Black hole temperature and entropy

Temperature first. The easiest derivation of black hole temperature follows the line of logic developed below. We'll build a black hole from scratch, one piece of information at a time.

What is the nature of that information? We need a simple bit of information without any internal structure (which would require extra information for its description). We choose photons (and ignore their polarization.) We'll build a black hole adding one photon, one bit of information, at a time.

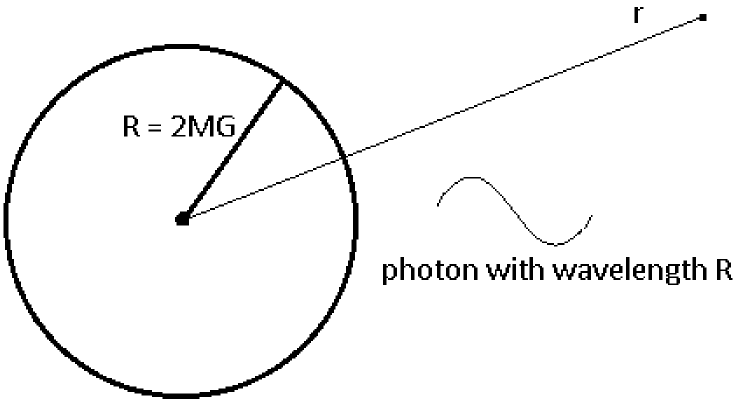

Figure 26.1. Parameters describing a black hole.

Next caveat, we add photons in a way such that we cannot determine where the photon entered the black hole. That would require further information about the process, a map showing where the photon crossed the event horizon. The photon must spread out over the entire event horizon, so we don't know where it entered. So we require photons with wavelength comparable to the radius of the black hole. We're all set. Here's the math.

Re-arranging the energy / entropy relation,

$$T = \frac{\Delta E}{\Delta S} \tag{26.1}$$

But we're building the black hole one bit of information, one photon, at a time, so $\Delta S = 1$ and

$$T = \Delta E \tag{26.2}$$

All we need is s suitable expression for E . It comes from the quantum relation between energy and wavelength.

$$\Delta E = hf = \frac{hc}{\lambda} \tag{26.3}$$

We have established that the wavelength $\lambda = \frac{2GM}{c^2}$, the radius of the black hole. So, finally,

$$T = \frac{hc^3}{2GM} \tag{26.4}$$

Note two features. The black hole temperature is quantized (evident by h in the equation), and temperature is inversely proportional to black hole mass. The smaller the black hole, the hotter it is.

And here lies a puzzle. Black holes are never in stable thermal equilibrium. The temperature of a black hole at the core of our Galaxy, for example, is very much lower than the temperature of the cosmic microwave background. Heat flows from hot to cold, so microwave photons fall into the black hole, increasing its mass and dropping its temperature even further.

But it's a race to the bottom. Even while the black hole grows, the universe is expanding, and along with expansion the cosmic background temperature drops. Eventually, it drops below the temperature of the black hole. Then the hole begins to radiate, losing mass, so it gets even hotter and radiates more vigorously . . . until it finally evaporates in a burst of gamma rays.

Return to Table of Contents