T-duality and branes

We've sketched the rationale for T-duality. Toroidal flat-space allows two string configurations that share the same energy spectra. Further analysis shows that T-duality requires an even more surprising extension of the theory: there must be other structures in the theory besides strings. Enter brane-world.

Here's the argument.

Kaluza-Klein strings have momentum

$$p_{circ} = \frac{n}{r} \tag{21.1}$$

Momentum is a function of velocity. If we arbitrarily assign mass $= 1$ and designate y as the axis circumferential around the torus, we can write the momentum associated with quantum number, n .

$$p_{circ} = \frac{\partial y}{\partial t} = \frac{n}{r} \tag{21.2}$$

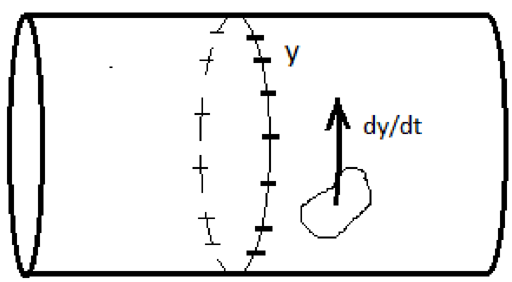

Figure 21.1. Momentum of a Kaluza-Klein string. y is the coordinate on the torus oriented around the circumference.

Solving for n

$$n = r \int \frac{\partial y}{\partial t} \tag{21.3}$$

where the integral is over all the points along the string.

Wound strings have energy

$$E = w2\pi r \tag{21.4}$$

where w is the winding number. E is the potential energy stored in the string and is proportional to how far the string is stretched. We can re-write

$$E = \frac{\partial y}{\partial \sigma} = w2\pi r \tag{21.5}$$

$\frac{\partial y}{\partial \sigma}$ measures how far the string is stretched from its relaxed, ground state. (Think of it as the ratio how many units of y fit in each unit σ of the string coordinates.)

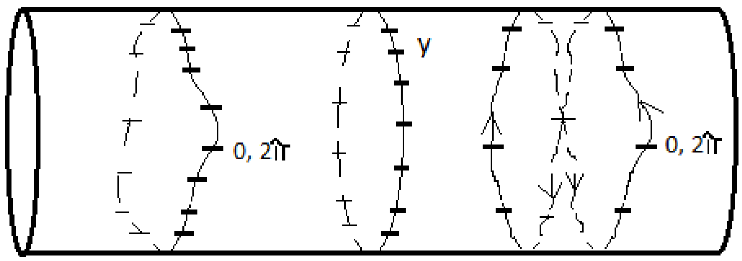

Figure 21.2. String with single winding on left. Note intrinsic coordinate σ has identity $0 = 2\pi$. String on right is wound twice. Note that the intrinsic coordinates have stretched such that each unit change in σ covers twice as many units y.

Dropping the constant coefficients and integrating all around the string

$$w = \frac{1}{r} \int \frac{\partial y}{\partial \sigma} \tag{21.6}$$

The equations for n and w are similar in form. We can transform one into the other by applying all of the following rules simultaneously.

$$n = r \int \frac{\partial y}{\partial t} \leftrightarrow w = \frac{1}{r} \int \frac{\partial y}{\partial \sigma} \tag{21.7}$$

if

$$n \leftrightarrow w$$

$$r \leftrightarrow \frac{1}{r}$$

$$\frac{\partial y}{\partial t} \leftrightarrow \frac{\partial y}{\partial \sigma}$$

This is T-duality.

And written in this form it requires new beasts in the zoo. We have established that open strings require the Neumann condition

$$\frac{\partial y}{\partial \sigma} = 0 \tag{21.8}$$

Applying T-duality to the Neumann condition,

$$\frac{\partial y}{\partial \sigma} = 0 \rightarrow \frac{\partial y}{\partial t} = 0 \tag{21.9}$$

Under this transformation, the ends of such strings can't wiggle. These strings must be anchored, and there must be some other structure that provides the anchor.

Enter the branes, which name derives from "membrane." Branes are the topological structures to which open strings attach. They are cataloged by their dimensionality.

D0 branes are points in space.

D1 branes are linear. The end of a string can slide along the one dimension, like the hondo on a cowboy's lariat.

D2 branes are sheets. The end of a string can move in two dimensions along the brane but cannot lift off the brane into the third dimension.

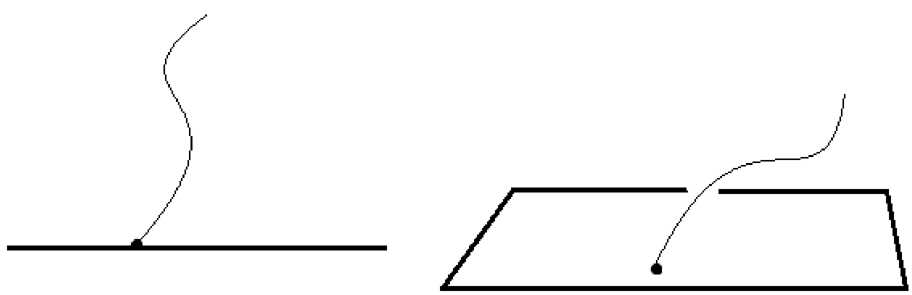

Figure 21.3. Left, string on D1 brane. Right, string anchored to D2 brane.

On D3 branes strings behave as if in the 3-space of our everyday experience, ends free to move in any of the three directions.

etc. into higher dimensions.

It's a brane new world!

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