Black hole entropy is due to strings

Now we can answer the question, what is the source of entropy in a black hole? If entropy represents hidden information, what are the bits and pieces of information hidden in a black hole?

Here, as often in our exposition of the logic, we'll start with dimensional analysis, and we'll work in units $c = h = 1$ so that we can concentrate on the core ideas without so many symbols in the way.

Here's what we'll need:

- $R = \frac{2GM}{c^2}$ reduces to $R = GM$. We ignore the coefficients and the c's in the black hole radius.

- $[M] = \left[\frac{1}{L}\right]$. The dimensions of mass (therefore also dimensions of energy) are inverse length.

- $l_P = \sqrt{\frac{Gh}{c^3}}$ reduces to $l_P{}^2 = A_P = G$. The Planck area is proportional to G.

- $l_P = g l_{str}$. Some explanation here. This equation assigns a relation between the Planck length and the characteristic string length using the interaction coefficient, g. g is the probability that two strings will interact if they bump into each other, and so it measures the strength of gravity. It has a small value, less than one.

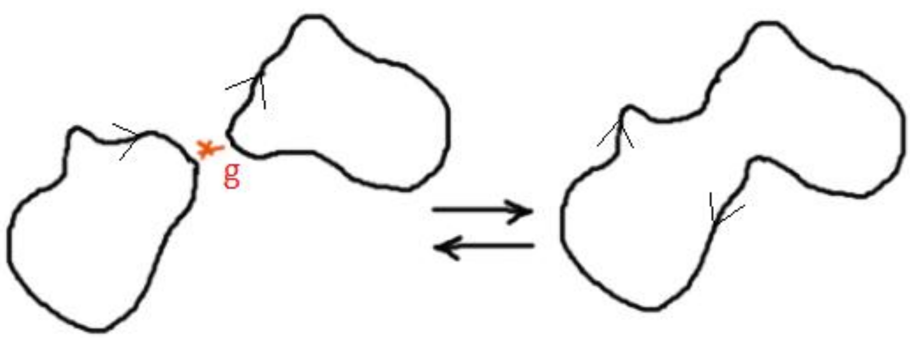

Figure 28.1. g measures the probability of string interaction. The interaction between fundamental closed strings, gravitons, is the gravitational interaction.

Thinking geometrically – gravity as the geometry of curved spacetime – strings are playing on the grid of spacetime, which has length scale l_P. Their interactions, as they wiggle and jostle, only bend distant parts of the grid, of the (larger) scale l_{str}. If the string scale was same as the Planck scale, gravity would be much stronger, i.e. bending and stretching and shrinking the fabric of spacetime on the very smallest (Planck) scale.

Our strategy is to compare the black hole entropy we calculated classically with the entropy of a string. We already found the classical, Bekenstein – Hawking entropy:

$$S_{BH} = \frac{A_{BH}}{4\pi A_P} \tag{28.1}$$

Using dimensional analysis and the relations above, this simplifies to

$$S = \frac{A_{BH}}{G} = \frac{R^2}{{l_P}^2} = \frac{G^2M^2}{{l_P}^2} = M^2{l_P}^2 \tag{28.2}$$

We have ignored constant coefficients, and we use = in the sense of logical assignment, not numerical equality.

What about the entropy of a black hole made of string? Where's the hidden information in a string? To figure that out, we imagine that the string exists on a lattice, like a vine on a trellis.

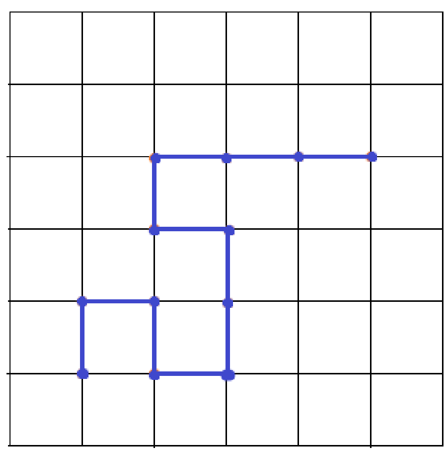

Figure 28.2. String segments, each about l_{str} in length, on a lattice.

Imagine building the string one lattice segment at a time. At each node, you have four choices: place the next segment above, below, left, or right. There's our information – the direction taken by each successive segment. Total entropy of the string is proportional to the number of segments, n.

We define a characteristic measure of string length, l_{str} . Then $S_{str} = n = \frac{L}{l_{str}}$, where L is the total length of the string. But this doesn't look promising: S_{BH} was proportional to a ratio of lengths squared, but S_{str} is proportional just to the ratio of lengths. Moreover,

$$S_{str} = \frac{L}{l_s} = \frac{M}{\mu} = \frac{\mathrm{M}}{{}^{1}/_{l_s}} = Ml_s \tag{28.3}$$

Here μ represents the mass density of a string segment and M the total mass of the string. The result is similarly disappointing: the mass and length terms are linear, not squared as in the equation for S_{BH}. It appears we're stuck. String entropy doesn't look like the Bekenstein-Hawking entropy of a classical black hole.

But we're not out of tricks yet. Assume we can "tune" the strength of string interactions and thereby adjust the strength of gravity in such a way that the entropy of the black hole remains constant. With such an adjustment, we can start with a Bekenstein-Hawking black hole and end up with a stringy black hole of the same entropy. What's the smallest black hole we can manipulate in this experiment? As before, we choose R, the black hole radius, of string length scale. Then,

$$S_{BH} = M^2 l_P{}^2 = M^2 G = M^2 \frac{R}{M} = M l_{str} = S_{str} \tag{28.4}$$

Voila! Classical Bekenstein-Hawking entropy is equivalent to string entropy. Or, put the other way, black hole entropy results from strings.

We've already shown that all the information is stored on the event horizon. In this picture of black hole entropy, we can think of the event horizon as covered with a "fuzz" of strings.

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