Compactification

If strings occupy extra dimensions, where are they? We experience only three dimensions of space and one of time.

Even with all the extras, there can only be one time dimension. Otherwise energy would vary along extra time dimensions, in violation of the conservation law. So any extra dimensions must be spatial. And we assume they must be compactified. That is, they are too small for us to measure with our present technologies (although experiments are underway to try to detect them). What do we mean, compactified?

As an example, consider a garden hose. From a distance it looks like a line, one-dimensional. We can determine the position of any point on the hose by measuring its distance from one end. But if we study the hose up close, we see that it is two-dimensional. An ant on the hose can crawl along lengthwise, but it can also crawl around the hose circumference. Up close, we need two variables to describe the ant's position – how far from one end and how far around the circumference. Seen from a distance, that second dimension is not evident – it has been "compactified" from our larger frame of reference.

We can simplify the topology (and therefore the math) by slicing the hose lengthwise.

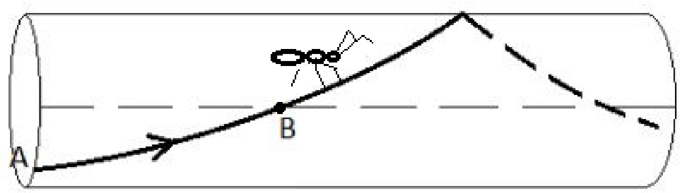


Figure 12.1 Hose with lengthwise slice.

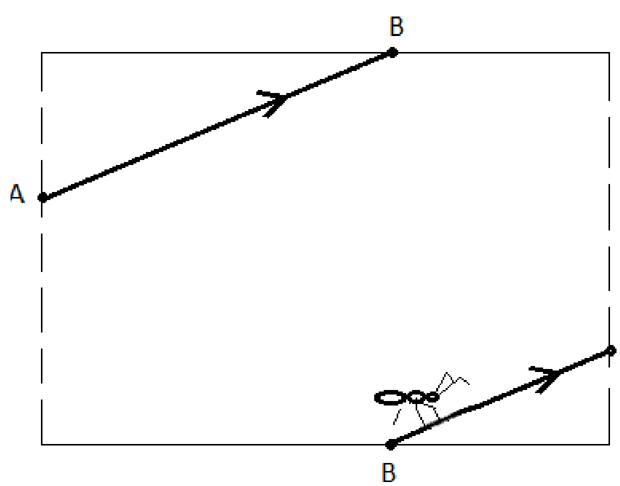


Figure 12.2. Section of sliced hose, assuming it extends indefinitely to right and left.

The hose becomes a flat sheet and the path of the ant a series of diagonal lines. So that the topology is the same in both pictures, we must identify points along the slice. Point B at the top "edge" in the figure below is the same B at the bottom. The ant "leaves" the top edge at B and immediately appears at the bottom B.

Similarly, we can illustrate the compactification of three dimensions into one. As illustrated in the figure below, we identify the top face of a parallelepiped with the bottom face, left face with right face, front face with back. Then every (x, y, z) point along the path of an object moving through the space can be mapped to a single point (the distance from a reference point) on the one-dimensional path.

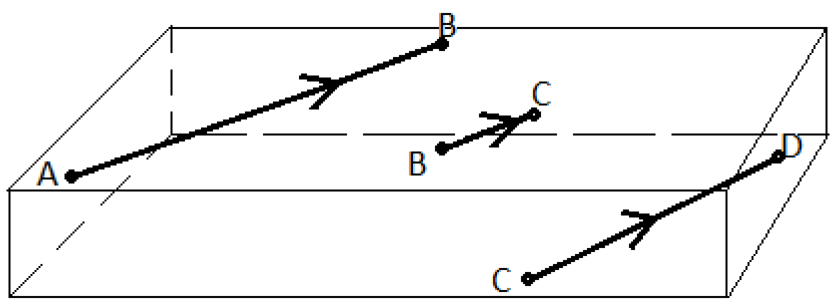


Figure 12.3

For our purposes discussing strings, we will use toroidal compactifications.

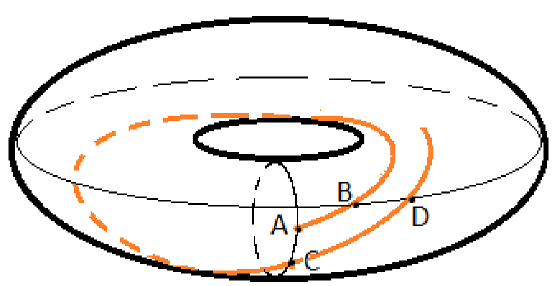


Figure 12.4. Path of an ant on a torus. Slices run through A – C and B – D.

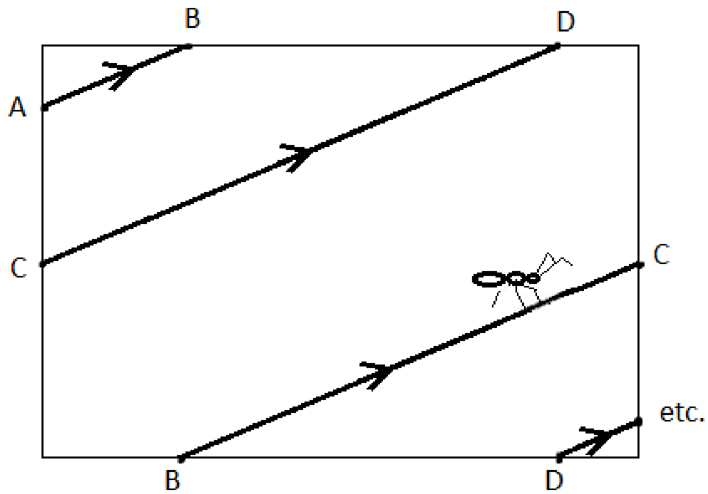
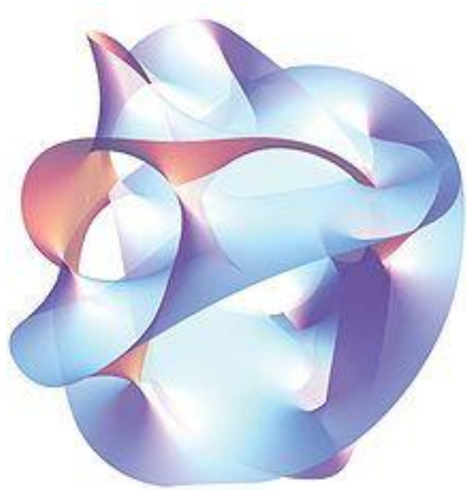


Figure 12.5. Sliced torus opened up to flat surface, showing path of ant and identifications along the slices.

Real strings are thought to lie on Calabi-Yau spaces, which include all the extra dimensions.



Projection of a Calabi-Yau space. (A projection is the "shadow" of the multidimensional structure cast onto familiar 3-space.) http://demonstrations.wolfram.com/CalabiYauSpace/

Just as sunlight projects a 2-D shadow of your 3-D body, this image projects the "shadow" of the extra dimensions onto our 3-D world. The strings would weave around the "handles" in the Calabi-Yau space.

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