## Maximum entropy and the holographic principle

Now we're getting to the really cool stuff. Black holes provide doorways to new physics and new ways of doing physics. In this chapter we calculate the maximum entropy in a volume of space. (It's proportional to the area of the black hole exactly occupying that space.) That discovery leads us to the holographic principle: physics in a spacetime of dimensionality D can be described exactly in the mathematics of a spacetime with $(D-1)$ dimensions.

Entropy, first of all. What is it? After all is said and done, it is the amount of information hidden in the microscopic degrees of freedom of a system. For example, it's the information in the ginormous catalog (hidden from us) of all the momenta and positions of all the buzzillions of water molecules in a glass of water. Or the catalog of all the positions and momenta of all the electrons and protons in the plasma puff-ball of a star.

Classically, we measure entropy as the amount of information in a region of space, the information in a volume. Imagine, for example, a giant lattice – a three-dimensional spreadsheet – with $n$ microscopic cells. Each cell contains one bit of information, a $1$ or a $0$. The entropy of the system is proportional to the number of possible states, $2^n$. Each of the $n$ cells can be in one of the two quantum states. By definition, the entropy is the logarithm, so

$$S = n\,log2 \tag{29.1}$$

$n$ in a region of space is $V_{region}/V_{cell}$. So the entropy is

$$S = \left( V_{region}/V_{cell} \right) log2 \tag{29.2}$$

The entropy is proportional to volume.

Now think about this. Fill a spherical region of space, radius $R$, with uniformly distributed mass just less than the mass of a black hole with radius $R$. Then outside that region build another spherical shell with just exactly the extra mass required to make a black hole inside region $R$. Allow the shell to collapse onto the region. Presto! We produce a black hole with radius $R$.

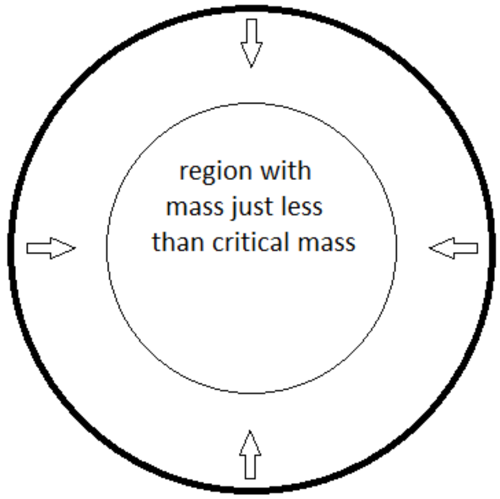


Figure 29.1. Shell with mass just sufficient to produce black hole collapsing on spherical region with just that mass deficit.

But wait. We've shown that the entropy of a black hole is proportional to its area.

$$S = \frac{A}{G} = \frac{A}{{l_p}^2} \tag{29.3}$$

A black hole has entropy given by the number of Planck areas on its horizon. That's where the information is distributed on the black hole – a patchwork of bits on the horizon. And that's the maximum entropy of a region of space. Add any more bits, and $R$ must increase.

There we have it. The maximum information in a *volume* of space is proportional to the *surface area* of the black hole that would occupy that region. Hence the holographic principle. We can calculate the area of the event horizon and figure out how much information there is inside the black hole. We can use 2D mathematics to describe 3D physics.

The holographic principle rescues physics (and physicists). It has been discovered, for instance, that the mathematics of higher-dimensional anti-de-Sitter space (of general relativity) corresponds to the maths of one-lower dimensional conformal field theory (good ol' quantum field theory). In some circumstances, it's a whole lot easier to calculate with the equations of anti-de-Sitter space. In other circumstances, it's easier to calculate using the equations of conformal field theory. For example, in otherwise intractable problems describing strong interactions, the calculations simplify considerably in 5D gravity. More on that later.

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