Having established that strings live in extra dimensions and that the "surfaces" in those extra dimensions must be Ricci flat, we proceed to explore the consequences. We will adopt the torus as a typical Ricci flat surface. Here we encounter the "dualities" of string theory. It turns out that strings on a torus can exist in different configurations that produce the same energy spectra; different configurations of string produce the same physics. By extension, the dualities apply to other Ricci flat surfaces, especially Calabi-Yau spaces, the favorite candidates for the string multiverse. Furthermore, the dualities demand other structures in the theory besides strings. Strings are tied to branes.

Strings on tori

Enter T-space, toroidal space. Our tori are Ricci-flat. You can cut along one curvature of the torus and straighten it to a tube. Slit the tube and you have a flat sheet. Flat space.

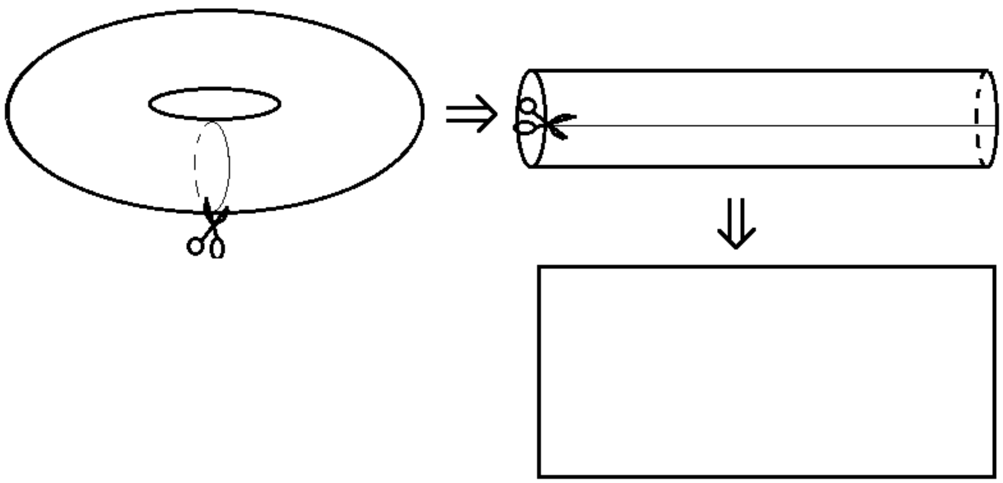

Figure 20.1. Dissecting the torus to show its surface is flat.

For simplicity, we'll draw strings on the tube, as in the middle image in Figure 20.1. Remember, though, they're moving on a torus.

Strings, both open and closed, have two components of motion on the torus, one lengthwise along the tube, the other circumferentially around the tube. Momentum along the tube is plain ol' linear momentum. But circumferential momentum behaves like angular momentum and is quantized.

$$p_{circ}r = L = n\hbar \tag{20.1}$$

where, as usual, n is mode number and we will set $\hbar = 1$. r is the radius of the tube. Then

$$p_{circ} = \frac{n}{r} \tag{20.2}$$

Particles described by this momentum are called Kaluza-Klein particles, after the physicists who first postulated extra dimensions in physical systems.

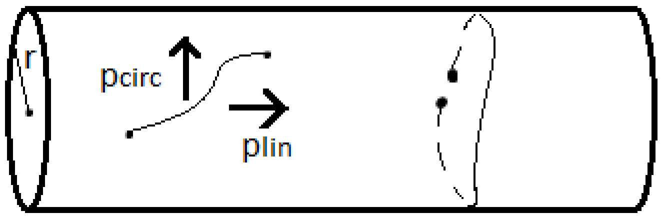

Figure 20.2. Open string on left, arrows showing linear and circumferential momentum. To the right is an open string wrapping around torus, ends about to join.

Energy is proportional to momentum (by $E = pc$), so the (quantized) energy spectrum for circumferential motion is widely spaced for small r, closely spaced for large r.

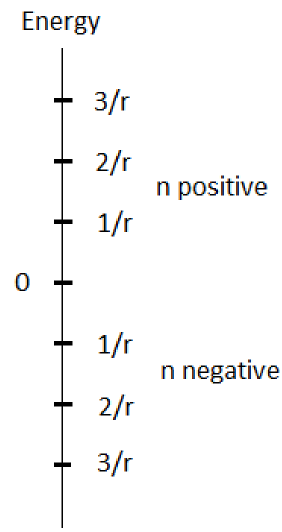

Figure 20.3. Spectrum of Kaluza-Klein string on torus with small r, $E = \frac{n}{r}$, is the same as the spectrum of a wound string on torus with large r, $E = w2\pi r$ (see below).

Strings have another option, though. Ends of an open string can join to form a loop around the tube, or a closed string can stretch around the tube and fuse to form a double loop. As always, orientation must be preserved when strings join.

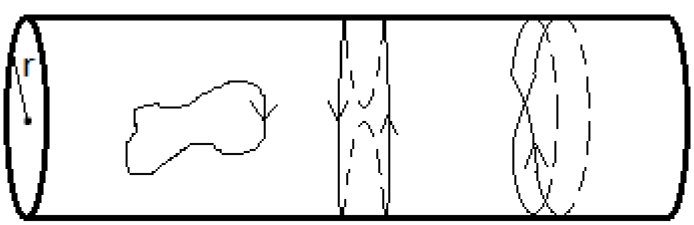

Figure 20.4. Closed strings on torus. Middle string wrapping around torus, ends about to fuse and form two wrapped strings. Right string wrapped twice around torus. Arrows show string orientation.

The wound string has a characteristic tension. The greater the radius of the tube, the more the string is stretched and the more energy is stored in the string.

$$E = w2\pi r \tag{20.3}$$

where w is the winding number and string energy is proportional to how far (how many times around the circumference) the string is stretched. The energy spectrum in this case is spaced proportional to r.

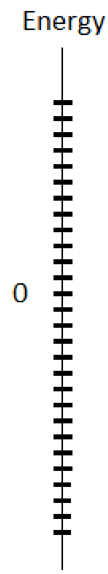

Figure 20.5. Spectrum of Kaluza-Klein string on torus with large r is identical to the spectrum of a wound string on torus with small r .

Curious. On the one hand, a Kaluza-Klein string fluttering around the circumference has energy spectrum inversely proportional to r . On the other hand, a closed string wound around the tube has energy spectrum directly proportional to r. At some r the energies of the two different string configurations are the same, and as r increases the different strings slide across onto the other's spectrum.

This is T-duality (T for "toroidal"). As you increase the circumference of the tube, the energy spectrum of the wound string looks like the energy spectrum of a string with circumferential momentum around a shrinking tube. And vice versa. The same physics describes both kinds of string, and we can use the same equations to describe either.

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