Black hole entropy

Having found the temperature of a black hole, we can calculate its entropy. Start with the equations for temperature and the general relation between energy and entropy.

$$T = \frac{hc^3}{2GM} \tag{27.1}$$

$$\Delta E = T\Delta S = \frac{hc^3}{2GM}\Delta S \tag{27.2}$$

$$\Delta S = \frac{2GM\Delta E}{hc^3} \tag{27.3}$$

We substitute the relation between energy and mass, $E = Mc^2$.

$$\Delta S = \frac{2GM\Delta Mc^2}{hc^3} = \frac{2GM\Delta M}{hc} \tag{27.4}$$

Next convert to units of R, the radius of the black hole, $R = \frac{2GM}{c^2}$.

$$\Delta S = \frac{R\Delta Mc}{h} \tag{27.5}$$

Now, from the same equation for black hole radius,

$$\Delta M = \frac{\Delta Rc^2}{2G} \tag{27.6}$$

so

$$\Delta S = \frac{R\Delta Rc^3}{2Gh} \tag{27.7}$$

But the surface area of a black hole is proportional to the square of its radius, so $\Delta A = 2\pi R\Delta R$, the increment in surface area of the black hole when we add one extra bit of information. Rearranging,

$$R\Delta R = \frac{\Delta A}{2\pi} \tag{27.8}$$

and

$$\Delta S = \frac{\Delta Ac^3}{4\pi Gh} \tag{27.9}$$

This is the increment in entropy when we add one bit of information to the black hole. Total entropy of the black hole is

$$S = \frac{Ac^3}{4\pi Gh} \tag{27.10}$$

We gain further insight with one last substitution. $\frac{Gh}{c^3}$ is the Planck area. The total entropy of the black hole is proportional to the number of Planck areas quilting the black hole horizon!

$$S = \frac{A_{BH}}{4\pi A_P} \tag{27.11}$$

This is the Bekenstein – Hawking entropy, named after its discoverers, Jacob Bekenstein and Stephen Hawking.

We've arrived at one of the great insights of modern physics, the holographic principle. By all common sense, we would expect the information content of a black hole to be proportional to its volume. Matter and energy in the form of gas and dust and stars and encyclopedias, with all their information content, fall through the event horizon into the volume of the black hole. But this equation for black hole entropy tells a different story. All that information is piling onto the surface of the black hole, onto the event horizon. The information content is proportional to area, not volume.

And we have here another duality. Just as a free-fall observer will probe the interior of a black hole while a stationary observer outside the event horizon sees objects splatting (over infinite time) onto the horizon, we find that we can measure the information content of the black hole as if it was tiled onto its surface.

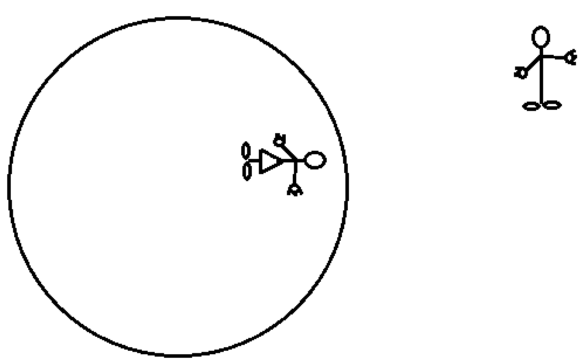

Figure 27.1. Two observers, Alice, inside the black hole, and Bob hovering outside the event horizon, with dual frames of reference on the black hole.

Return to Table of Contents