CHAPTER 5

QUANTUM MECHANICS

We turn now to the second great development in modern physics, quantum mechanics. Quantum mechanics describes events on the smallest scale. It outlines rules governing the behavior of atoms and their constituent elementary particles. Studying quantum mechanics, we find a realm filled with surprises.

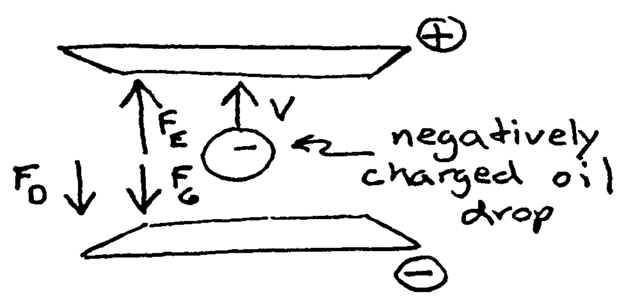
While relativity theory followed from a logical series of deductions, quantum mechanics developed ad hoc, to explain experimental results. In this chapter, we discuss the basic concepts of quantum mechanics and the experimental evidence from which they evolved. Particularly, we consider nature's quantum structure, the wave attributes of matter, the uncertainty principle, and the random nature of events at the atomic scale. We will also consider a few of the practical spin-offs of quantum theory: semiconductor technology, lasers, magnetic resonance imaging, and superconductors.

QUANTA OF ELECTRIC CHARGE

The term "quantum" infers a "packet" or "parcel." The macroscopic Universe is built from discrete bits, or "quanta," and there are fundamental quanta -- i.e. smallest bits which cannot be further divided. Moreover, all aspects of the subatomic realm are quantized. As examples, we shall discuss evidence for the quantization of electric charge, mass, spin, and energy.

Robert Millikan proved the quantization of electric charge with his "oil drop" experiment. He found it possible to suspend a microscopic drop of oil between two charged metal plates and to determine the electric charge on the oil drop by observing how it drifted in relation to the plates.

Three forces act on the drop: gravity, the electric force due to charge on the plates, and a "drag" force due to friction of collisions with air molecules. These forces determine the drift velocity at any given time.



F_D = drag force = Dv

$$Dv - mg = 0 \quad \text{(field off)}$$

$$q_1 E - mg - Dv_1 = 0 \quad \text{(field on)}$$

Therefore

$$q_1 = \frac{mg}{E}\left(\frac{v + v_1}{v}\right)$$

where m = mass of oil drop
v = free fall velocity
v_1 = drop's velocity in field when drop has charge q_1

Millikan noticed the drift velocity changed abruptly from time to time, and the abrubt changes occurred more frequently if he ionized the air between the plates (e.g. by placing a radioactive source nearby). Presumably the sudden change resulted when the drop acquired an extra negative charge (i.e. an electron) or extra positive charge (i.e. an atom stripped of one electron).

$$q_2 = \frac{mg}{E}\left(\frac{v + v_2}{v}\right)$$

where v_2 = drop's velocity when it has new charge q_2

Therefore

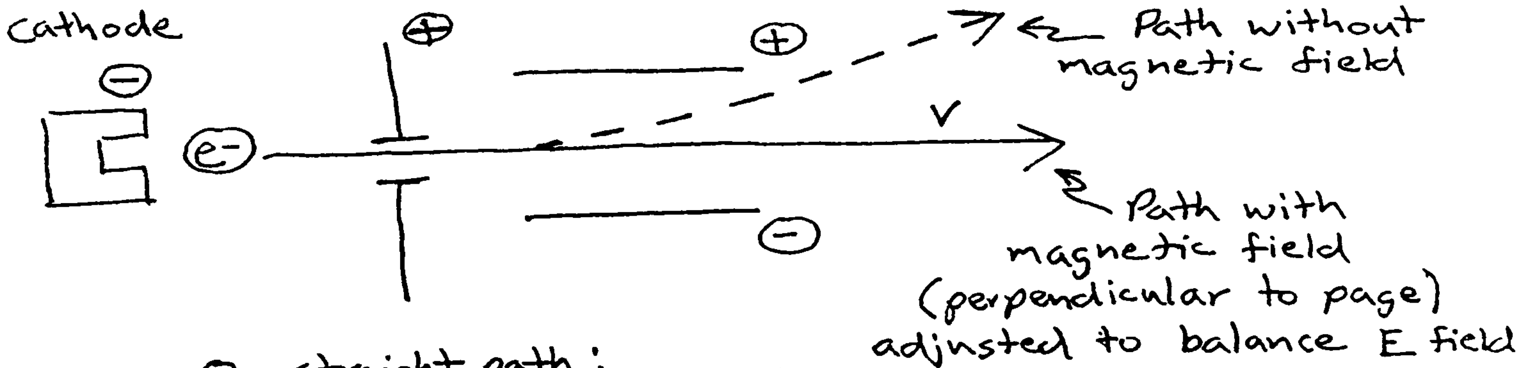
$$\frac{q_1}{q_2} = \frac{v + v_1}{v + v_2}$$

In a painstaking series of measurements, Millikan determined that the ratio of the charge before an acceleration to charge after the acceleration was always a ratio of small integer numbers (e.g. 2/3, 5/9, 3/4) -- direct evidence that charge is quantized. Knowing the mass of the oil drop (from other measurements) and the magnitude of the electric field between the plates, he could calculate the charge on an individual electron. It is about 1.602 X 10^-19 coulomb.

Atoms are exactly electrically neutral, to the best experimental accuracy. A carbon atom, with six protons in its nucleus and six electrons surrounding the nucleus, will not drift in an external electric field: it has net zero electric charge, so it feels no force. The charge on the proton, then, must be exactly equal in magnitude, but opposite in sign, to the charge on the electron.

QUANTA OF MASS

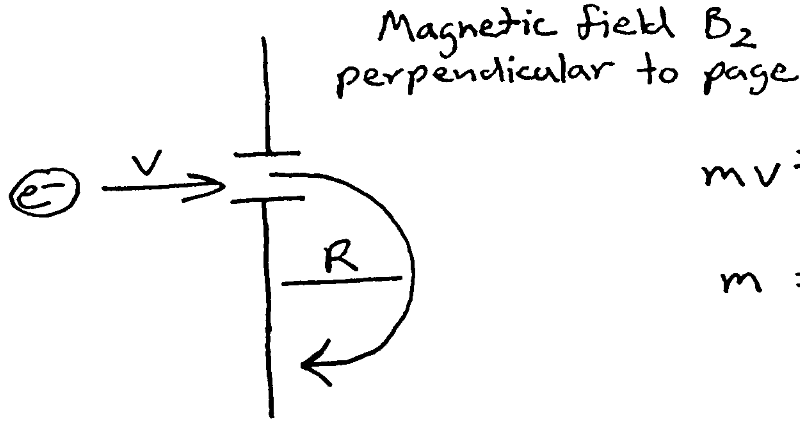
Knowing the charge on an electron, scientists can measure its mass: Accelerate electrons down a known electric potential, and direct them through a region between charged plates that is also permeated by an adjustable magnetic field oriented perpendicular to the electron beam and to the electric field. This region selects electrons with a particular velocity.



On straight path:

$$qE = qvB$$
$$v = E/B$$

Now channel the electrons into another magnetic field, but without an electric field, and measure the radius of curvature of the electron beam. The centrifugal force must equal the magnetic force on the electrons.



$$mv^2 = qvB_2$$
$$m = \frac{qRB}{v}$$

We know the electric charge (from Millikan's experiments). We know the velocity, from the calculations above, and we can measure the magnetic field and the radius of curvature. We can, therefore, calculate the mass of the electron: It is about 9.11 X 10^-28 gm.

Another, independent determination of the electron mass comes from studies of the electron's rest-mass energy. In every electron/positron annihilation, exactly the same energy is released per particle -- 0.511 mEv (assuming the annihilating particles are at relative rest). By the equivalence of energy and mass (E = mc^2), the mass of any electron (or positron) must equal the mass of every other electron. (0.511 mEv corresponds to a mass of 9.11 X 10^-28 gm.)

$$e^- + e^+ \longrightarrow 2\gamma$$

each γ ray has an energy of 0.511 million electron volts

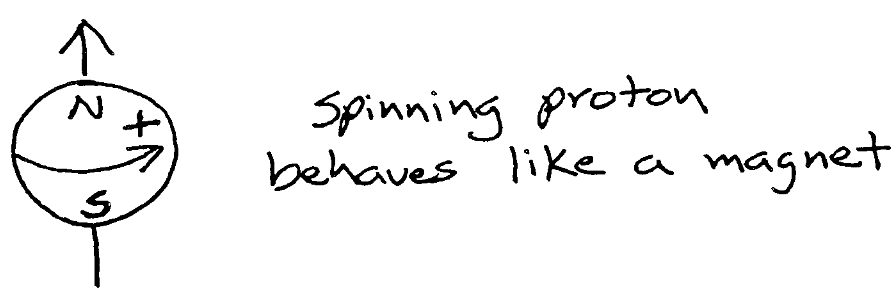
QUANTA OF SPIN

Spin, also, is quantized. Subatomic particles behave, in a way, like spinning tops (they produce an associated magnetic field, as would an electrically charged top), and their angular momentum -- their spin -- is quantized. Electrons, for instance, spin either + or -h/2. The factor 1/2 indicates an electron may spin with its axis pointed one of two directions -- up or down. (By convention, if we wrap the fingers of our right hand in the direction of spin, the thumb points along the spin axis.) The factor h, Planck's constant, represents the "fundamental quantum," the smallest possible parcel -- in this case, the smallest possible parcel of spin.

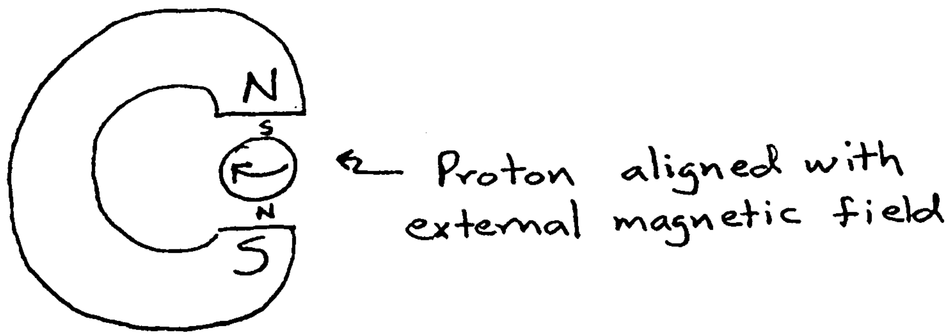
APPLICATIONS OF QUANTIZED SPIN: MAGNETIC RESONANCE IMAGING

It's possible to use the phenomenon of quantum spin in practical devices. For instance, MRI -- magnetic resonance imaging -- allows doctors to see soft tissue structure inside the body. (Standard X-rays also image structures inside the body, but X-rays only define edges -- air/water or water/bone -- between tissues of grossly different density.)

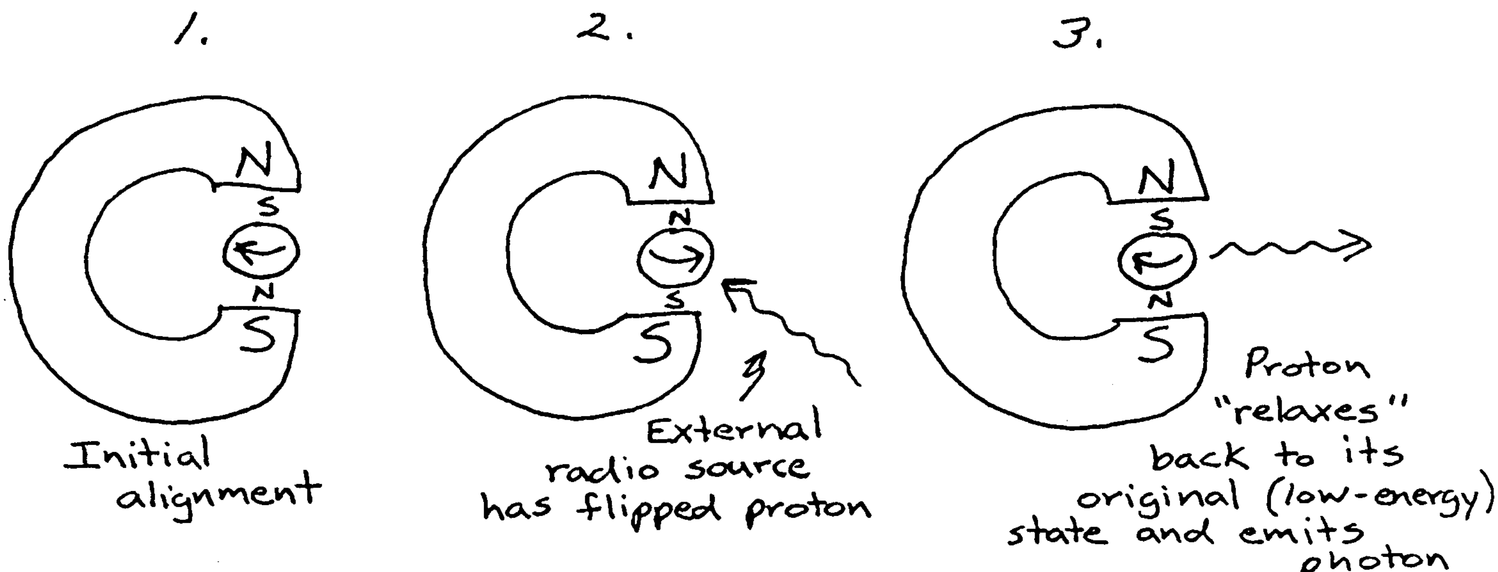
MRI works on the following principle: A spinning proton produces a magnetic field, as does any moving charge.



Placed in an external magnetic field, a proton tends to align itself with the field.



Radio waves at the appropriate frequency will flip an aligned proton to a polarity opposite the external field, but the proton quickly flips back into alignment with the field. When it does so, it emits a radio wave of the same frequency (and energy) as the one it had absorbed. (It takes work to turn the proton opposite to the field, so energy is released when it re-aligns with the field.)

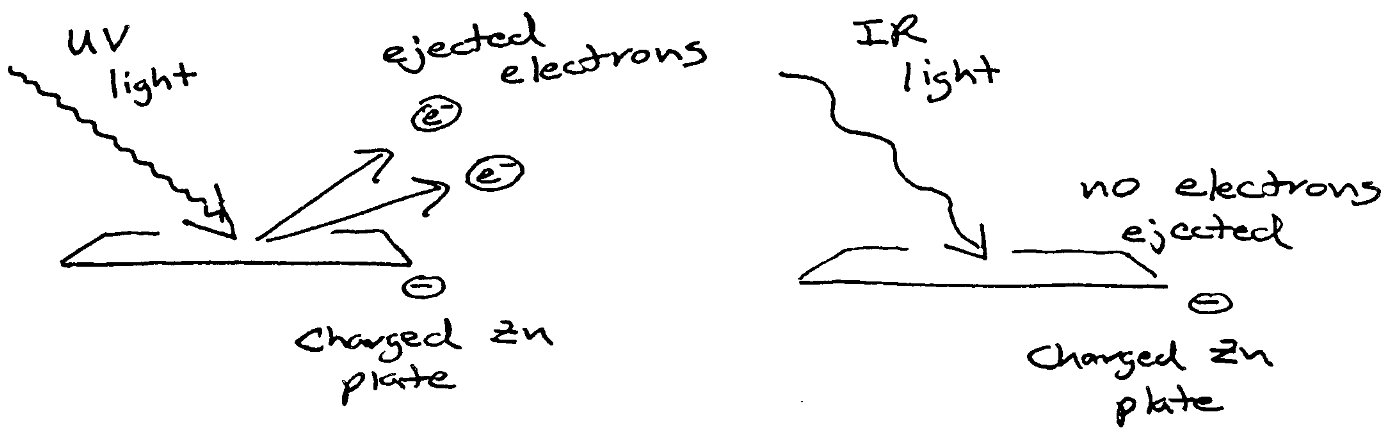


The amount of energy released in the transition depends on the strength of the external field and on the proton's magnetic environment -- the magnetic fields produced by its neighboring neutrons, protons, and electrons. Protons in the molecules of different body tissues have different "neighborhoods" and therefore emit photons of different frequency. The transition frequencies, then, reflect the character of the "neighborhood."

In practice, a person is placed in a very strong (and safe) magnetic field. A tunable radio transmitter then sweeps the tissues under study, and a radio receiver captures photons produced in proton spin-flip transitions. By measuring the photon energies, it's possible to map the distribution of biological molecules inside the body -- and create a detailed image of internal anatomy. Such devices have revolutionized medical diagnosis and obviated much diagnostic surgery.

QUANTA OF ENERGY

Historically, the photoelectric effect provided the first clue to the quantum nature of energy. If you shine ultraviolet light on a negatively charged zinc plate, electrons boil off the plate. The greater the intensity (the brighter the light), the more electrons, but red light or infrared produces no electrons at all.



It turns out there's a minimum frequency of light required to eject electrons from the zinc. (The minimum frequency differs with different metals.) Below that frequency, no electrons emerge. At frequencies above the minimum, electrons emerge with kinetic energies proportional to the frequency: the greater the frequency, the greater the kinetic energy.

Physicists puzzled over this phenomenon until Einstein realized the photoelectric effect could be explained if light came in quanta -- called "photons" -- with energy depending on frequency. He found the relation, $E = hf$: That is, a photon has an associated energy equal to its frequency times a constant, h, now called Planck's constant. In the photoelectric effect, a minimum energy, hence a photon with a minimum frequency, is required to eject an electron,

PLANCK'S CONSTANT

What is Planck's constant, and what does it represent?

In numerical terms, Planck's constant, h, is 6.6256 X 10^{-27} erg-seconds. An erg-sec, (energy (ergs) multiplied by time in seconds) represents a unit of "action." Interestingly, these are the same units as angular momentum and the same as the product of momentum times distance. Note that Planck's constant is a very small number. Note also the use of Planck's constant satisfies the parameters of the equation, $E = hf$, converting frequency to energy. (f is measured in units of cycles per second, 1/sec. Multiplying h times f gives units of energy, ergs.)

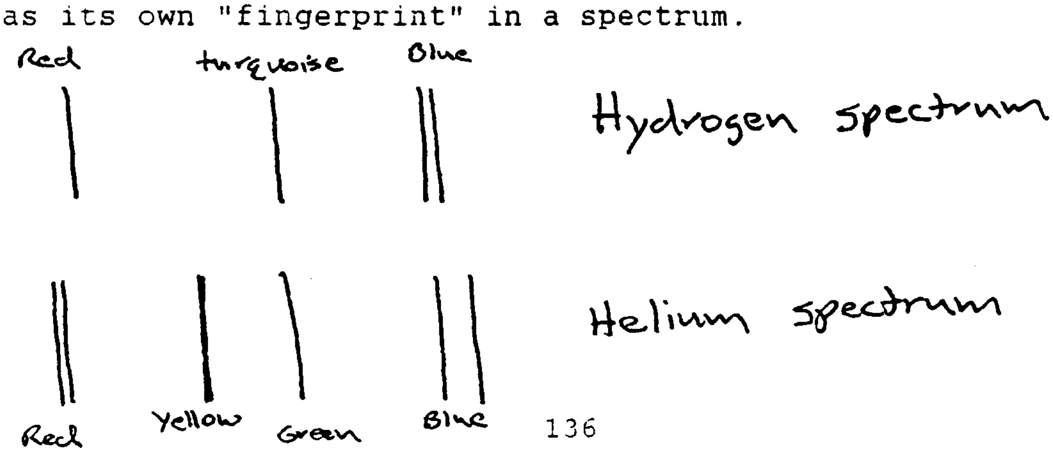
In physical terms, we might think of Planck's constant as the measure of the fundamental quantum -- the fundamental grain in an immense Universe built from infinitesimal packets of mass and energy.

ENERGY LEVELS IN ATOMS

Atomic spectra also demonstrate the quantum nature of energy, look at a hydrogen or helium lamp through a diffraction grating. The grating acts like a prism and spreads the light into its component colors -- its spectrum. You can see bright lines of color shining dramatically against the background.

Different elements -- neon, argon, mercury, sodium, etc. -- produce different spectral lines. Different elements differ in the energy levels available to their electrons. Electrons in sodium, for example, feel different electromagnetic fields than the electrons in hydrogen, because sodium has more protons in its nucleus and carries a larger nuclear charge. An electron in a sodium atom also feels the electromagnetic fields of the other ten electrons surrounding the nucleus, while the hydrogen electron dances alone.

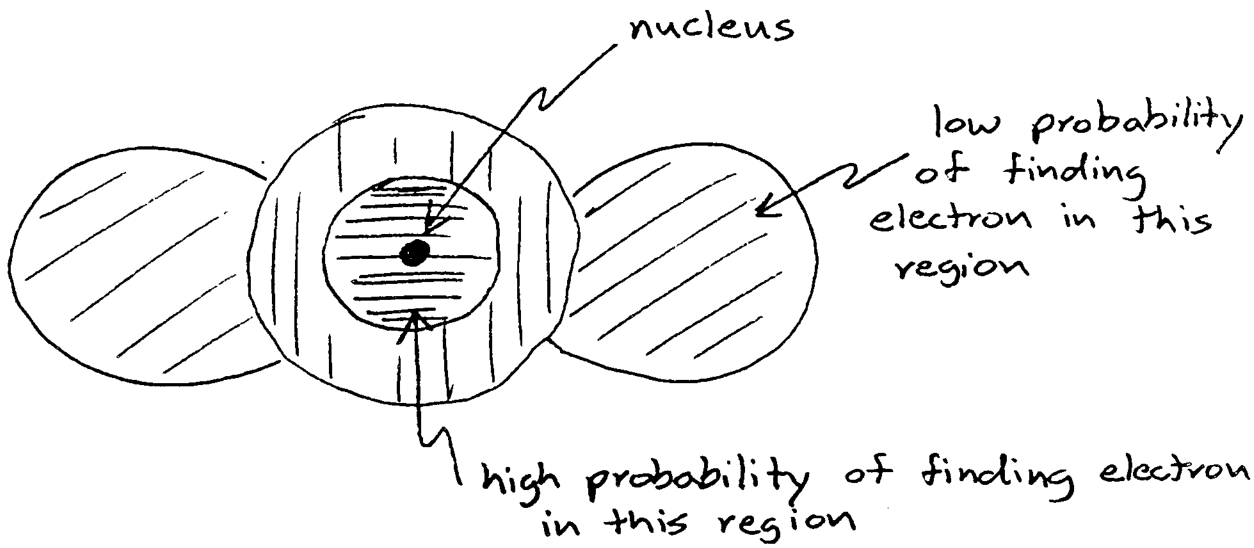
Since their electron energy levels differ, each element has its own "fingerprint" in a spectrum.



These lines represent quanta of energy released from atoms jostling in the fluorescent lamp. The flow of electric current (i.e. the flow of electrons) through the lamp knocks electrons to higher energy levels on the atoms of gas inside. When an electron falls back to its initial energy state, it releases a specific quantum of energy -- light with a particular frequency. If that frequency lies in the visible portion of the spectrum, we see evidence of the energy transition as a bright line in the spectrum.

MODELS OF THE ATOM

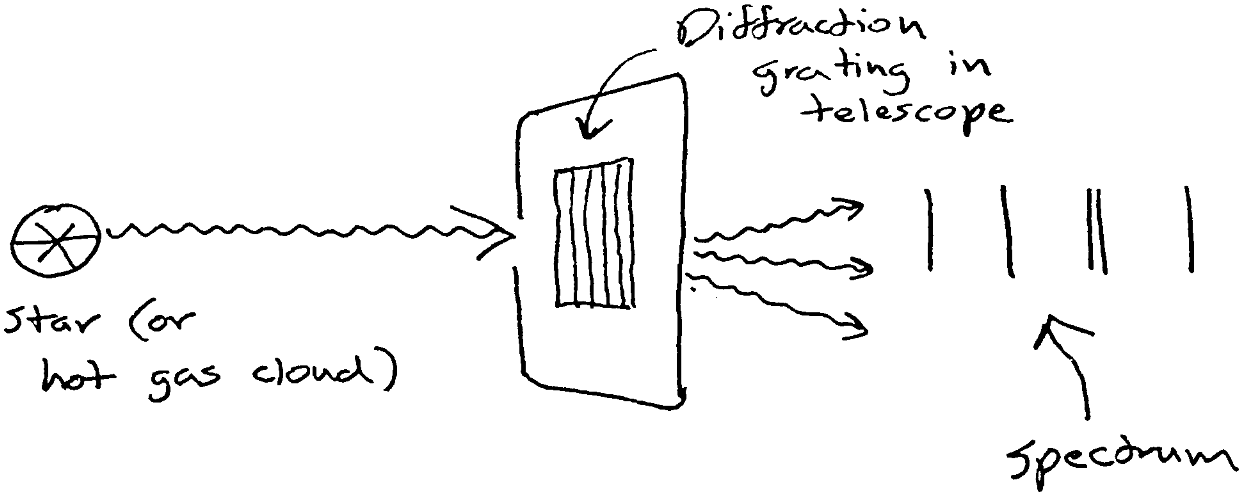
We are using the Bohr model of the atom, depicting electrons as if they were tiny planets orbiting a sun, to illustrate energy levels of electrons around a nucleus. In many ways this is a useful model, but it's a bit misleading. As we shall see, the electron can best be described as a wave and its position in the atom as an "amplitude." That is, the electron is probably here, but it may be over there, and in fact there's a chance it could be clear out yonder.



ATOMIC FINGERPRINTS: THE SPECTRUM

Astronomers routinely analyze spectra to determine the chemical composition of stars and gas clouds in space. To obtain the spectrum of a celestial object, they focus its light through a diffraction grating incorporated into a telescope.

Gas near the surface of a star or in an excited gas cloud produces an emission spectrum -- like a fluorescent light -- in which the excited frequencies appear as bright lines.



Similarly, atoms in a cold gas cloud absorb specific quanta of energy and later re-emit those same quanta. When it re-emits a quantum of energy, however, the atom likely will emit the photon in a direction other than along the line of sight to Earth. This effectively subtracts the transition frequencies from the starlight that reaches us, and the "missing" frequencies appear as dark bands in the spectrum of the starlight.

Note that frequencies in the absorption spectrum are the same as the emission bands for each element.

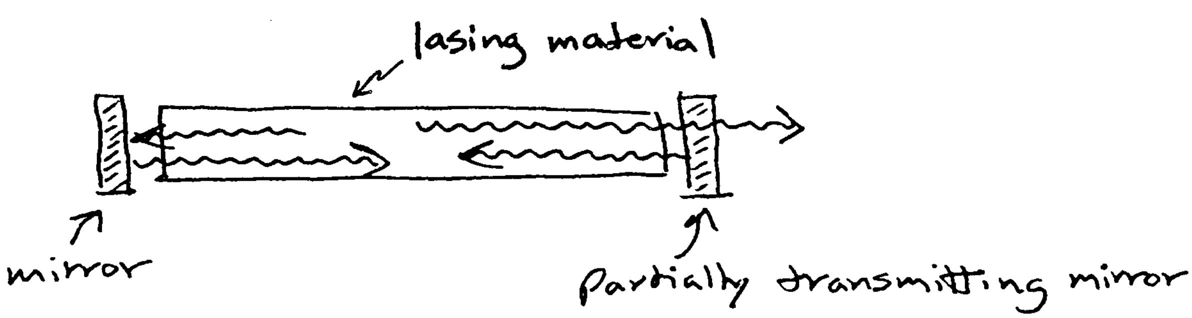
The sky itself demonstrates a quantum phenomenon. Blue sky represents the emission spectrum of nitrogen gas in the atmosphere: nitrogen molecules re-radiate blue light from the solar energy they absorb.

LASERS

Lasers exploit atomic energy transitions to produce intense light of a single frequency that is also collimated: that is, it remains confined in a narrow beam and does not disperse as it travels. The word "laser" is an abbreviation of the phrase "light amplification by stimulated emission of radiation."

In operation, an outside energy source drives electrons in the lasing material to a higher energy level. When one of the electrons decays (falls to a lower energy level), the photon it releases may interact with another high-energy electron, causing it also to decay and to release another photon of the same frequency. (This is the "stimulated emission" of one photon by another.)

Two mirrors, one of which is partially transmitting, bracket the lasing material. As photons ricochet back and forth between the mirrors, they generate a cascade of photons all of the same frequency. Some of the photons leak out, through the partially transmitting mirror, and that's the laser "beam" we see.

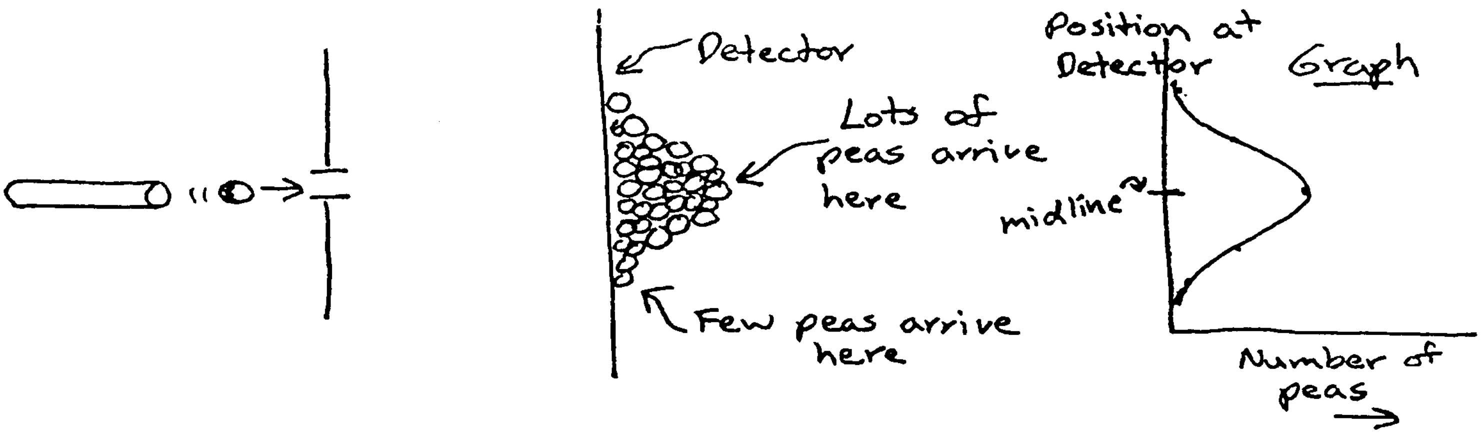


Physicists can choose from many different materials to build lasers, and different lasers produce beams of different frequency and intensity. They serve a multitude of functions -- from eye surgery to surveying, to welding, and even as optical "tweezers" to manipulate and study individual atoms.

THE WAVE NATURE OF MATTER *

Like the Michelson-Morley experiment used to illustrate aspects of special relativity and the imaginary spaceship with which we demonstrated some of the ideas of general relativity, the "double slit" experiment contains the essence of quantum mechanics. It models behavior of elementary particles.

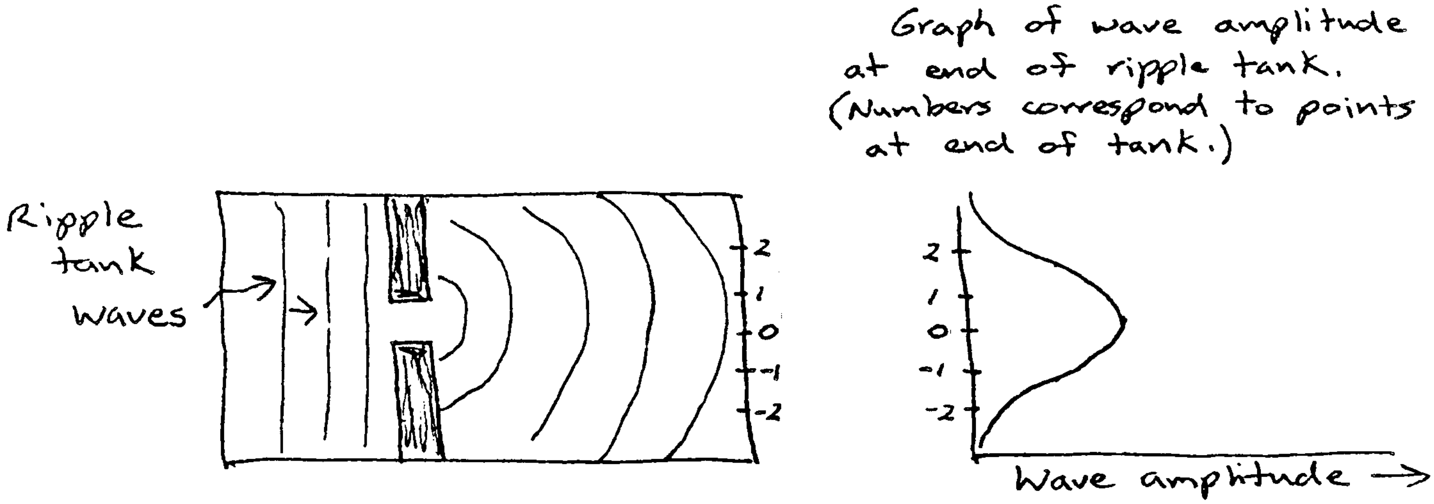
Imagine shooting peas from a pea shooter toward a single slit in an otherwise impenetrable barrier. Peas can pass through the slit to a detector, which counts them. If we record the number of peas arriving at different points on the detector, we find the following distribution:



Most of the peas that reach the detector pass directly through the center of the slit, in line with the slit and the shooter. Some, however, strike the edge of the slit and ricochet away from the direct line, arriving at the detector some distance from the mid-point.

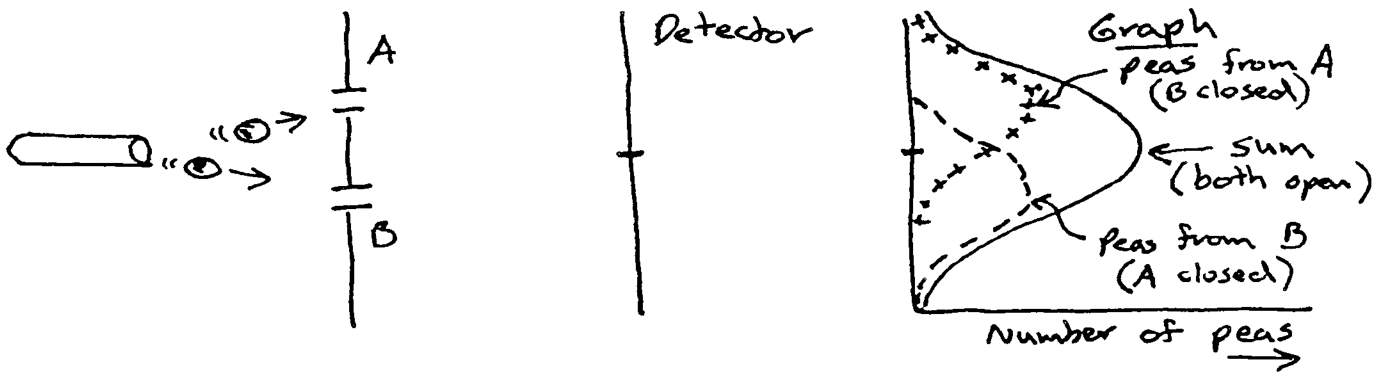
Now let's repeat the experiment with waves instead of peas. We'll measure the amplitude of the waves arriving at different points on the detector. As we have seen previously with the ripple tank (see p.), amplitude varies across the end of the ripple tank in a manner similar to the distribution of the peas: wave amplitude decreases with the distance the wave travels. With a single slit, then, wave amplitude distributes like particles.

* Following argument adapted from Feynman

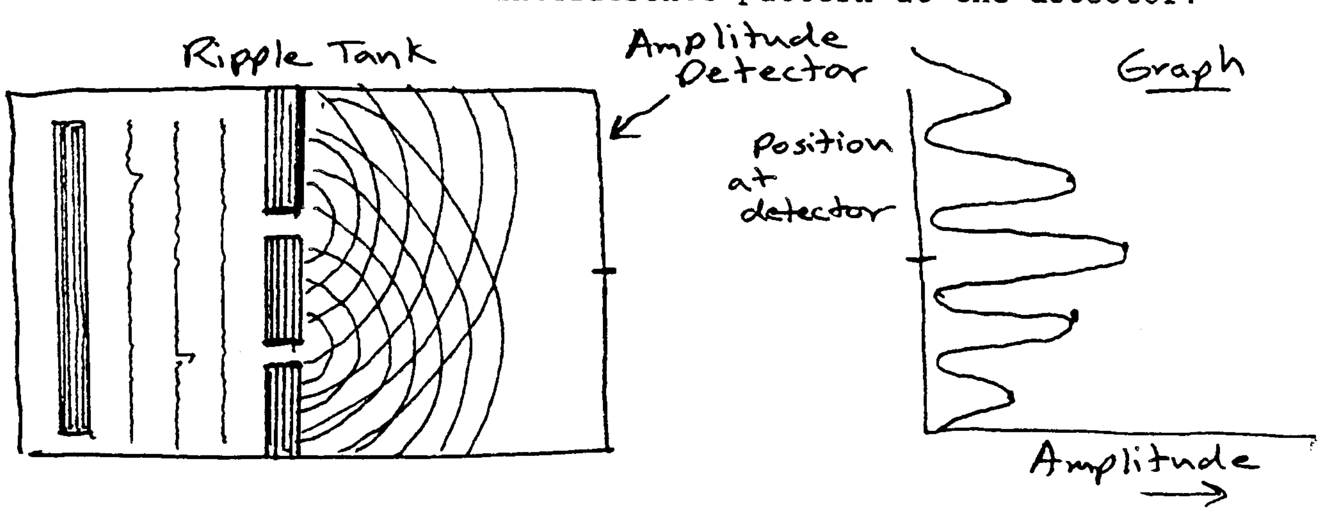


Now let's experiment with two slits rather than one.

Shooting peas toward a double slit, we find the distribution at the detector is the sum of the distributions of peas passing through each of the two slits separately.



With waves, however, the distribution looks completely different: we see an interference pattern at the detector.



A double slit experiment, then, can distinguish waves from particles: waves interfere.

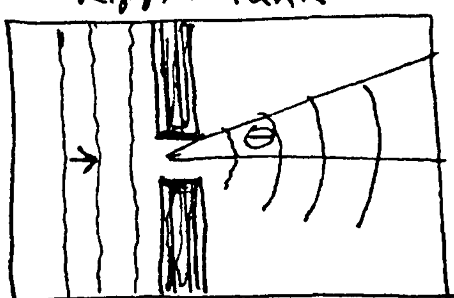
One of the great surprises in modern physics was the discovery that electrons, classically described as "particles," behave like waves in a "double slit" experiment.

DIFFRACTING ELECTRONS

A "double slit" experiment with electrons requires different apparatus than that described above, but the idea is the same. Instead of slits cut through a solid material, physicists use crystals. Only at the atomic dimensions of crystal planes are slits narrow enough to diffract electrons.

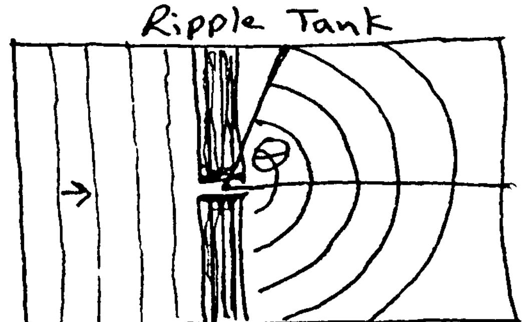
The angle of diffraction (θ) is proportional to the ratio of wavelength to slit width: $\theta = \lambda/w$, where W represents slit width, and λ is the wavelength. The greater this ratio, the greater the angle of diffraction.

Ripple Tank



Wide slit, small θ

Ripple Tank

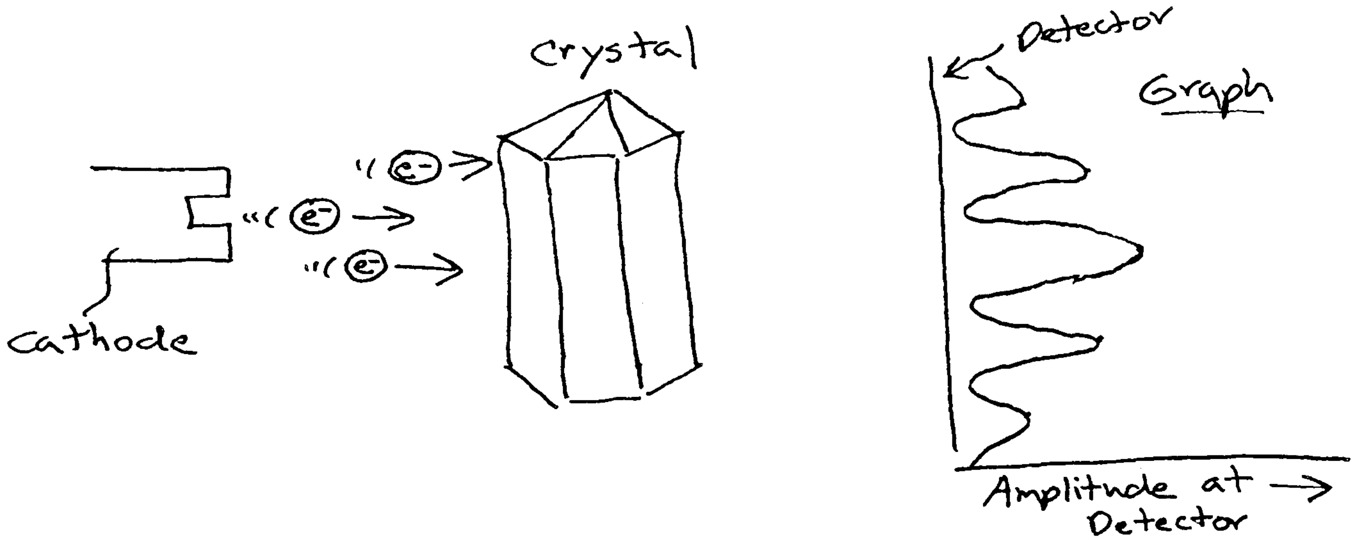


Narrow slit, large θ

As we shall see, electrons can be regarded as wave packets, and they have a very short wavelength. Only very narrow slits will diffract them. Experimenters use quartz crystals for the diffraction grating: narrow planes between atoms in the crystal structure form the slits.

<u>ELECTRON INTERFERENCE</u>

When we set up an electron gun (a cathode such as in a TV) and shoot electrons at the crystal, we find an interference pattern at the detector.

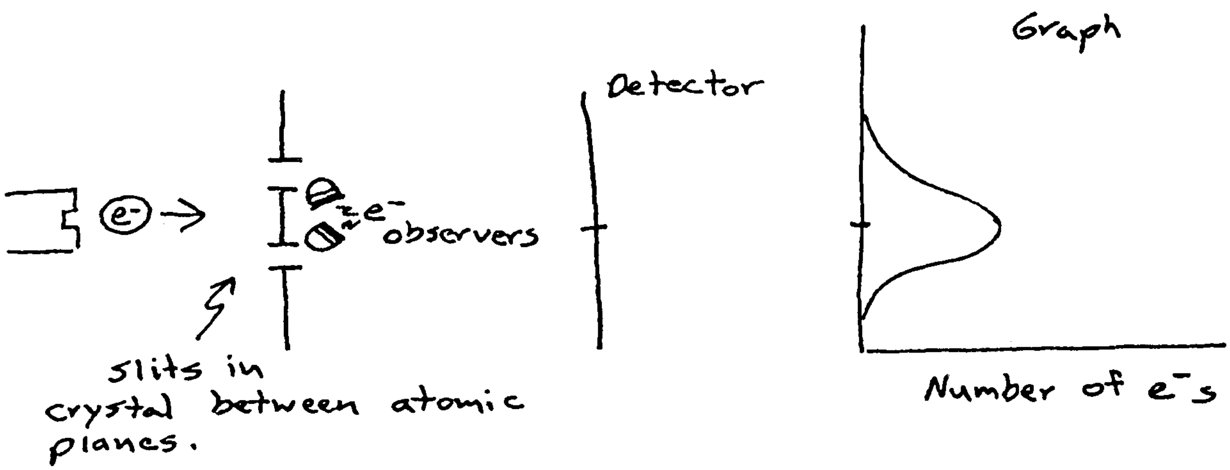


The electrons, described classically as particles, behave like waves! Particles, we would assume, should distribute themselves in a summation pattern, but electrons passing through a crystal show an interference pattern -- a wave phenomenon!

What's going on here?

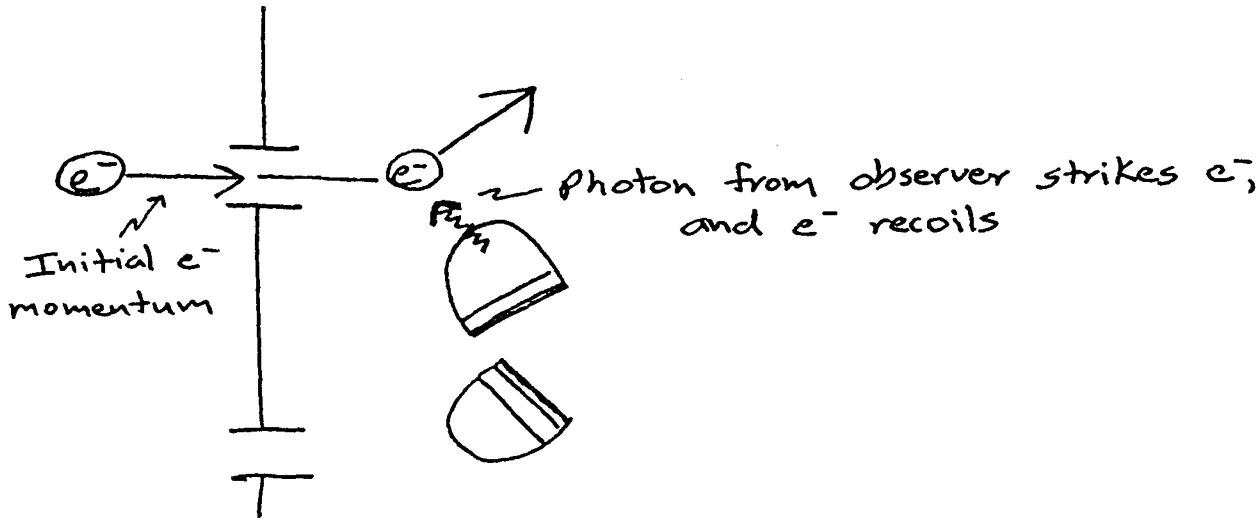
THE EFFECT OF THE OBSERVER

We need some more information. Let's watch the electrons more closely. We'll add a device to observe the electrons as they traverse the slits.



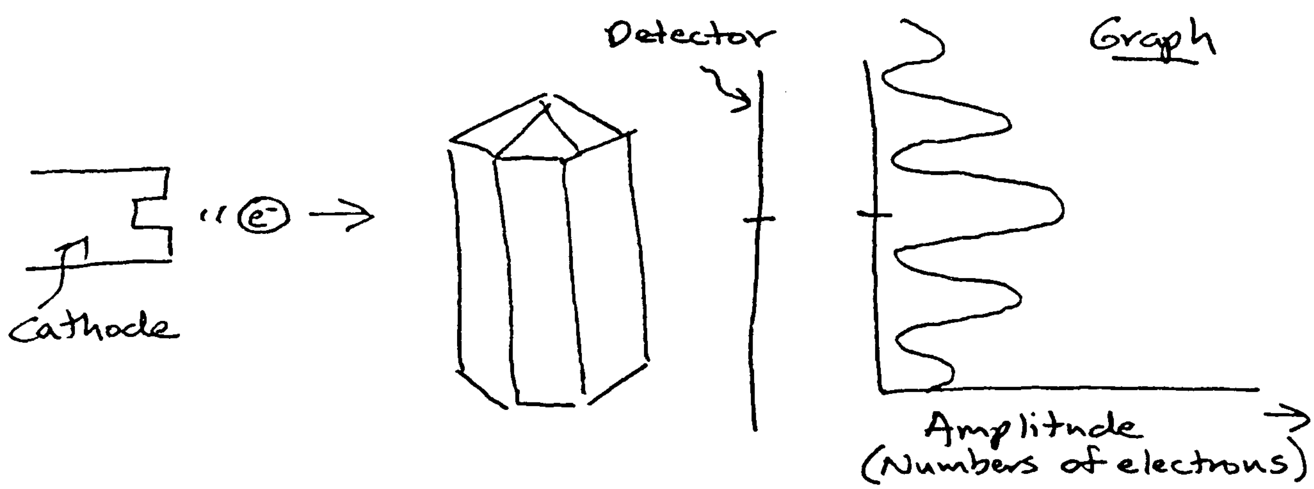
When we run the experiment, watching the electrons as they pass through the slits, our detector records a smooth distribution, like the distribution of peas. The electrons' behavior changes! Somehow, the act of observing the electrons affects the outcome of the experiment.

We might explain this change in electron distribution as follows: when we observe an electron traversing the slit, we "bounce" a photon off of it. Momentum transferred from the photon changes the electron's momentum and smears out the interference pattern, producing the bell-shaped summation curve.



How else can we explain the interference pattern produced by "particles?" Perhaps large numbers of electrons travel in waves, just as large numbers of water molecules form waves. If so, if we shoot one electron at a time through the apparatus, we should observe a particle distribution (bell curve) at the detector.

We set up the experiment -- one electron at a time:

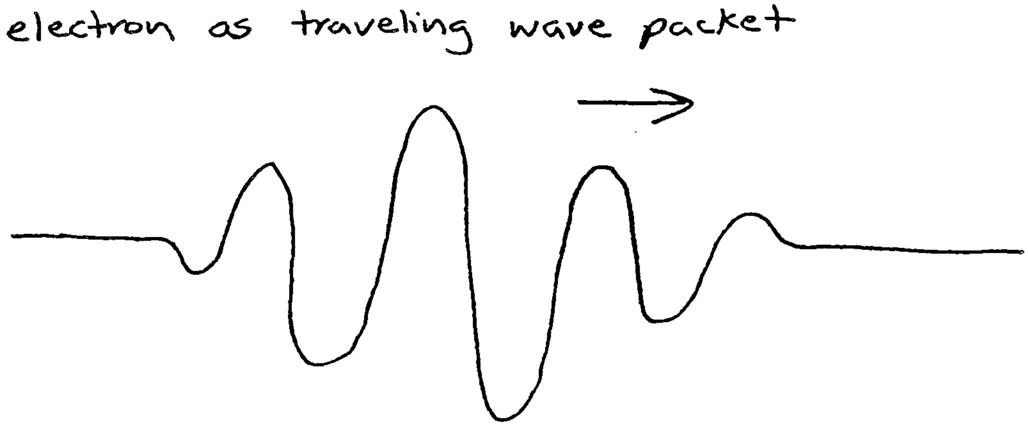


We find an interference pattern! It appears that each electron interferes with itself!

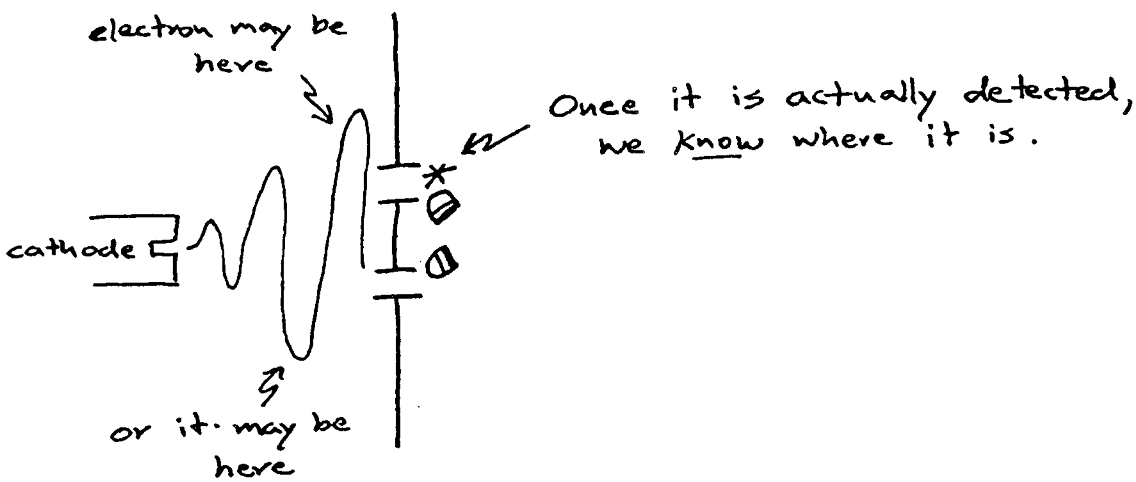
QUANTUM INTERPRETATION OF ELECTRON INTERFERENCE

Quantum mechanics offers this interpretation of electron diffraction and interference:

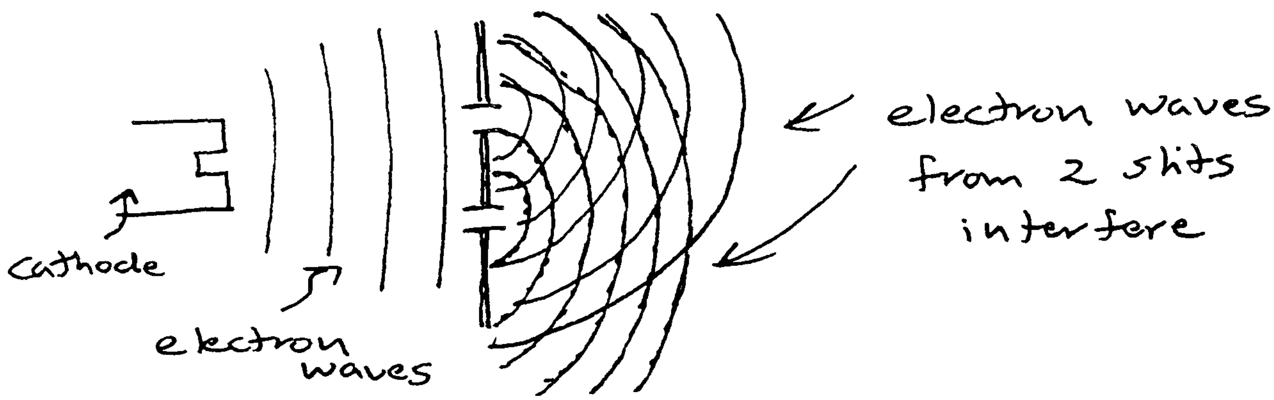
An electron behaves like a wave in that there is a certain amplitude we will find it in our detector at a particular time and place. The amplitude varies with time and position, like other waves.



There is a certain amplitude we will find it here, a certain amplitude we will find it there, but until we actually <u>detect</u> it, we don't know exactly <u>where</u> it is.



The interference pattern produced by electrons -- or any other particle -- in a diffraction experiment represents interference of amplitudes.

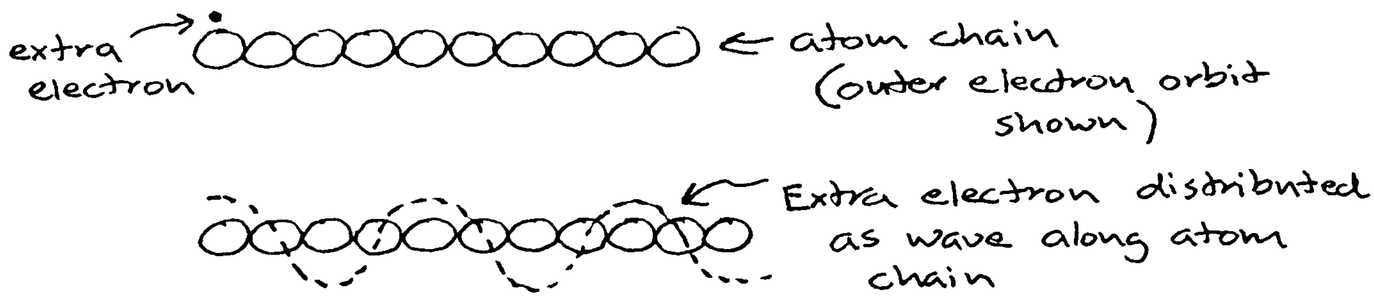


PRACTICAL APPLICATIONS

Many practical devices exploit the wave nature of electrons. Microchip electronics -- the components of computers, TV's, radios, car ignition control systems, etc. -- use "semiconductor" materials, especially silicon and germanium, as electron waveguides to control the flow of electric current. Superconductors, similarly, exploit the de-localized nature of electron waves to accomodate resistance-less flow of electricity.

A regular crystalline lattice, as available in pure crystals of silicon or germanium, allows the flow of electrons as if down a wire.

More precisely, when an extra electron is added to the semiconductor crystal, the electron wave distributes itself all along the atomic chain. In a steady state (no other electrons added, and no voltage applied along the crystal), the electron behaves like a standing wave.

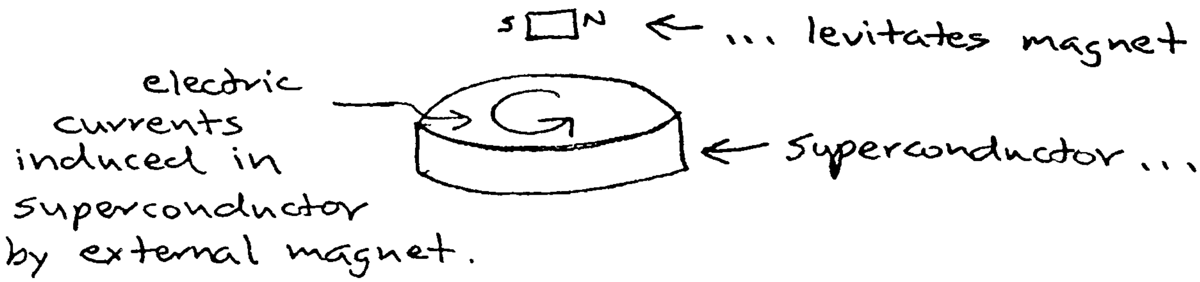


By adding more electrons and applying a voltage (attaching one end of the crystal to the negative pole of a battery and the other end to the positive pole), it's possible to generate traveling waves, i.e. the flow of electrons down the atomic chain. If there are no defects in the crystal, and if the atoms of the crystal lattice are not vibrating too violently, interfering with the electron waves, the device is a superconductor.

Superconductors carry an electric current (the flow of electrons) without resistance. Currently available superconductors must all be cooled to low temperatures in order to suppress disruptive atomic vibrations: even "high-temperature" superconductors recently developed must be refrigerated to the temperature of liquid nitrogen -- about 170 degrees below zero, Fahrenheit! Even so, many practical devices use superconductors, including superconducting magnets in particle accelerators and magnets in magnetic field imaging devices used in diagnostic medicine.

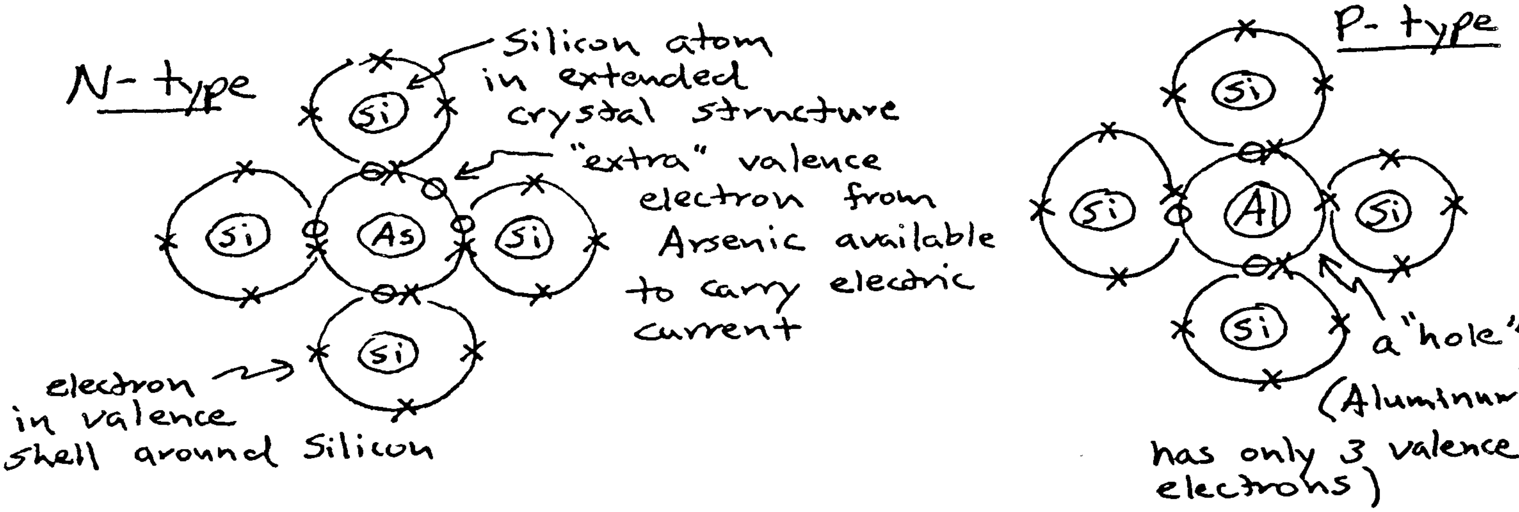
Potentially, superconducting wires could transport electricity through the nation's electric grid with minimal power loss.

Superconductors also expel any external magnetic field. As a consequence, a magnet will float above a superconductor. Transportation systems may eventually exploit this magnetic effect to "levitate" high-speed trains.



SOLID STATE TECHNOLOGY

The electrical characteristics of a semiconductor can be controlled by "doping" the crystal with "impurity" atoms containing one more or one fewer valence electrons than the silicon atoms of the crystal matrix. (A valence electron is an electron in the outer electron shell of an atom, most distant from the nucleus.) A semiconductor doped with atoms such as arsenic, containing one more valence electron is called an "N-type" (negative-type) semiconductor. A semiconductor doped with atoms such as aluminum, containing one fewer valence electron is called "P-type" (positive-type).

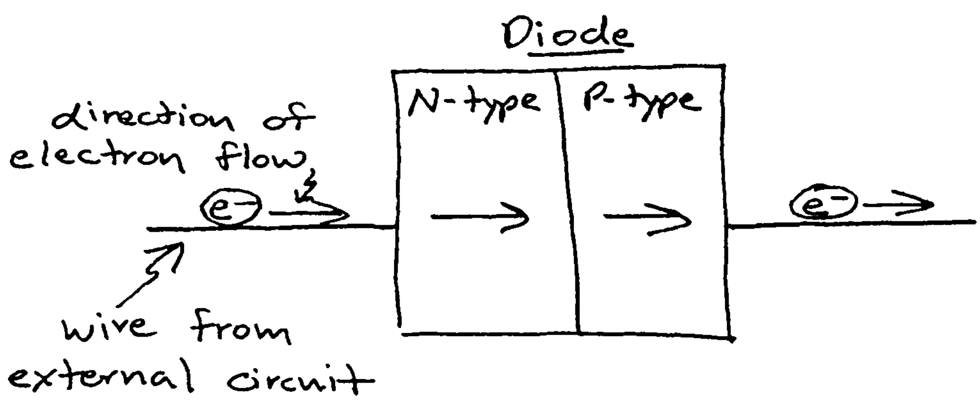


The "extra" electrons in the N-type material behave as mobile negative charge carriers, and the "holes" in the P-type material -- the "vacant" spots in the valence shells -- act as mobile positive charges.

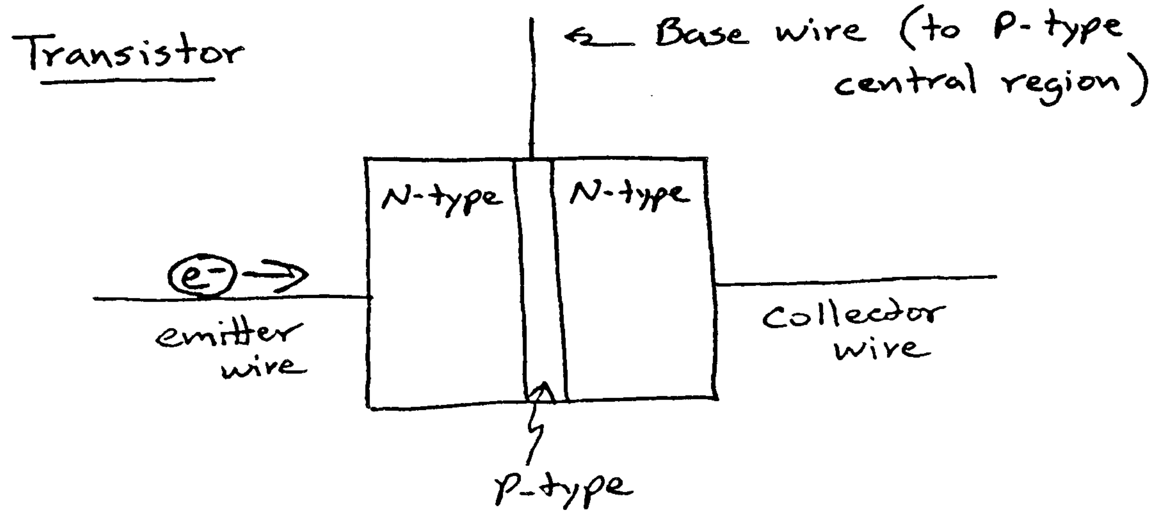
The electronics industry manufactures useful devices such as diodes and transistors by alternating layers of N-type and P-type material. A diode, or "rectifier," for instance, allows the flow of electricity only in one direction.

When we anneal a P-type semiconductor to an N-type (by growing crystal layers of the N-type directly on the surface of the P-type), electrons and holes redistribute themselves across the boundary layer: electrons move from the N-type to fill holes in the P-type, and holes move from the P-type to "swallow" electrons in the N-type. This redistribution of electrons and holes stops when the static electric charge in the N-type material (now positive, having lost electrons) restrains other electrons from crossing the boundary. Similarly, the accumulated negative charge in the P-type material prevents more holes from crossing the boundary in the opposite direction.

Now if we insert the diode in an electric circuit, it allows the current to flow in only one direction. Electrons from the circuit enter the N-type, pulled by the positive charge, then cross into holes in the P-type and on down the electric potential in the circuit.



A transistor is a three-layer device, a sandwich with a thin center of one type (e.g. P-type) between thicker layers of the opposite type (e.g. N-type). Current in the main circuit runs from an emitter wire to a collector wire. A third wire, the base, controls the charge in the central layer, which acts as a kind of gate: "opening" the gate -- by decreasing the negative charge in the central P-type -- allows electrons to flow through the device, from the emitter to the collector. Furthermore, the transistor acts as an amplifier: small changes in current to the base cause large changes in the current from emitter to collector.



Such devices allow us to do all kinds of useful things. For instance, by controlling the flow of electrons, we can store and retrieve information in a computer: pressing the "1" key on the computer opens a transistor gate and stores an electron in the working memory of the computer (a capacitor, a device for storing electric charge), for retrieval later on.

THE UNCERTAINTY PRINCIPLE

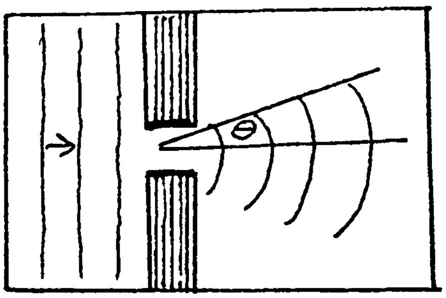
We've already hinted at the next great principle of quantum mechanics: if we put a detector at the slits in our double slit experiment, so we know which slit the electron traverses, we change the electron's momentum and blur out the interference pattern. Let's demonstrate, more formally, this "uncertainty principle:" one cannot know precisely, at the same time, a particle's position and its momentum. (Werner Heisenberg first elucidated the uncertainty principle, and it bears his name.)

First, consider once again an analogy from the ripple tank. Waves traversing a single slit diffract, and the angle of diffraction is proportional to the wavelength of the incident waves and inversely proportional to the slit width:

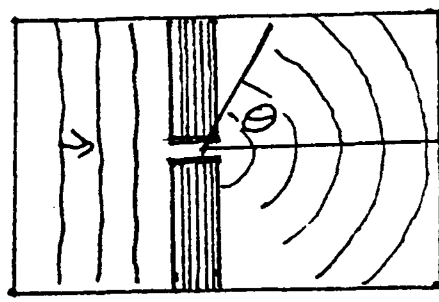
$$\theta = \lambda / w$$

Increase the wavelength, or narrow the slit, and the angle increases.

Ripple tank



wide slit, small θ

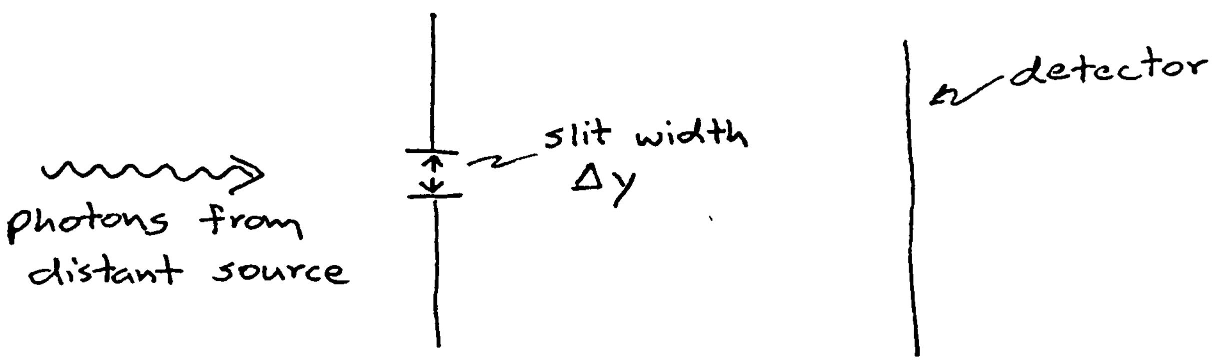


narrow slit, large θ

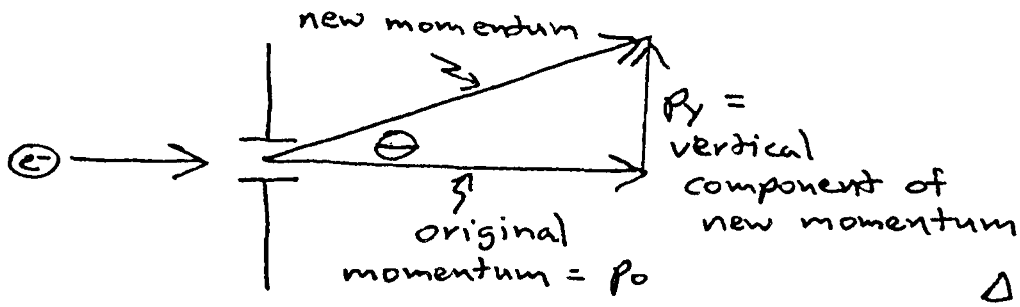
Other waves -- such as light and electrons -- behave like the waves in the ripple tank.

Imagine a light source far removed from a single slit. The photons arriving at the slit travel a horizontal path, with no vertical component to their momentum.

Call the width of the slit Δy. The photons have an associated wavelength, λ. We want to determine the position and momentum of the photons after they traverse the slit.



We know their vertical position at the slit to an accuracy Δy, but as they traverse the slit, some are diffracted at angle θ. We've lost information about their momentum: some of the photons, by diffraction, have acquired a vertical component of momentum, p_y. By trigonometric analysis:



For small θ,

$$\sin\theta \cong p_y/p_o$$

$$\sin\theta \cong \theta$$

$$\theta \cong p_y/p_o$$

$$\Delta p = p_y \cong p_o\theta$$

where Δp is the change in the vertical component of momentum

Since the angle of diffraction is proportional to wavelength and inversely proportional to slit width, we derive the equation:

$$\Delta p = p_y \cong p_o(\lambda/\Delta y) \qquad (1)$$

We can show that $p_o\lambda$ is equal to Planck's constant:

By the classical expression of momentum,

$$p_o = mv \qquad (2)$$

Substituting relativistic rest mass, m = E/c^2,

$$p_o = Ev/c^2$$

For a photon traveling at c, $p_o = Ec/c^2$ (3)

and $p_o = E/c$ (4)

Since E = hf, and c = λf, $p_o = hf/\lambda f$ (5)

Therefore $p_o = h/\lambda$ (6)

and $p_o\lambda = h$ (7)

Substituting into eq.1 $p_y \cong h/\Delta y$

and, finally, $\Delta p\Delta y \cong h$

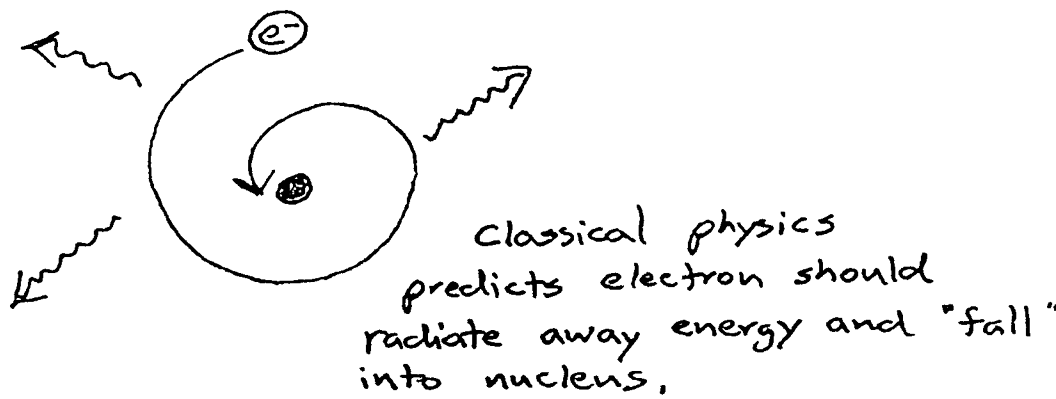
That is, the product of uncertainty in momentum and uncertainty in position equals Planck's constant. As Δy, the uncertainty in position, decreases, Δp must increase, and vice versa. If we know the position of a particle precisely, we cannot at the same time know its momentum. If we know the momentum precisely, we cannot know the particle's position.

In fact, $\Delta p\Delta y = h$ is the optimum condition -- the absolute limit to our knowledge. Practically, our measurements are always less accurate, and $\Delta p\Delta y \geq h$.

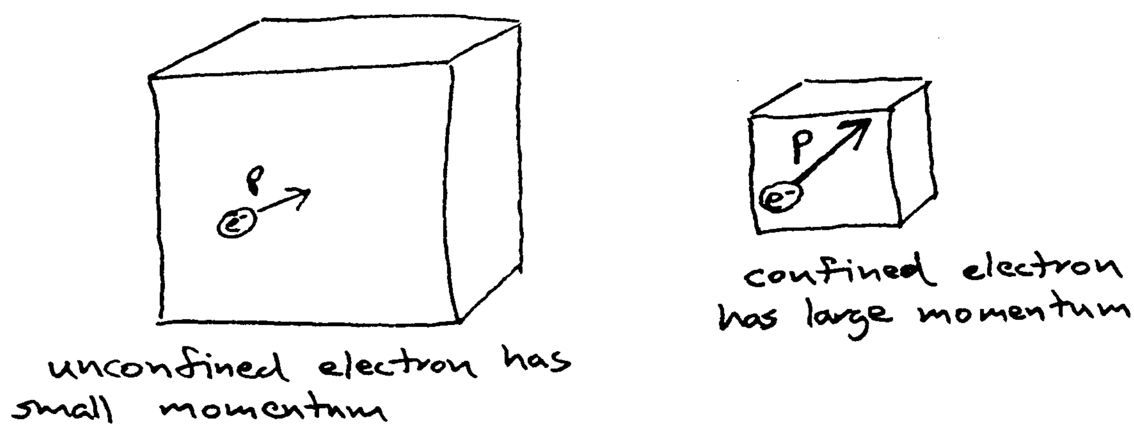
THE UNCERTAINTY PRINCIPLE AND THE STABILITY OF ATOMS

The uncertainty principle solved a longstanding puzzle: classical physics predicted electrons would fall into atomic nuclei. Atoms demonstrably do not collapse -- or we wouldn't be here to testify -- but no one could explain their stability until Heisenberg and his uncertainty principle.

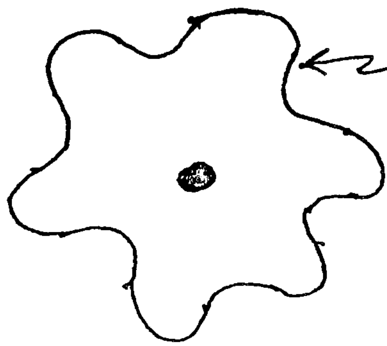
According to classical physics, electrons must radiate continually: an electron is always accelerating in its orbit around the nucleus, and accelerated electric charges radiate. The radiation robs energy from the electron, so it should lose velocity and spiral into the nucleus, pulled by the positive charge there.



The uncertainty principle explains why this collapse does not occur: if an electron's orbit approaches the nucleus, there is less uncertainty in its position (it is confined to a smaller region). But if there is less uncertainty in position, its momentum must increase. The increased momentum prevents it from plunging into the nucleus.



We can regard the atom's stability from another perspective, as well. The electron is not a point particle, but a wave packet distributed over some region of space (its uncertainty in position). Under normal conditions of temperature and density, it is not possible to confine that wave packet to a region so small as the nucleus, and it distributes itself around the nucleus.



UNCERTAINTY OF ENERGY AND TIME

The uncertainty principle applies to energy and time as well as to momentum and position:

$$\Delta E \Delta t \geq h$$

(The derivation is straightforward:

$$\frac{c}{c}(\Delta p \Delta y) \geq h$$

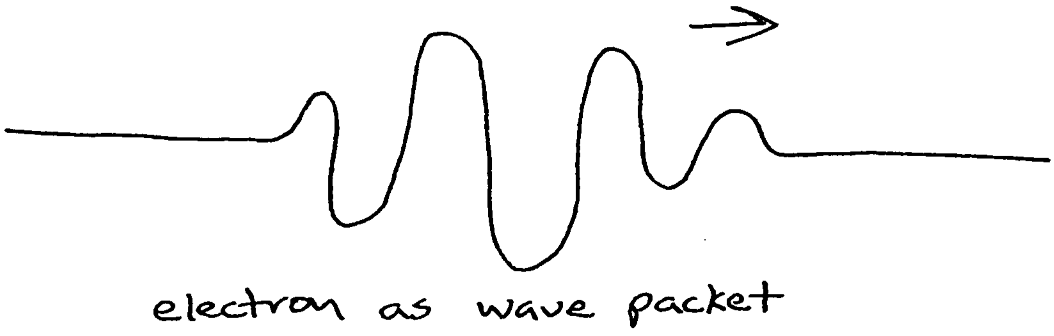
$$c\Delta p \left(\Delta y / c\right) \geq h$$

Since $c\Delta p = \Delta E$ and $\Delta y / c = \Delta t$

$$\Delta E \Delta t \geq h \quad)$$

The product of the uncertainty in energy of a particle and the uncertainty in time during which the particle is observed is greater than or equal to Planck's constant. We can never know with absolute certainty the energy of a particle at an instant in time.

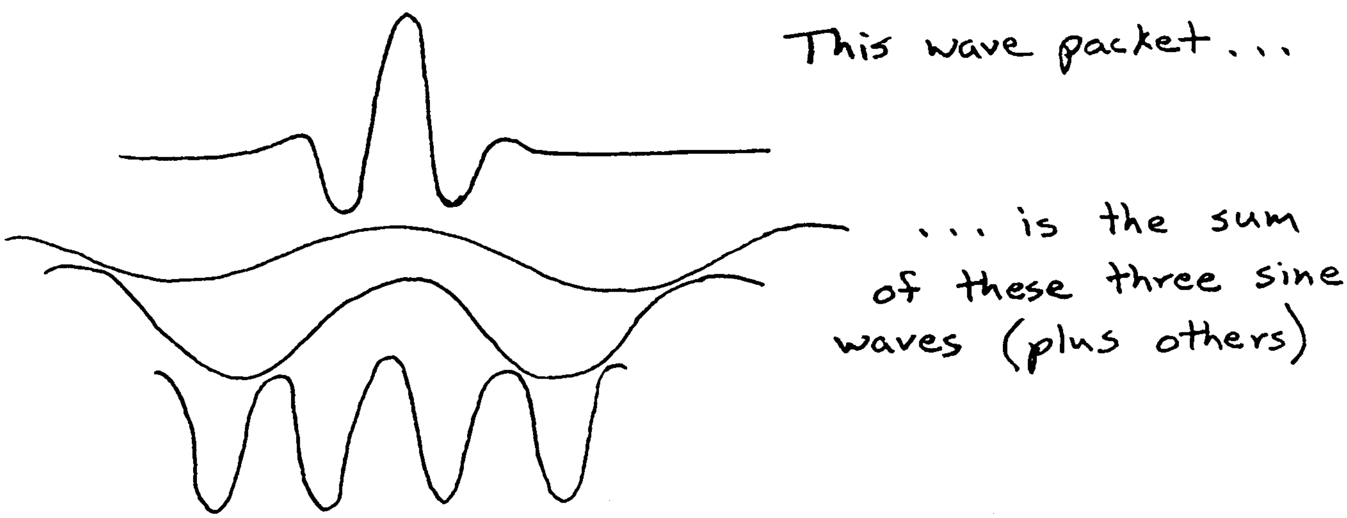
To understand this constraint, imagine an electron (or other particle) as a wave packet.



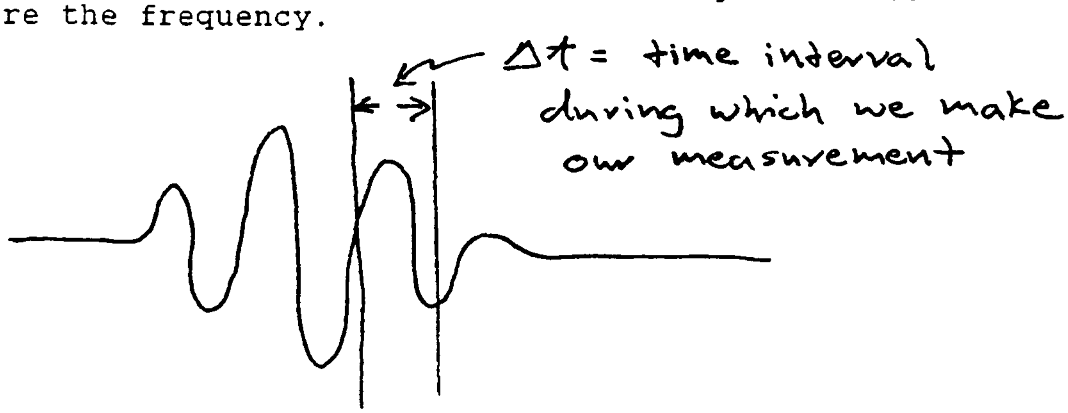
FOURIER ANALYSIS

The wave packet can be described mathematically, by Fourier analysis.

Fourier analysis, essentially, states that any wave form (such as an electron, or any other particle) can be analyzed as the superposition of sine waves of different frequency, amplitude, and phase.



The energy of the wave packet is proportional to the frequency. If we try to measure the frequency during a very short time interval, we cannot determine it accurately: too short a time interval may not include enough waves to measure the frequency.

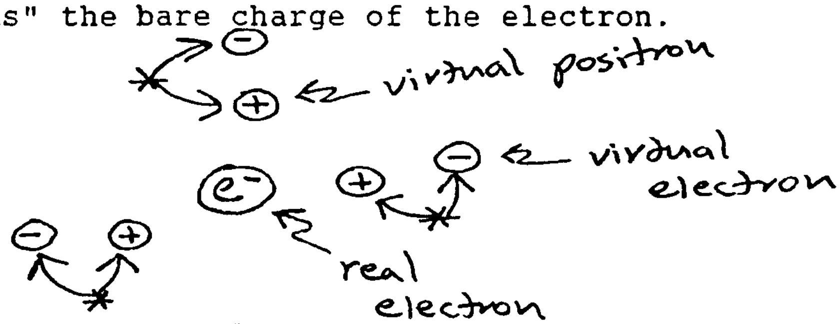


On the other hand, if we want to determine the energy of the wave packet precisely, we require a prolonged time interval for exact Fourier analysis.

VIRTUAL PARTICLES

The energy/time relation of the uncertainty principle, $\Delta E \Delta t \geq h$, invites a surprising prediction: in a short time interval, nature can "borrow" enough energy to produce particles -- called "virtual particles" -- out of "nothing." (More precisely, nature creates virtual particles out of the "vacuum," which, for our purposes here, we can think of as the substance of spacetime.) The virtual particles, produced as particle/anti-particle pairs, exist for an immeasurably brief instant of time and then annihilate.

Virtual particles cannot be observed directly, but scientists can detect their effects: Probing electrons with particle accelerators, physicists find the charge becomes more negative as they delve closer to the electron itself. It is thought this occurs because a cloud of virtual electron-positron pairs surrounds the "real" electron. Positrons among the virtual pairs are attracted to the electron, and virtual electrons are repelled. In effect this "shields" the bare charge of the electron.



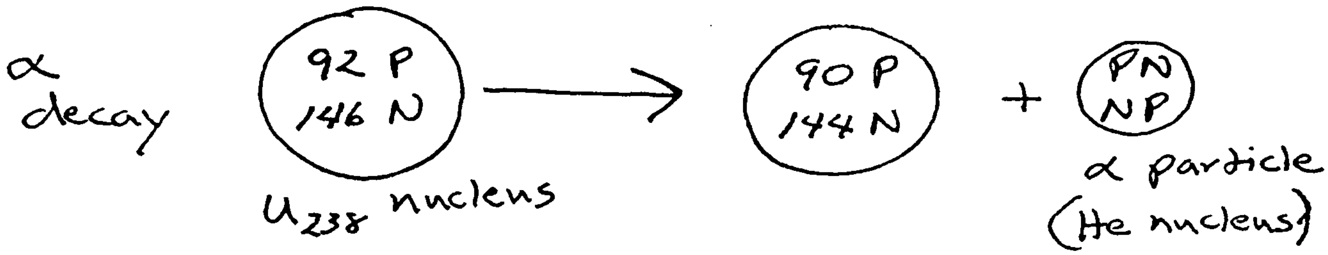
Virtual particles are important because they transmit the forces of nature -- our topic in Ch.7.

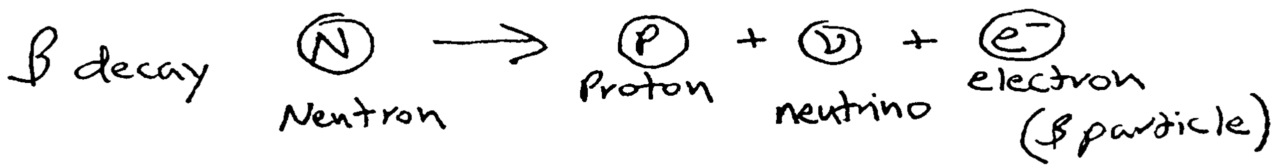
RANDOMNESS

The quantum world, as we have seen, is probabilistic, not deterministic. More strange, it is random. One cannot predict exactly what an electron, or photon, or any other particle will do.

Nuclear decay illustrates this randomness: The half life of uranium 238 is 4.5 billion years. That is, after 4.5 billion years, half of a lump of uranium will have decayed to lighter elements. We can measure this half-life accurately. We cannot know, however, exactly when any <u>single</u> uranium nucleus will decay: it may decay in the next minute, or not for a trillion years.

Nuclear decay involves changes in the numbers or types particles in an atomic nucleus. In beta decay, a neutron decays to a proton, an electron (beta particle), and an anti-neutrino. In alpha decay, the nucleus emits an alpha particle (a helium nucleus -- two protons and two neutrons).





Popcorn models random processes nicely. We know that, at a given temperature, all but a few kernels of a particular brand of popcorn will pop within a certain time. But take the lid off the popcorn and watch individual kernels (and duck!). We cannot predict when any one kernel will pop. It may pop within a few seconds -- or not at all.

Randomness rules at the quantum level. A given electron, pumped to a higher energy level in a fluorescent light, may fall to a lower level immediately and release a photon, or it may remain at the higher energy indefinitely. After an applied radio-frequency field has flipped its spin state, a proton may return to its initial state immediately, or not for some time.

What is truly amazing is that we find an orderly world built out of this fundamental chaos. Nature deals with such immense numbers of random events that the outcome, on the large scale, is predictable. All the electrons in our fluorescent light could, conceivably, cascade at once to a lower energy level and release a flash of light, but they don't. All the molecules in a wall, conceivably, could move aside just so, and -- voila! -- we walk through walls. But the probability of such molecular rearrangement is vanishingly small.

CAUSALITY AND OBJECTIVITY

Look how quantum mechanics has changed our view of the Universe:

Newton described a clockwork universe (see Ch. 1). According to the Newton, knowing the initial position and momentum of all the bits and pieces of the Universe would allow perfectly accurate description of the past and perfectly accurate prediction of the future. Furthermore, in Newton's view, the Universe existed of and by itself: Planets orbited stars, uranium decayed, trees fell in the forest whether or not anyone was there to observe them.

Quantum mechanics changed that world view. The quantum world is inherently unpredictable. Events occur at random. We can calculate probabilities of events, but only probabilities. We cannot say for certain that such and such will occur.

Furthermore, the observer participates in the Universe, and the very act of observing affects the outcome of natural processes. If we go looking for wave phenomena, we find wave phenomena. If we go looking for particles, we find particles. We are part of the Universe we observe, and we affect the systems we study.

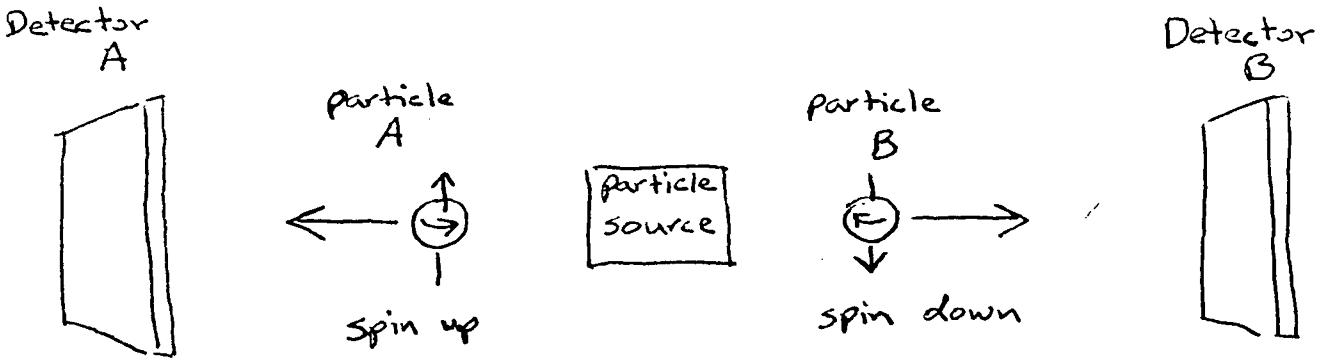
BELL'S INEQUALITY

This quantum view of the world is unsettling, and many brilliant thinkers in the mid-twentieth century -- most notably Einstein -- argued that quantum mechanics is incomplete: if only we had finer tools -- more accurate probes and better experiments -- they said, we could elucidate the inner workings of the elementary particles, and we would discover, on this more fundamental level, a predictable, cause-and-effect Universe.

John Bell's theoretical work and experiments (most notably by Alain Aspect) testing "Bell's inequality" dashed these hopes. Over the past ten years, a number of experiments have shown that there can be no hidden cause-and-effect in the quantum world.

Bell, working at CERN (the European Center for Nuclear Research), proposed the following experiment to test the predictions of quantum mechanics:

Suppose we produce a pair of particles the members of which are exactly opposite in some characteristic. For example, it's possible to produce proton/anti-proton pairs with opposite spin moving from the experimental apparatus in opposite directions. We set up detectors to determine the spin of each particle. If our logic and our classical view of the Universe are accurate, measuring the spin of one particle tells us the spin of the other.



For example, if detector A measures a particle spin up, we know particle B (A's anti-particle) must be spin down -- <u>if</u> classical reasoning and classical mechanics hold true.

This line of reasoning makes three assumptions:

1. Particles have an objective reality, existing outside the machinations of the observer.

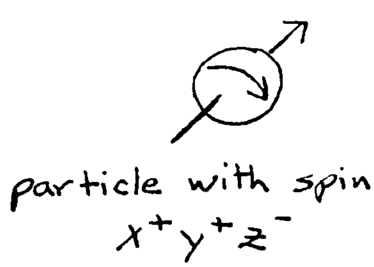
2. Inductive reasoning is valid. That is, we can make accurate predictions based on past measurements.

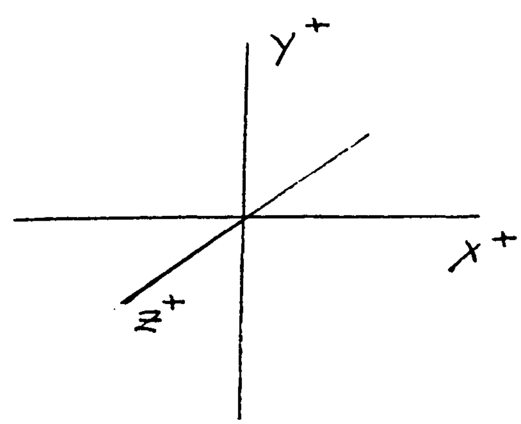
3. Particles cannot communicate faster than the speed of light. (If they did, one particle upon arrival at a detector might instantaneously change the spin of the other and upset our measurements.)

THE MATHEMATICS OF BELL'S INEQUALITY

Bell's argument goes like this:

We measure the spin axis of the particles and anti-particles in 3-space -- that is, in relation to the three spatial coordinates, x, y, and z.





Just as an observer in the double slit experiment alters the state of an electron if he observes it traversing the slit, so the detectors in this experiment alter the spin state of the particles and anti-particles passing through. However, since the spins of the particle/anti-particle pair are exactly opposite, we can get added information about one particle by measuring the spin of the other.

When we measure the spin axis of large numbers of particles, if our classical assumptions hold true, we expect to find that

$$N(x^+y^-) = N(x^+y^-z^+) + N(x^+y^-z^-) \qquad (1)$$

That is, the number of particles with spin axis pointed in the positive direction along the x axis and the negative direction along y includes particles (x+,y-,z+) oriented along positive z and (x+,y-,z-) oriented along negative z.

Extending this argument,

$$N(x^+z^-) \geq N(x^+y^-z^-) \qquad (2)$$

The number of particles oriented with axes along the positive x axis and the negative z axis is greater than or equal to the number of pasrticles oriented (x+,y-,z-), because the set (x+,z-) also includes (x+,y+,z-).

Similarly, $N(y^-z^+) \geq N(x^+y^-z^+)$ (3)

Adding equations 2 and 3,

$$N(x^+z^-) + N(y^-z^+) \geq N(x^+y^-z^-) + N(x^+y^-z^+) \qquad (4)$$

The right hand side of equation 4 is the same as the right side of equation 1. Therefore, comparing the left hand sides of those equations, we find

$$N(x^+y^-) \leq N(x^+z^-) + N(y^-z^+) \qquad (5)$$

This is Bell's inequality.

Since we can measure only one spin component of each particle in an actual experiment, experiments test another, equivalent statement of Bell's inequality:

$$N(x_A^+y_B^+) \leq N(x_A^+z_B^+) + N(y_B^+z_A^+) \qquad (6)$$

That is, if a particle arrives at detector B with y spin +, its partner must have arrived at detector A with y spin -.

All the above arguments rely on the classical assumptions of a predictable, well-bahaved Universe. But as we have seen, the quantum world is inherantly unpredictable.

Quantum theory predicts random events will scramble the spins of the particle/antiparticle pairs in our experiment, and

$$N(x_A^+ y_B^+) \geq N(x_A^+ z_B^+) + N(y_B^+ z_A^+)$$

Here we have the opportunity for a head-to-head experimental test, quantum mechanics <u>vs</u> classical theory.

Several experimental groups have measured the behavior of various particle systems, and the results indicate

$$N(x_A^+ y_B^+) \geq N(x_A^+ z_B^+) + N(y_B^+ z_A^+)$$

Quantum mechanics wins. Quantum mechanics describes events in the subatomic realm more accurately than classical theory. There's something wrong with our classical assumptions: there are no hidden gears and levers to make particles behave nicely. The quantum realm is, indeed, unpredictable.

<u>SUMMARY</u>

Quantum mechanics turns upside down some of our classical notions about how Nature behaves. On the quantum level:

Nature comes in discrete packages. Mass, energy, charge, spin -- all the measurable attributes of our Universe are quantized.

Particles behave like waves, and their attributes can be described in terms of wave parameters (frequency, wavelength, amplitude.) Energy is proportional to frequency ($E = hf$), and momentum is inversely proportional to wavelength ($p = h/ $).

The observer changes the system she observes.

There are limits to our knowledge: we cannot know, to arbitrary precision, a particle's position and momentum or its energy and the span of time it had that energy.

Events in the quantum realm are random and indeterminate. We can only predict the <u>probabilities</u> of events on the quantum scale. The macroscopic world <u>is</u> predictable because there are such enormous numbers of quanta participating that we see a kind of average behavior.

With this introduction to the quantum realm, we proceed to a discussion of particle physics -- the search for the ultimate constituents of matter and the forces that govern their interaction.

CHAPTER 5 QUESTIONS
QUANTUM MECHANICS

1. Cite evidence that mass comes in discrete quanta.

2. Use the Bohr model of the atom to describe the mechanism by which spectral lines are produced.

3. The wavelength of the hydrogen "alpha transition," one of the common lines seen in stellar spectra, is 6.65 X 10^-5 cm. How much energy is released in this transition?

4. If a star is moving away from us, what happens to the lines in its emission spectrum?

5. Cite evidence that electrons are waves.

6. How is the energy of a wave related to its amplitude? To wavelength?

7. Give an example illustrating how the observer affects the system he/she observes.

8. Electron microscopes provide amazing detail of objects approaching the size of atoms. Most samples observed in such microscopes are coated with a heavy metal to preserve them. Why? What does this say about the act of observing?

9. What is the photoelectric effect, and what does it tell us about light?

10. Find the value of Planck's constant based on the following data, which were obtained in a hypothetical experiment involving the photoelectric effect.

frequency of incident light	kinetic energy of ejected electron
1×10^{16}/sec	6×10^{-11} erg
2×10^{16}/sec	12×10^{-11} erg
3×10^{16}/sec	18×10^{-11} erg

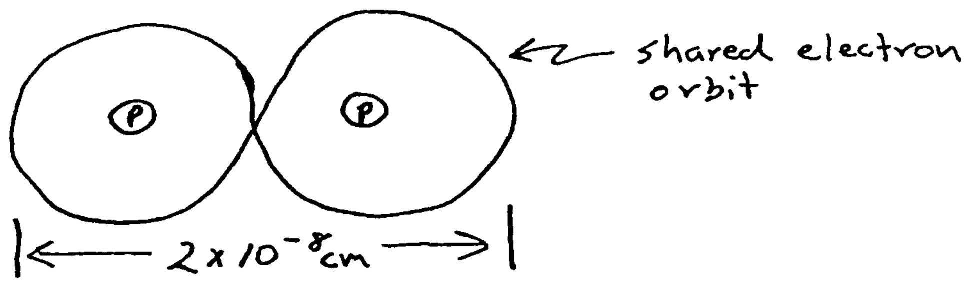
11. A photon of wavelength 3000 angstroms (3 X 10^-5 cm) collides with an electron and recoils (reverses direction). After the interaction, the photon has a wavelength of 6000 angstroms. Assuming the electron is initially at rest, what is the velocity of the electron after the interaction? (The

electron has a mass of 9 X 10^-28 gm. Use the value of h you obtained above and the relation p = h/λ .)

12. Use the single slit experiment to derive the uncertainty principle qualitatively.

13. Use the uncertainty principle to calculate the velocity of an electron in the lowest energy level of a hydrogen atom. The diameter of the atom is about 10^-8 cm, and you can consider this the uncertainty in position. Use the electron mass given in number 10 above.

14. Use the uncertainty principle to calculate the velocity of an electron shared by two protons in molecular hydrogen. Assume the electron has an equal probability to be in the vicinity of either proton, i.e. it could be anywhere in a total diameter of 2 X 10^-8 cm.



Calculate the kinetic energy of an electron in molecular hydrogen, and compare it to the kinetic energy of an electron in atomic hydrogen. (KE = 1/2(mv^2).) If "nature seeks the lowest energy state," why is most hydrogen found in the molecular form, at low temperatures?

15. Derive the time/energy uncertainty relation qualitatively, using the example of the two tuning forks.

16. What are virtual particles? Where do they come from? How does the uncertainty principle predict their existence?

17. What is the role of virtual particles with regard to the forces of nature?

18. Which has the shortest de Broglie wavelength, a proton or an electron? Why?

19. Describe how a laser works. Why is the laser light of a single frequency? Why is the beam collimated?

20. Describe the mechanism of superconductivity. What are the practical uses of superconductors?

21. Describe the operation of a diode. What are the practical applications of diodes?

22. We have described "particles" as two different kinds of waves -- probability waves and wave packets. Distinguish between these two models.

23. Einstein (and others) argued that if we had better understanding of particle physics and better tools to measure the particles we would find underlying cause-and-effect relations in the quantum world. How would you respond to this argument?

24. What determines when a uranium nucleus will decay?

25. Why do heavy nuclei emit alpha particles? (What force "drives out" the alpha particles?)

26. Why is it possible to predict the half-life of a lump of uranium, but not when any individual nucleus will decay?

27. Computers rely on electron flow and storage for their function. What happens to the logic of a computer as its components approach the size of an electron? (Hint: remember the uncertainty principle.)

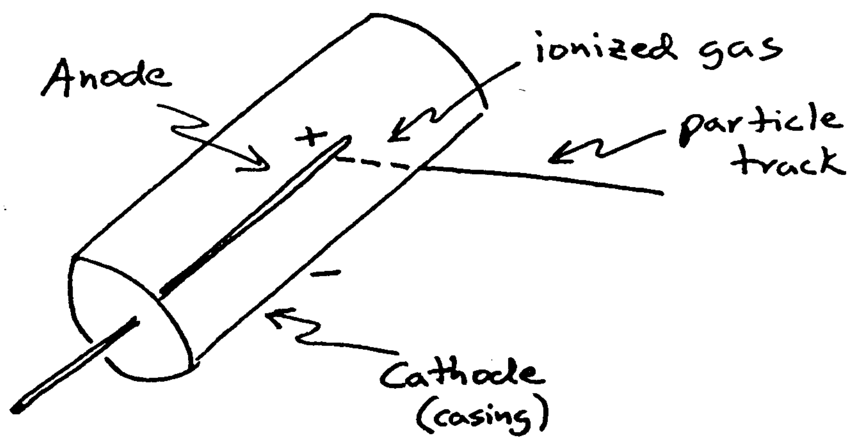
DEMONSTRATIONS
CHAPTER 5

1. The Geiger counter:

The geiger counter detects energetic subatomic particles, the most common of which, in everyday counter use, are gamma rays and high-energy beta particles (electrons). It is a simple analog to some of the sophisticated detectors used in modern particle accelerators.

The detector comprises a metal tube filled with gas. A battery gives the tube (the cathode) a net negative charge, and a metal rod (the anode) in the axis of the tube is given a net positive charge. Whenever a high energy particle enters the tube, it ionizes gas along its track, so a spark jumps between the anode and the cathode. The counter records the spark, and we hear the spark, amplified by the detector, as a click.

Geiger detector



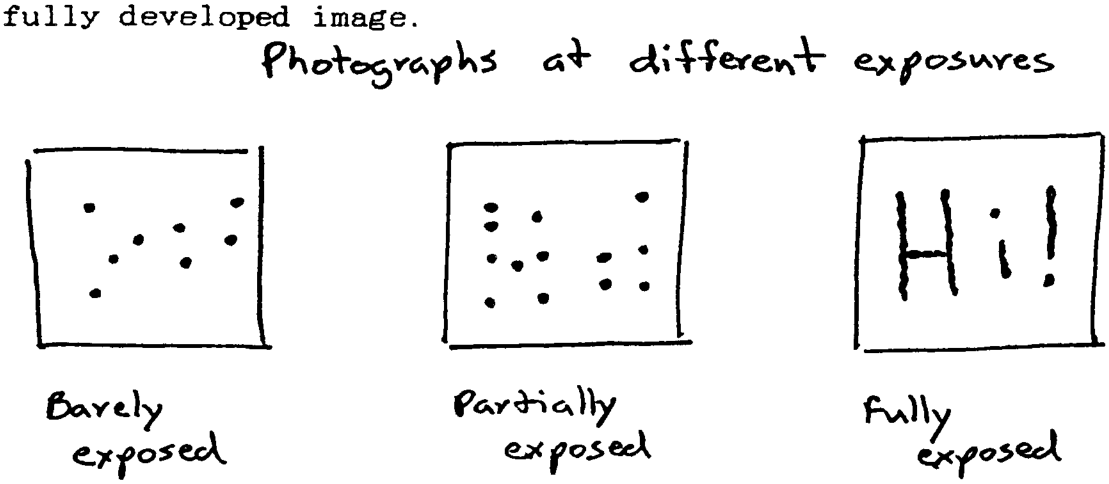
Listen to the random clicks of a geiger counter as it records the "background" radiation, including cosmic rays (high energy particles generated in outer space), the secondary particles they produce by collisions with atoms in the atmosphere, and radioactive sources in the geophysical environment -- uranium in the ground and in building materials, radon gas, etc.

Then place the counter near a radioactive source. Knowing the weight of the source, the atomic weight of the elements in the source, and the decay rate recorded by the geiger counter, can you determine the half life of the source?

2. Quantum nature of light:

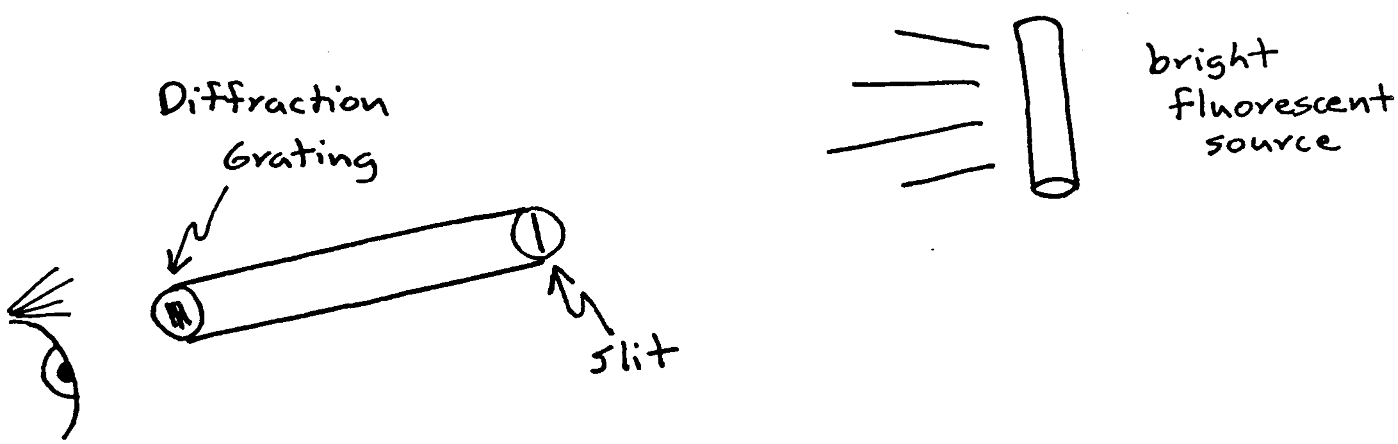
That light comes in "packages" is evident in under-developed film. Take a series of photographs of the same subject. Deliberately under-expose the first few photographs (i.e. don't let enough light reach the film to fully expose it). Gradually increase the exposure time in successive photographs until the final picture is fully exposed.

In the under-exposed photographs, you will see "grains" (dots) on the film where photons (quanta of light) have exposed light-sensitive molecules on the film. In more adequate exposures, the grains blend together into a smooth, fully developed image.



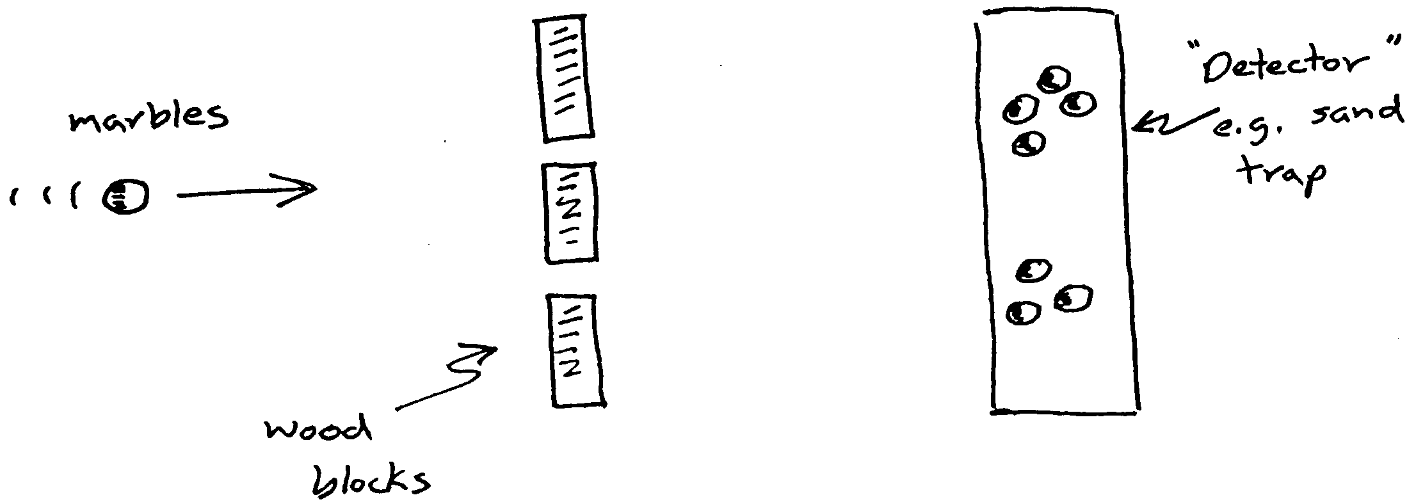
3. Diffraction grating spectra:

Find a diffraction grating. (Science laboratories, including school labs, usually have gratings available.) Look at different fluorescent lights (e.g. hydrogen, helium, mercury vapor, sodium vapor, argon, neon) through the grating. How do the emission lines of the different sources compare?



4. Particles and waves:

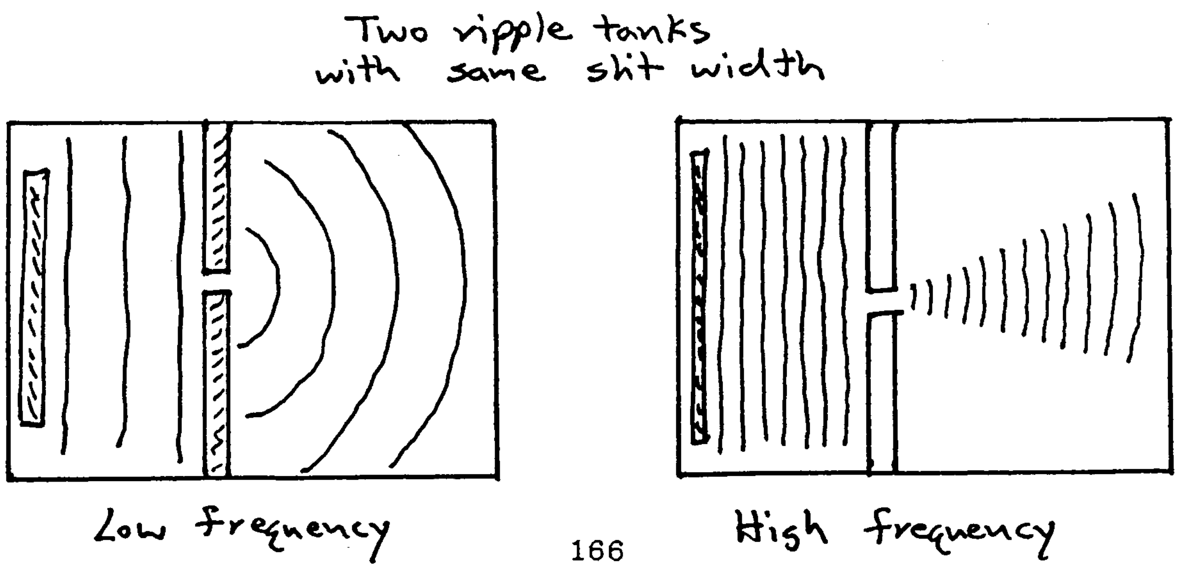
Set up a double slit experiment on a smooth, flat surface with marbles and blocks of wood. Record the numbers of marbles arriving at different points on a detector placed at some distance from the slits.



Now set up the double slit experiment in a ripple tank. See diagram on p. .) How does the wave distribution at the detector compare with the distribution of marbles?

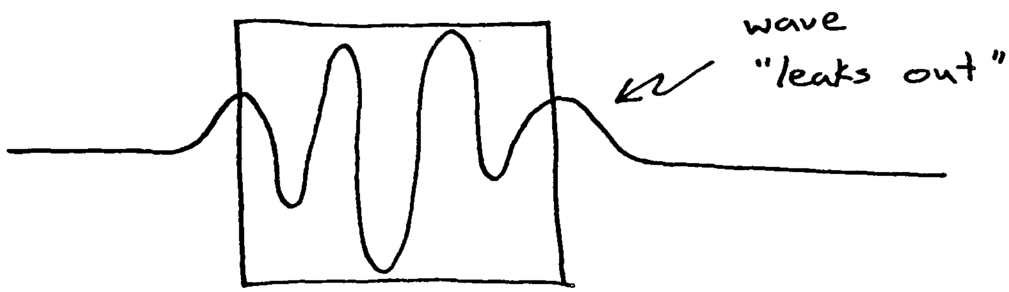
5. Angle of diffraction depends on slit width and wavelength:

Set up a single slit in a ripple tank. Vary the slit width and the wavelength (by varying the frequency). What is the relationship of the angle of diffraction to the slit width? To the wavelength?



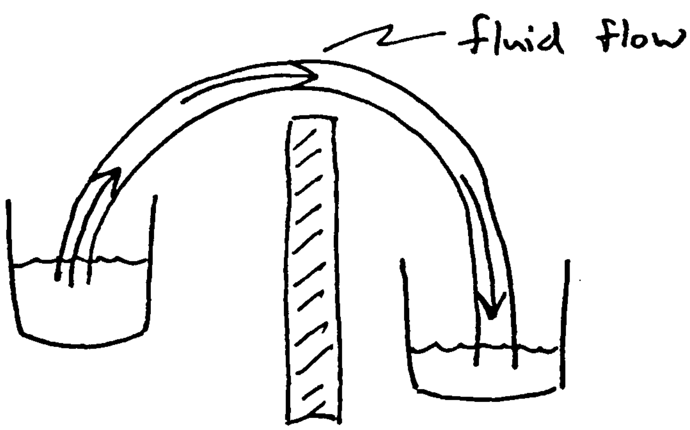
6. Tunneling:

One consequence of the wave nature of matter is a phenomenon called "tunneling." Because a de Broglie wave (matter wave) is delocalized, it has a certain amplitude to tunnel through classical barriers: part of the wave "leaks" through the barrier.

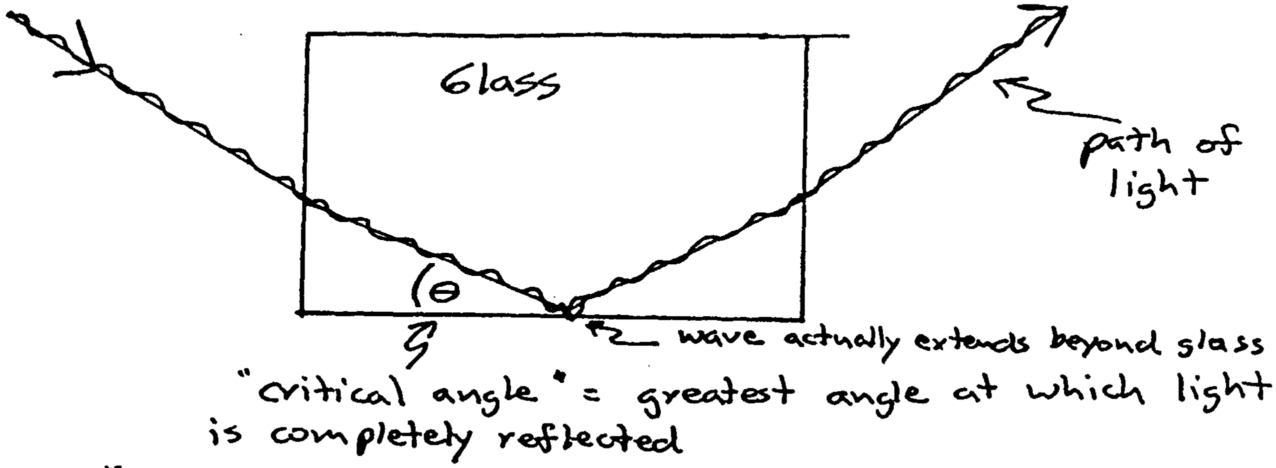


electron wave packet "confined" in some volume (e.g. an atom) actually extends outside the volume

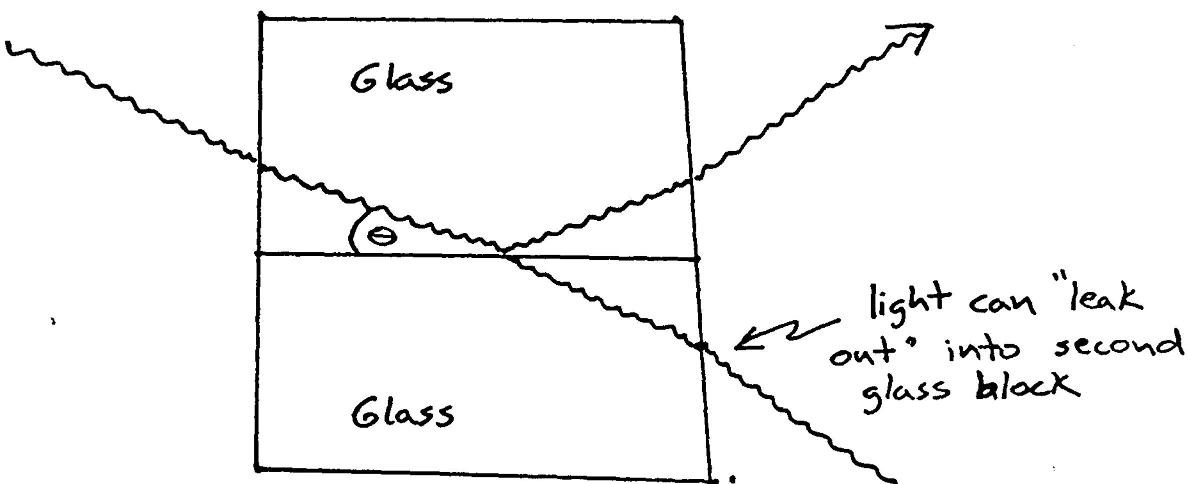
A siphon offers one analogy of tunneling: A siphon can raise fluid over an otherwise "impenetrable" barrier, allowing it to flow to a lower energy state on the other side.



A more accurate demonstration of tunneling comes from the phenomenon of total internal reflection. Shine a laser beam into a block of glass. Vary the angle at which the beam strikes the inner surface of the glass until it is just reflected from the inner surface and no light passes through the glass.



Now, with the beam still striking the inner surface at the critical angle, juxtapose another block of glass. Part of the beam will pass into the second block. This occurs because the light waves actually extend beyond the glass of the first block and are available to travel through the second block as soon as it is juxtaposed.



7. The effect of the observer on reality:

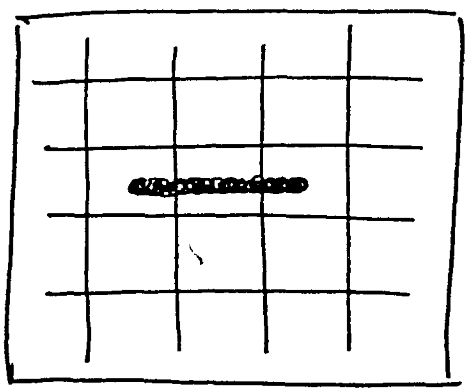
Here's an interesting game, based on the game "20 questions," demonstrating how an observer can affect the system she/he observes.

From a group of people, send one person (the "observer") out of the room. Choose an object or living thing ("animal, mineral, or vegetable") and invite the observer back into the room. Asking each person in the group, one after another, a "yes" or "no" question, the observer has twenty questions to decide what object the group has chosen.

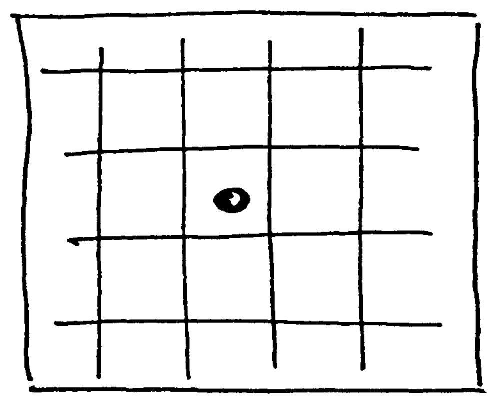
Now send another observer from the room. This time, each person in the group chooses his own object or living thing and does not tell the other members of the group. The observer returns to the room and asks questions as before. Each person answers according to his own choice of object. The members of the group change their choice of object as the questioning progresses in order to accomodate answers by other members of the group. What happens? To the direction of the questioning? To the objects chosen by the group?

8. Uncertainty principle:

Take photographs of a rapidly moving projectile (for instance a baseball). First photograph a uniform square grid, black lines on a white background. Then take a double exposure (superimposed on the grid in the first photo) while tracking the ball with the camera. For the second set of pictures, photograph the grid as before, then double expose the flight of the ball with the camera fixed on a tripod. With the camera fixed, the projectile appears as a blur against the grid. With the camera tracking the projectile, we obtain a sharp image of the projectile. How does this system model the uncertainty principle? Why is this macroscopic system not exactly analogous to the quantum world?



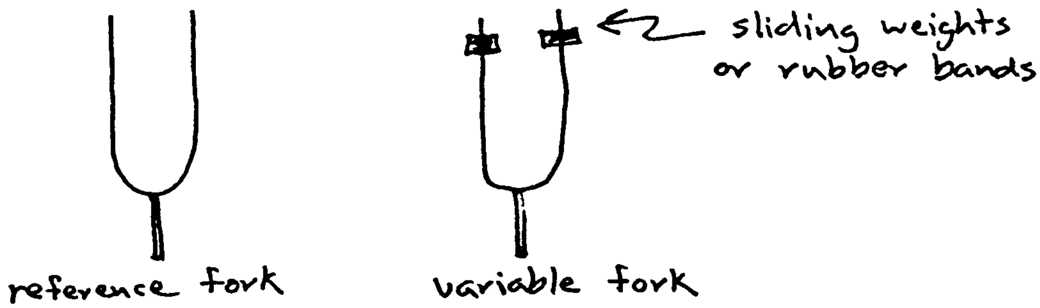
Picture taken with camera fixed and ball moving rapidly left to right



Picture taken with camera tracking ball

9. Time/energy uncertainty:

Set up the paired tuning fork system discussed on p. . Imagine that you have no other measuring devices except the two tuning forks, and you want to use the forks to determine frequency and to measure the flow of time. The fixed-frequency fork is the reference fork for your measurements, and the other fork is variable.



In such a system, you will hear "beats" (oscillations in loudness) when the forks are slightly out of tune. When the two forks are ringing at precisely the same frequency, there will be no beats. You know, then, that the variable fork is exactly in tune with the reference fork when there are no beats.

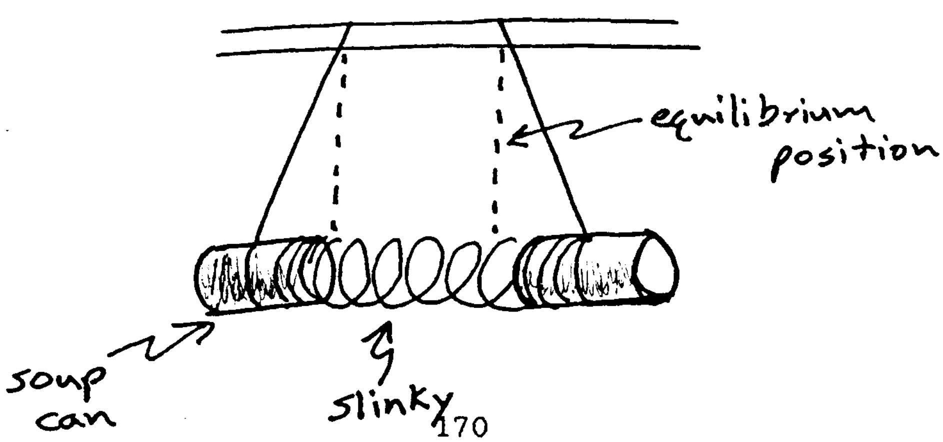
On the other hand, in order to build a clock from the two forks, you must have beats: the beats are the "ticks" of the clock. Furthermore, the more beats, the more accurate the clock: you can measure time more accurately if you divide it into smaller intervals.

With only the two tuning forks, then, you can <u>either</u> measure frequency accurately (no beats) <u>or</u> you can measure time accurately (many beats), but you cannot measure both together. Since frequency is proportional to energy in quantum mechanics, the tuning fork system privides an analogy to the energy/time uncertainty relation.

10. Coupled pendulums as a model of atomic energy states:

Hang two pendulum bobs on strings of equal length, and connect them with a weak spring. (Soup cans as bobs connected by a slinky serve well.) Pull both cans slightly to the right, the same distance from their equilibrium point, and let them go. What happens? Determine their frequency (oscillations per minute).

Now pull one can to the left and the other to the right, the same distance, and let them go. Count their frequency. How does it compare with the frequency you found above?



Now leave one can at equilibrium but pull the other slightly away from the equilibrium position, and let it go. What happens?

These coupled pendulums behave similarly to coupled atoms, for instance in a laser. The different frequencies represent different energy states available to the atomic system.

If you have more pendulums and springs available, try building extended systems, with three, four, or more pendulums.

11. Randomness:

Pop several batches of popcorn, one after another, in a pyrex (transparent, heat-tolerant) container. Use constant heat, and use the same number of kernels in each batch. Record the time it takes for half of each batch to pop. What is the average "half life" for a batch of popcorn?

Pick an individual kernel in each batch, and predict when it will pop. How do your predictions compare with the actual time at which the kernel pops?