The Veneziano amplitude

We are in search of a model to predict scattering amplitudes – what particles will we find among the products of a particle collision, and what is their momentum? We've introduced the two "black boxes" of particle interactions, the S- and T- channels. With a little imagination and some logic, we can describe what's happening inside those channels.

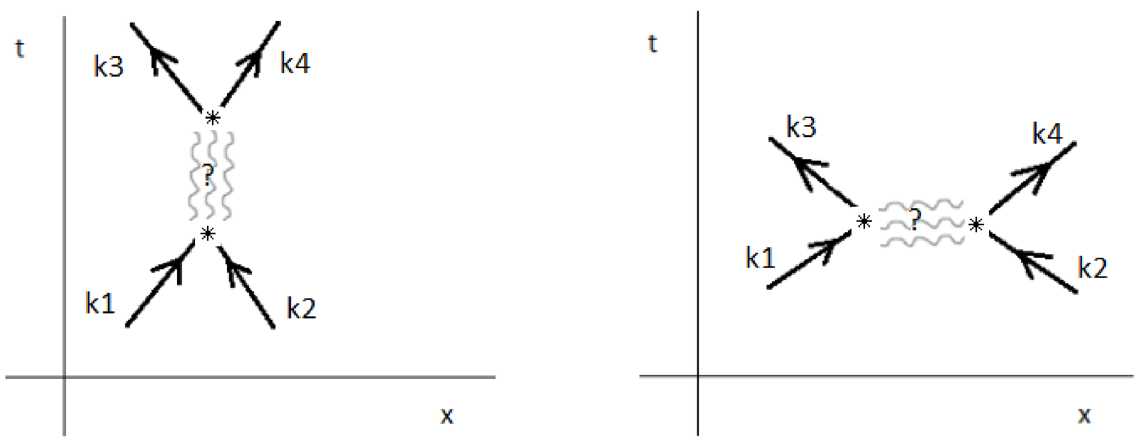

Figure 15.1. T- channel on left, and S- channel (right) particle interactions.

Presumably, inside the black box of the T-channel is a combination of propagators, loops, and vertices. Ditto inside the S-channel. Classical probability theory tells us

$$C_{S+T} = \frac{n_{S+T}!}{n_S!\, n_T!} \tag{15.1}$$

The total number of combinations of particle processes in both channels, combined, equals the factorial of all processes in the two channels divided by the product of the factorials of the processes in the two channels separately.

As a simple example, suppose you are given a bag containing three marbles, one red, one green, and one blue. Calculate how many different color combinations you can expect if you repeatedly pull two marbles out of the bag. Replace them before the next draw. Simple enough – you could remove a red and a green (RG), or RB, or GB. There are three possible combinations. (We ignore order – which marble was pulled out first, which was second.) Our calculation, sure enough, predicts three possible combinations.

$$C_{3,2} = \frac{3!}{2!\, 1!} = 3 \tag{15.2}$$

Good for marbles. But we're interested in the amplitude for particle interactions. To continue the logic, we'll think in terms of probabilities. (The probability of an event is the square of the amplitude for that event.) What is the probability for a particular outcome in a particle collision?

Well, the more combinations of processes there are in the S- and T- channels, the less probable it is that any one particular outcome will occur. The more combinations in the black boxes, the greater the number of different particles that we might see flying out of them. So the probability for any outcome is proportional to

$$\frac{1}{C_{S+T}} = \frac{n_S!\, n_T!}{n_{S+T}!} \tag{15.3}$$

Our rough argument is illustrative but incomplete. The complete expression for the interaction amplitude is

$$A = g^2 \frac{\Gamma_{-\mathrm{S}}\Gamma_{-\mathrm{T}}}{\Gamma_{(-\mathrm{S}-\mathrm{T})}} \tag{15.4}$$

This is the Veneziano amplitude, discovered by the Italian physicist Gabriele Veneziano. The gammas, Γ , are extended factorial operators with value $(n-1)!$, where the n's can be fractional. g , as usual, is the interaction coefficient, squared here because two interactions occur, one at each end of the black box. The minus signs on the channels represent the fact that S and T have units of negative energy.

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