The string interaction amplitude

Now we have all the pieces necessary to build the full interaction amplitude for open strings. The interaction world sheet can be mapped to a conformal disk. We know the amplitude for strings. All we need is to add the contributions from the k's , the momenta of the incoming and outgoing particles.

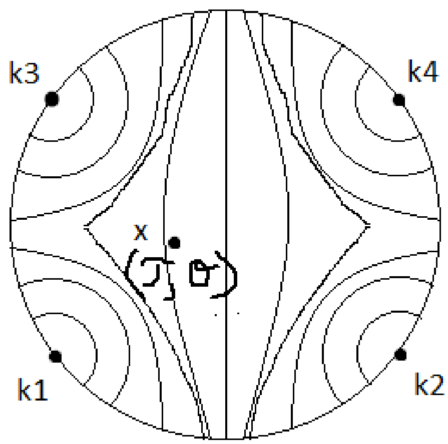

Figure 18.1. World sheet of interacting strings mapped to a disk. Mapping is conformally invariant. k1 and k2 are momenta of incoming particles, k3 and k4 momenta of outgoing particles.

We define a parameter, z , representing the interval of time during which the strings are joined. We can choose to study the interaction in a center-of-mass frame of reference, so the trajectories of incoming and outgoing particles are symmetric on the disk.

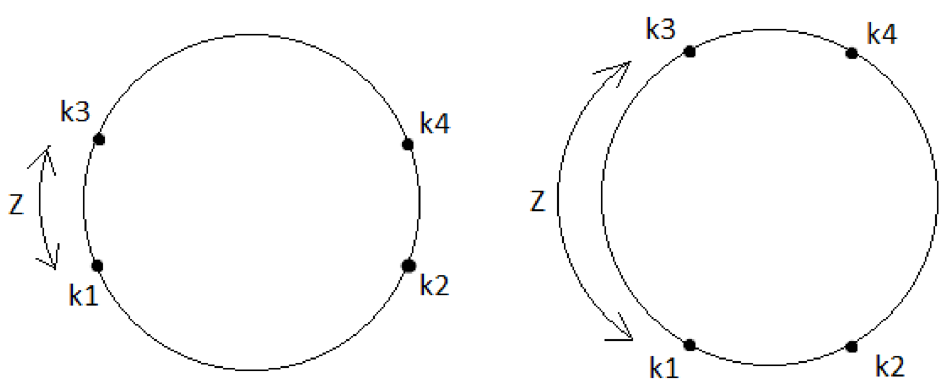

Figure 18.2. Short joining interval on the left, longer interval on the right

Just as there are an infinitude of loops and vertices and propagators in the particle model, there are an infinite number of possible joining intervals, z's , in the disk model. We sum over all z's in the final expression of the interaction amplitude.

$$A = \int_{all\ z} \int_{all\ surfaces} e^{-\int_{all\ modes} d\tau d\sigma \left(\left(\frac{\partial x}{\partial \tau}\right)^2 + \left(\frac{\partial x}{\partial \sigma}\right)^2 \right)} \Pi_i e^{k_i x^{\mu}(z_i)} \tag{18.1}$$

This is the grand result of our efforts. $e^{-\int_{all\ modes} d\tau d\sigma \left(\left(\frac{\partial x}{\partial \tau}\right)^2 + \left(\frac{\partial x}{\partial \sigma}\right)^2 \right)}$ accounts for all the energies of all the modes. $\Pi_i e^{k_i x^{\mu}(z_i)}$ factors in momenta of all possible incoming and outgoing particles.

Messy as it looks, the integral is solvable. Its solutions are mathematically analogous to solving for the electric field on a 2D surface given point charges (the k's) distributed on the rim. Easy – except we have to work in 26D for strings, as if solving the field for 26 charges at each k. Daunting, but doable!

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