String scattering amplitude

What's inside the black boxes of the T- and S- channels? We've assumed they are filled with propagators, loops, and particle interactions. But the picture isn't very satisfying. Can we build a better model?

Physicists fifty years ago started doodling. One of them, Leonard Susskind, was taken with the simple elegance of Feynman diagrams that showed interactions between extended objects, strings, rather than point particles.

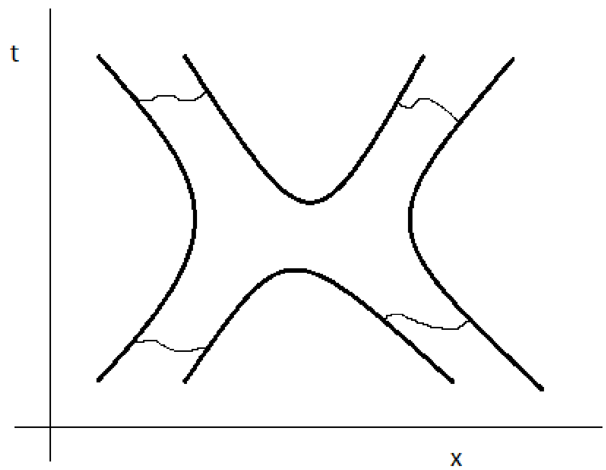

Figure 14.1. World sheet of interacting open strings. Incoming strings approach each other (bottom of the diagram), join briefly, then separate again.

The strings trace out a "world sheet." From bottom to top (earlier time to later time) they approach, join, and separate. In this picture, the distinction between T- and S- channels disappears. Turn the diagram on its side, and it looks the same. With some work, we will show that this model of particle interactions has several advantages.

Some refinements: In these diagrams, loops appear as holes in the world sheet of open strings.

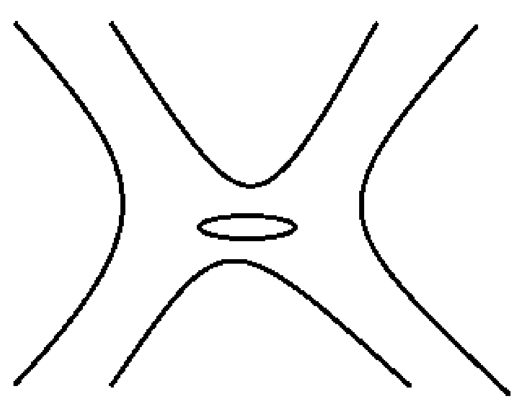

Figure 14.2. World sheet with a handle. We do not show the axes; assume the usual axes.

And closed strings form a “pant leg” diagram.

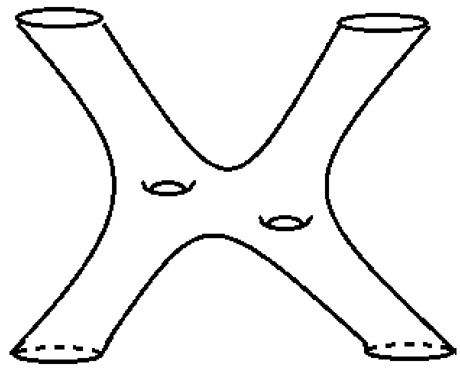

Figure 14.3. Interactions between open strings.

The basic mechanics of the open- and closed- string interactions are the same.

Here’s our strategy. First, we’ll write the amplitude for a string starting in one particular state and ending in another particular state, including terms for center of mass, position along the string, and momentum of points along the string. Then we’ll minimize the action of the string’s path. The action equations will allow us to express string interactions in terms of the topology of the world sheet. That topology has an invariance that allows us to summarize the interactions in much simpler form.

Quite a mouthful, all that. Put another way, we’re going to express the equations in pictures on a stretchy membrane (the world sheet) whose basic form is simple.

Here we go. We’ll work with open strings. First, we write the amplitude for a string to be found in state P at time t_1 and in state Q at time t_2.

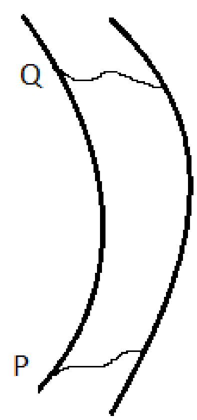

Figure 14.4. Two states along the world sheet of an open string.

$$A = \int_{all\ surfaces} e^{i \int_{all\ modes} d\tau d\sigma \left(\left(\frac{\partial x}{\partial \tau}\right)^2 - \left(\frac{\partial x}{\partial \sigma}\right)^2 \right)} \tag{14.1}$$

The terms in red are the string's internal kinetic and potential energies, but now with the minus sign of the Lagrangian. All the internal modes are integrated over time in the exponential to give the string action. Finally, the action is integrated over all possible surfaces. Let's think about this a bit.

The world sheet from state P to state Q can vary wildly between those states. For instance, the following world sheet still meets the criteria that the string starts in state P and ends in Q.

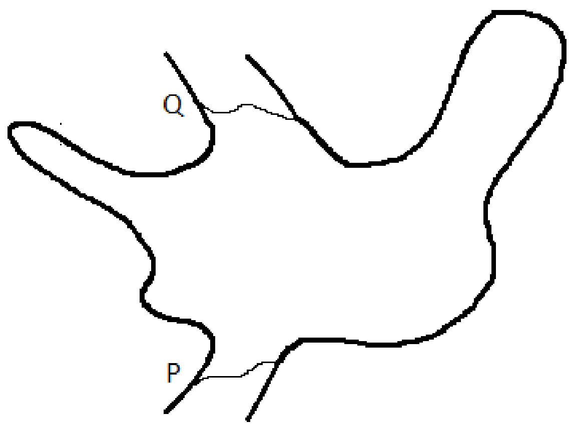

Figure 14.5. Another possible configuration of the world sheet between states P and Q.

Just as all possible loops must be included in calculating amplitudes for particle interactions in the standard model, so must the contributions of all possible world sheets in the string model. However, not all world sheets are equally probable. The probability is vanishingly small that a world sheet would stretch out to Alpha Centauri.

We can weed out the low-probability contributions with a mathematical trick. We re-parametrize our equation, substituting $-i\tau$ for τ. The effect is to rotate all the position and momentum vectors $90°$ around the complex plane, but the amplitudes stay the same.

$$A = \int_{all\ surfaces} e^{i \int_{all\ modes} d\tau d\sigma \left(\left(\frac{\partial x}{\partial \tau}\right)^2 - \left(\frac{\partial x}{\partial \sigma}\right)^2 \right)} \tag{14.2}$$

$$\Rightarrow \int_{all\ surfaces} e^{i \int_{all\ modes} -i d\tau d\sigma \left(\left(\frac{\partial x}{-i\partial \tau}\right)^2 - \left(\frac{\partial x}{\partial \sigma}\right)^2 \right)}$$

$$= \int_{all\ surfaces} e^{- \int_{all\ modes} d\tau d\sigma \left(\left(\frac{\partial x}{\partial \tau}\right)^2 + \left(\frac{\partial x}{\partial \sigma}\right)^2 \right)}$$

This is the Polyakov amplitude (first formulated by Susskind). Two points to note. The substitution results in a negative exponential, so wild fluctuations disappear (the exponential goes to zero). And the plus sign returns in the energy term. We have converted the Lagrangian (with the minus sign) into Laplace's equation (with the +). This recovers the total internal string energy we found earlier and, most importantly, allows us to represent the amplitude for string interactions as topology, the shape of the world sheet.

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