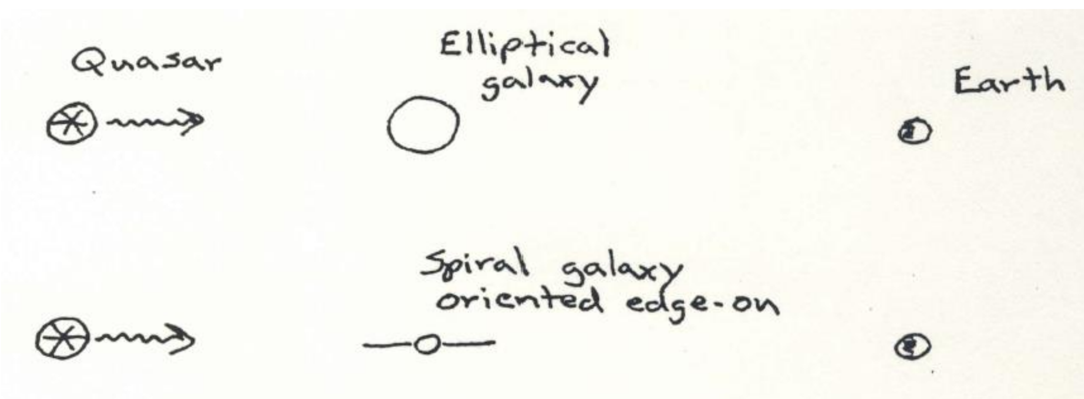
3. Describe a "thought experiment" (imaginary experiment) showing that light bends in a gravitational field.

4. Describe actual experiments proving light bends in a gravitational field and clock rates change in a gravitational field.

5. How far will light fall as it crosses a room 30 meters wide?

6. The sun has about a million times the mass of the Earth, and the sun's radius is about 100 times that of Earth. How far will light from a distant star fall as it traverses a distance of 30,000 km near the sun's surface?

7. Illustrate how a gravitational field can behave like a lens. How would you expect a distant quasar's light to refract in the vicinity of a spherical elliptical galaxy? Near an edge-on spiral? What images would we see on Earth?



8. How fast does a clock 100 meters above the surface of the Earth tick compared to a clock at ground level? Assume the change in clock rate is proportional to the change in the force of gravity.

9. Why are four "dimensions" required to measure events in the Universe? What are those four dimensions?

10. If there was no motion in the Universe, would we need clocks? Discuss.

11. What is "spacetime?" How is spacetime related to mass? Can energy warp spacetime? Why?

12. Discuss the statement, "light follows a straight line through curved spacetime."

13. Light escaping from a star is "red-shifted:" i.e. we see a lower frequency light (longer wavelength) than was emitted at the surface of the star. What is the mechanism of the "red shift?"

14. How much energy does light lose climbing to 500 km above the Earth's surface? To 500 km above the surface of a neutron star? (Assume the mass of the neutron star is three million times the mass of the Earth, i.e. about 6 x 10^{30} kg., and that the neutron star has a radius about 0.002 times Earth's radius. Potential energy near Earth's surface is *mgh*, where *g* is the acceleration due to gravity and *h* is the height above the surface.)

15. What happens to the rate of a clock at the event horizon of a black hole?

16. What is the metric? What factors contribute to the metric? In what way does it define the "latitude and longitude" of spacetime?

17. Why does the metric vary in time as well as in spatial position near a mass?

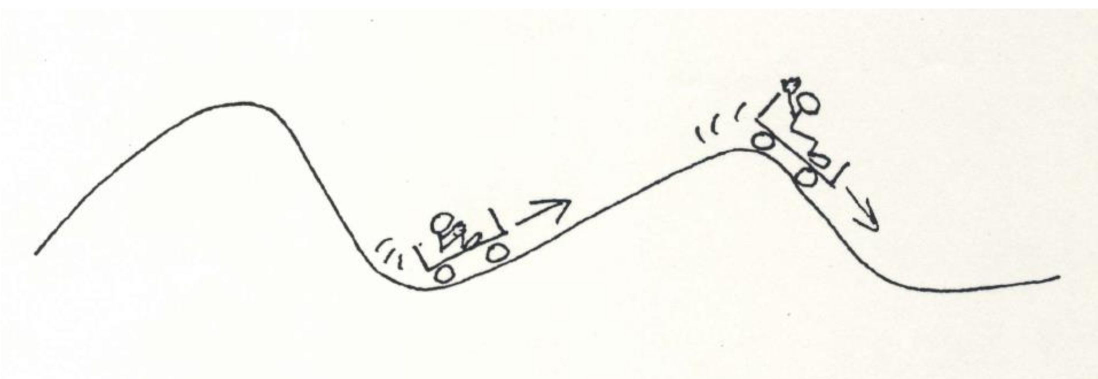
18. Describe a method by which we might measure the curvature of spacetime near a star.

19. Compare Einstein's concept of gravity with Newton's.

General Relativity
Demonstrations

Principle of equivalence

1. Ride a roller coaster. As your car bottoms out after a steep downhill run, how "heavy" do you feel? Why? As your car tops a rise at high speed and starts to fall down the other side, how "heavy" do you feel? Why?



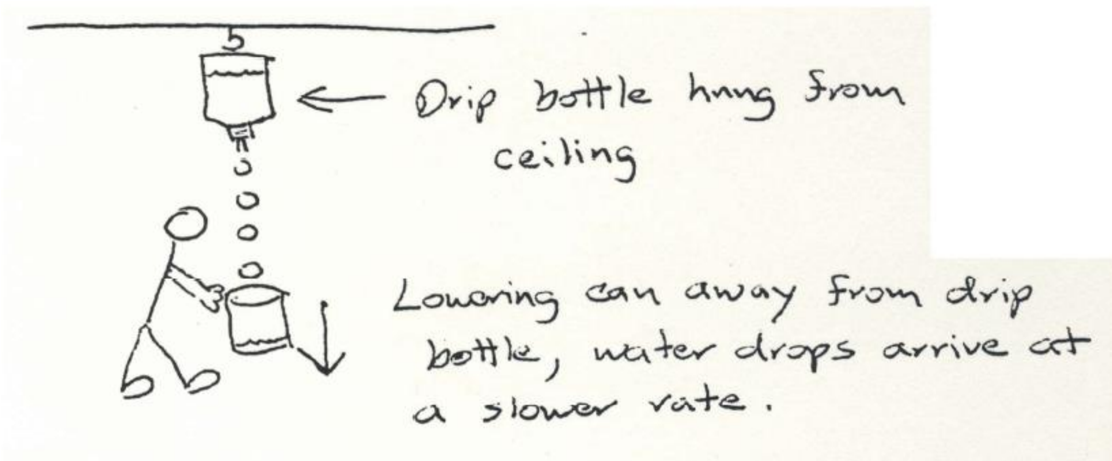
2. If you have ever flown in an airplane, you have experienced the principle of equivalence. On takeoff, as the plane accelerates and noses up off the runway, how heavy do you feel?

Bending light in a gravitational field

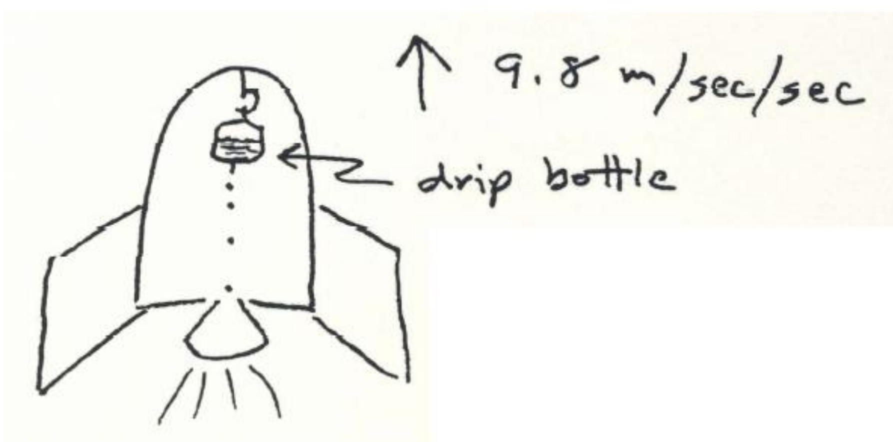
3. Using mirrors and a laser, see if you can design an experiment to test whether light falls in the Earth's gravitational field. What are the practical limitations of your experimental design?

Clocks in a gravitational field

4. A drip bottle models the effects of gravity on clocks. Hang a drip bottle from the ceiling. (Discarded IV drip bottles work nicely.) Set the drip rate at about one drop per second. Hold a tin can underneath the drip, and move the can upward toward the dropper. What happens to the rate at which the drops strike the can? Now move the can away from the dropper. What happens to the rate at which drops strike the can? Compare these results to the apparent changes in clock rates in a gravitational field as seen by an observer above or below the clock.

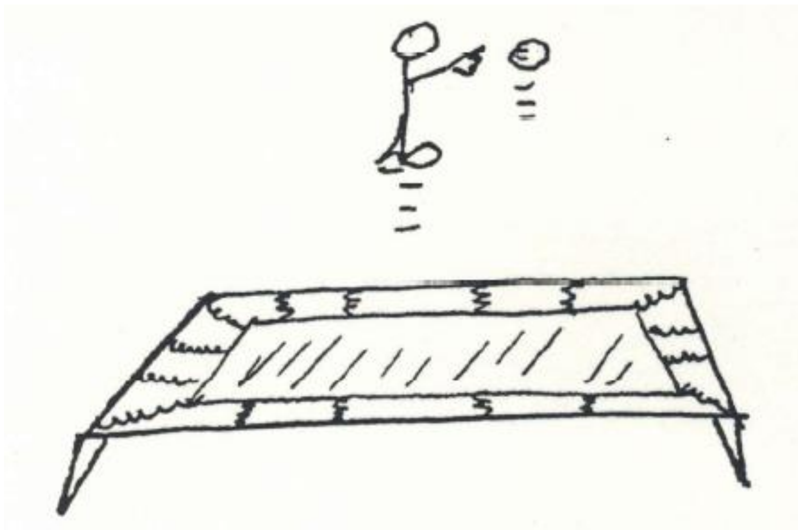


5. Set up the drip bottle demonstration, as described above. Would the demonstration appear any different if you were in a spaceship accelerating at 9.8 m/sec/sec?



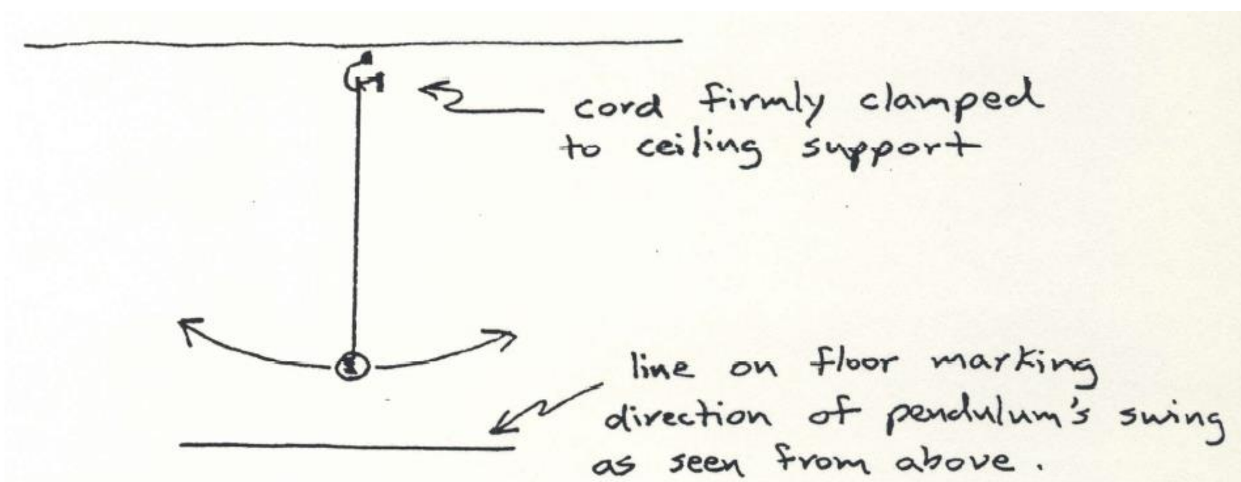
Spacetime trajectory

6. Carry a tennis ball on a trampoline. Let go of the ball while jumping, in mid-air, without pushing the ball. What path does the ball follow relative to you? Relative to the ground? How does the path of your tennis ball compare to the path of a ball carried on board a space shuttle in Earth orbit?

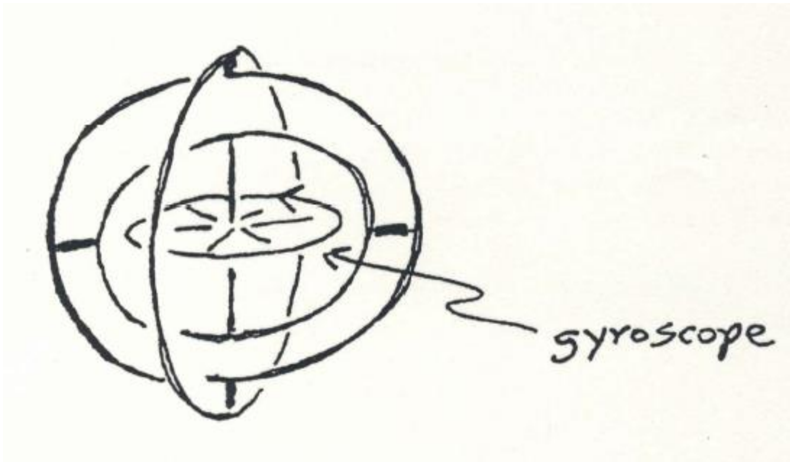


Foucault pendulum, gyroscope, and Mach's Principle

7. Hang a two pound pendulum bob from a high ceiling by a cord long enough that the weight is an inch or two above the floor. Start the bob swinging on a smooth arc so that it follows a straight line as seen from above. Mark the line of swing on the floor. Return after an hour and mark the line of swing. What has happened, and why?

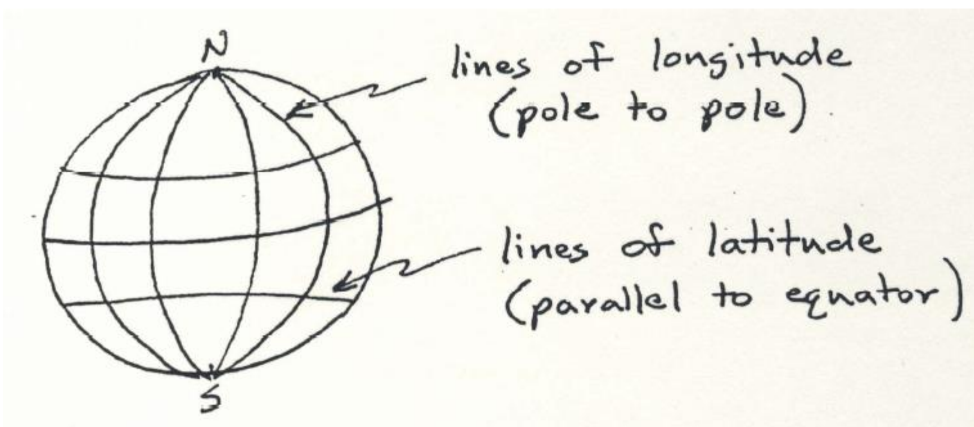


8. Find a gimballed gyroscope (one that can rotate freely about any axis). What happens to the axis of the gyroscope as you rotate the frame around it? Explain in terms of Mach's Principle.



Geodesics

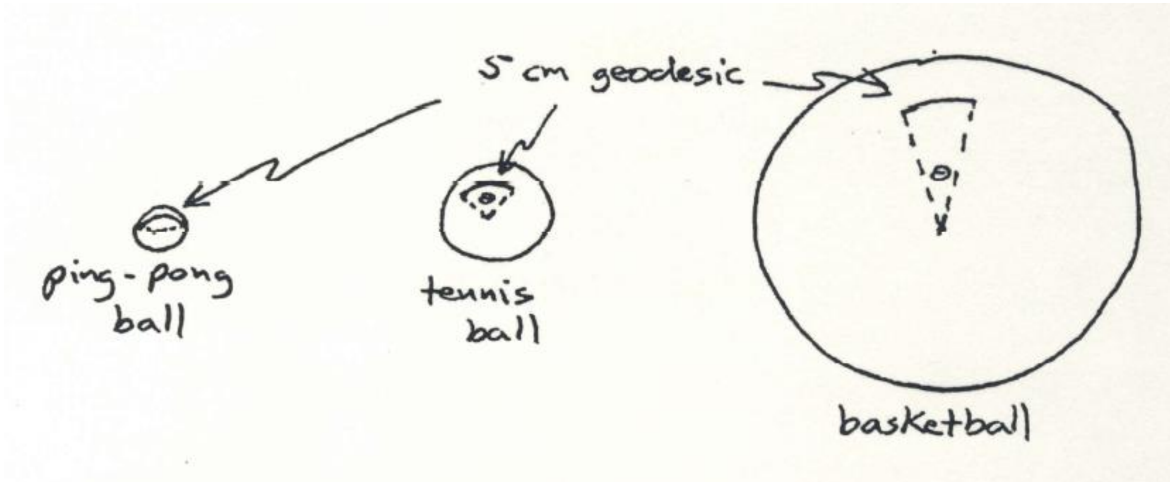
9. Find a globe (spherical map of the Earth). Observe the lines of latitude and longitude and the time zones. What happens to "parallel lines" on the surface of the Earth? What is the longitude at the North Pole? What happens to time and distance, measured relative to the lines of latitude and longitude, as you travel north from the equator?



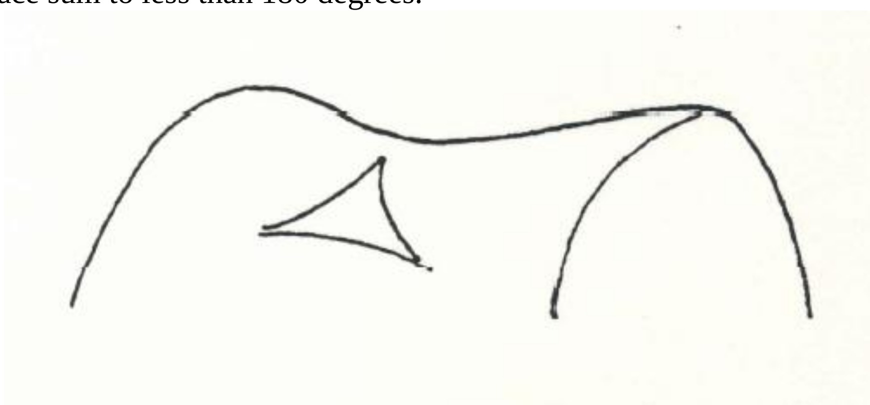
10. On a world globe, find the geodesic between New York and New Delhi. To do so, pull a string taut over the globe, holding one end at each city. Now trace the path of the geodesic on a flat map (e.g. Mercator projection) of the world. What does the geodesic look like on a flat map?

Curvature

11. Draw 5 cm geodesics on a ping-pong ball, a tennis ball, a basketball, and on the Earth's surface. Compare the angles subtended by the geodesic (the angle from the center of the sphere to each end of the geodesic) on each of these spheres. Which sphere has the greatest curvature? Which has the least?

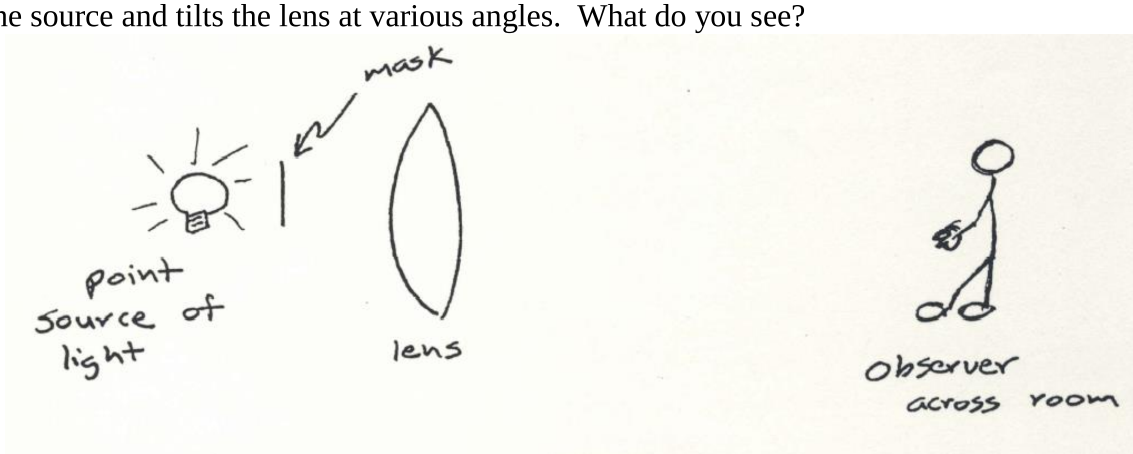


12. Convince yourself that the angles of a triangle drawn on a sphere sum to greater than 180 degrees. (Hint: draw a triangle with one vertex at the North Pole and the other two vertices on the equator.) Convince yourself the angles of a triangle following geodesics on a saddle-shaped surface sum to less than 180 degrees.



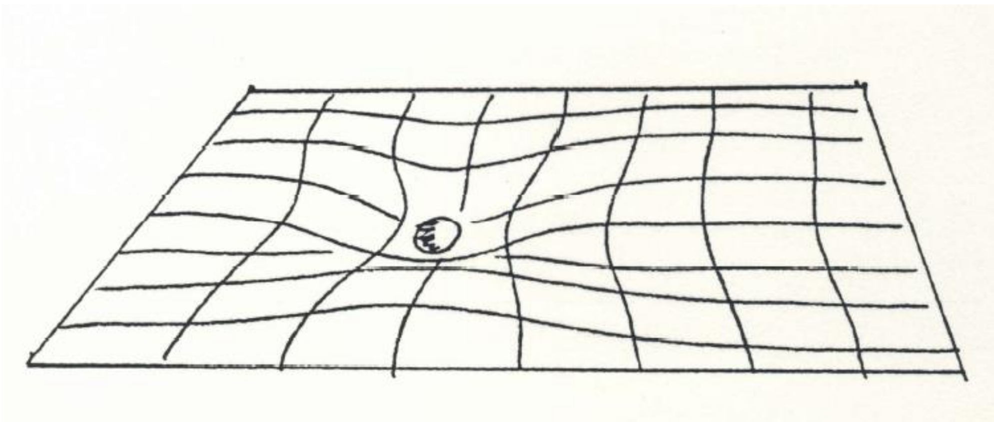
<u>A model of gravitational lenses</u>

13. Place a point source of light across the room (a bright flashlight bulb will do). Mask it with an opaque barrier just wide enough to cover the source as you look from across the room. Now place a large convex lens between you and the mask. The lens must be larger diameter than the mask. Stand across the room and observe while a friend holds the lens at various distances from the source and tilts the lens at various angles. What do you see?

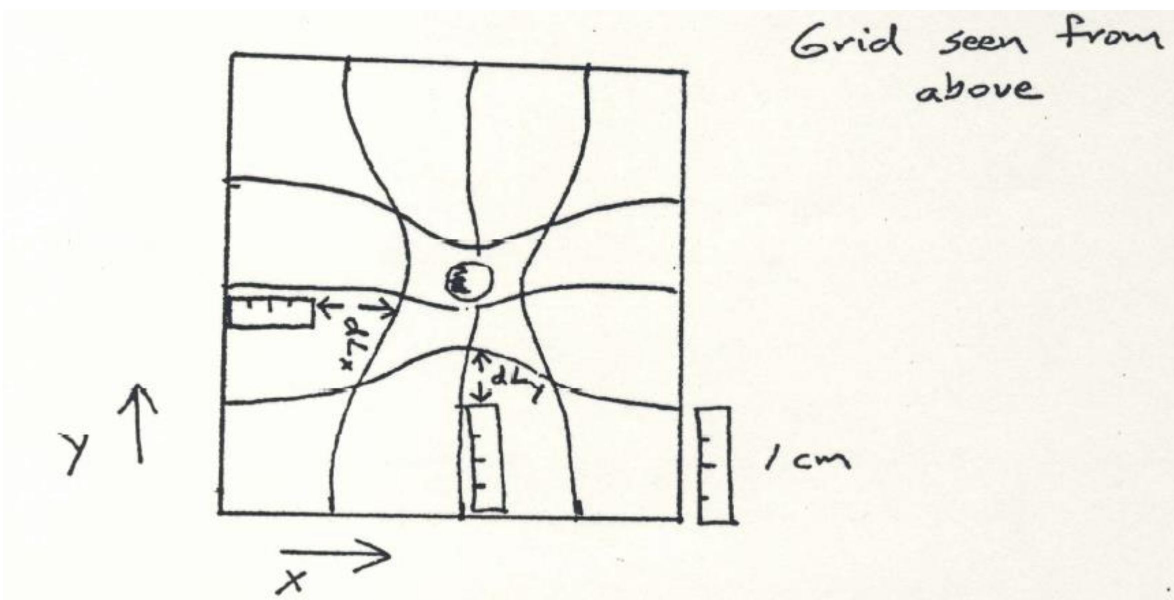


Two-dimensional model of spacetime

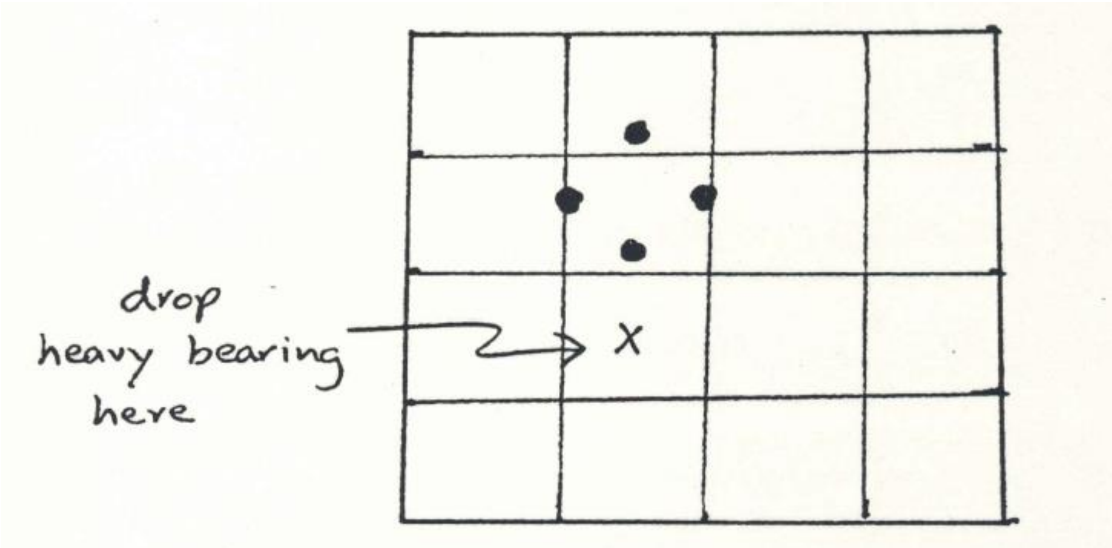
14. Build the drumhead model of Einstein's gravity: stretch nylon or a rubber sheet across a basin. Mark a grid (*x* and *y* coordinates equally spaced one centimeter apart) on the surface. Place bearings of different weights on the surface. How much do the different weights distort the gridwork?



Calculate the metric around different weights. For instance, place a heavy bearing on the surface, then measure d*Lx*/d*x* (the distance, in centimeters, between grid marks at different distances from the bearing along the *x* axis), and measure d*Lx*/d*y* (the distance, in centimeters, between grid marks along the *x* axis at different distances along the *y* axis from the bearing). How does d*Ly*/d*y* compare with d*Lx*/d*x*? Repeat the above measurements with a light marble.



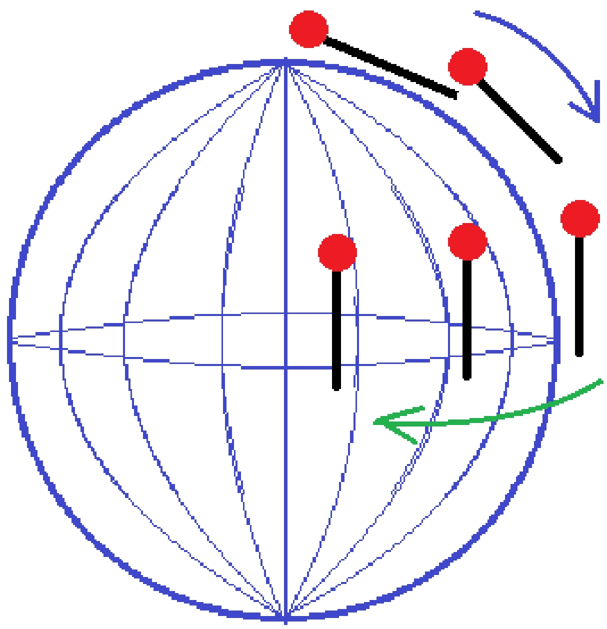
Draw dots in a diamond shape on the surface. Drop a heavy bearing on the surface near one of the dots. What happens to the relative positions of the four dots? Compare the curvature produced by equal masses with different diameters. How much does a 100 gm bearing distort the surface compared to a 100 gm tennis ball? How can you measure the distortion?



Measuring Curvature: The Riemann Curvature Tensor

15. There is another, equivalent, method of measuring curvature besides the metric. The Reimann curvature tensor measures change in orientation of a vector as it is transported around a closed path.

    Find a globe marked with lines of latitude and longitude. Place a match stick at the north pole, parallel to one of the lines of longitude, say the Greenwich meridian. Now "parallel transport" the match down the meridian to the equator (i.e. move the match so that it is always parallel to the meridian line). Then transport the match westward along the equator to longitude 90 degrees west, always keeping the match perpendicular to the equator. Then transport the match back north to the pole at the same angle as it started along the 90^{th} meridian. Notice that, when you return to the pole, the match points 90 degrees from its original orientation relative to the Greenwich meridian, as if it had been rotated 90 degrees clockwise.



Parallel transport along geodesics starting on the Greenwich meridian at the north pole (matchstick at the top). If the match is now transported northward from the equator up the 90^{th} meridian (represented by vertical line to left of last drawn position of the match) it will end up at the north pole perpendicular to its starting orientation.

Notice, in the exercise described above, that the match always takes its traveling orders from the "great circle" on which it sits – from the local line of longitude along which it moves or from the equator. These lines are among the geodesics of the globe, lines marking the shortest path (on the surface of the globe) between two points. (Note that the lines of latitude are <u>not</u> geodesics: they are not great circles, centered on the center of the earth, and they do not represent the shortest paths between points. To see that this is so, place a piece of string along the fortieth parallel north, between Denver and Paris, say. Measure the length of string between the two cities. Now arc the string slightly north of the fortieth parallel, so that it curves up from Denver, then back down to Paris along a great circle. You will find that this great circle route, the geodesic, provides a shorter path between the cities. In fact it is the shortest possible path.)

Notice also that the deviation of the match around the closed path is proportional to the surface area enclosed by the path: transporting the match down the Greenwich meridian, then 45 degrees west, and back to the pole produces only half the angular deviation as before.

Measuring angular deviation of a vector, here represented by the match, provides a method of measuring curvature because – and here is the main point of the argument – the area enclosed by a parallel transport is inversely proportional to the curvature; i.e. the greater the curvature (on a smaller sphere), the smaller the area. If you transport the vector around the same path on two different-size globes, pole to equator then 90 degrees west and back to the pole, the smaller globe (with greater curvature) has smaller area enclosed by the transport path.

Vector deviation can be described mathematically by the Riemann curvature tensor. The larger the terms in the tensor, the greater the deviation of a test vector around a closed path, and the greater the curvature.

Note that parallel transport on a flat USGS topographical map, which shows just a small portion of earth's surface, ends up with the vector still pointing north. Measuring the effects of general relativity requires large transports (equator to north pole) or very fine measuring instruments.