Slowing down the string

We've got a problem. Strings are so very tiny that quantum jitters cause enormously rapid internal motion. We have to slow them down, so that we can study them.

Enter special relativity, to the rescue. If we can boost the string to high velocity in a direction perpendicular to its length, clocks slow and internal jitters quiet down.

Our goal is to find the internal energy of the string – energy due to its internal motions and due to tension along the string. From the expression for energy, we'll be able to tease out an understanding of the string using all the known energy relations: the classical mechanics sum of kinetic and potential energies, or the energy expressions in the Lagrangian or Hamiltonian formulations, or the quantum $E = hf$, etc. Here goes.

We apply the convention that the speed of light $c = 1$. (More on this later. Our equations are much simplified if we use such natural units.) The relativistic expression for energy in this convention is

$$E = \sqrt{p^2 + m^2} \tag{2.1}$$

We choose the direction of motion to be along the z axis, so we can model internal motions of the string wiggling along the x and y axes. Our strategy is to set the string speeding along the z-axis, slowing down its internal motions so that we can study them.

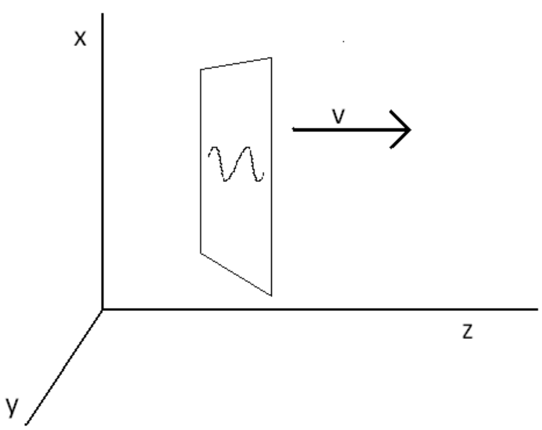

Figure 2.1. String boosted to high velocity along z-axis. Motions along x and y slow because of relativistic time dilation.

We separate the relativistic and non-relativistic components of momentum in equation 1.

$$E = \sqrt{{p_x}^2 + {p_y}^2 + {p_z}^2 + m^2} = p_z \sqrt{1 + \frac{{p_x}^2 + {p_y}^2 + m^2}{{p_z}^2}} \tag{2.2}$$

Since the ratio $\frac{{p_x}^2+{p_y}^2+m^2}{{p_z}^2}$ is very small (p_z , the momentum along the boost direction, is much larger than momentum along the axes perpendicular to the boost), we replace the square root with the approximation

$$E = p_z\left(1 + \frac{1}{2}\left(\frac{{p_x}^2 + {p_y}^2 + m^2}{{p_z}^2}\right)\right) = \left(p_z + \frac{{p_x}^2 + {p_y}^2 + m^2}{2p_z}\right) \tag{2.3}$$

So

$$E - p_z = \frac{{p_x}^2 + {p_y}^2 + m^2}{2p_z} \tag{2.4}$$

and, finally,

$$(E - p_z)p_z = \frac{{p_x}^2 + {p_y}^2 + m^2}{2} = \frac{{p_x}^2 + {p_y}^2}{2} + \frac{m^2}{2} \tag{2.5}$$

Now some sleight-of-hand. The p_z factor contributes energy of motion of the entire boosted string, the center of mass motion. We're interested in the internal energy of the string. The term with $p's$ on the right represents the string's internal kinetic energy, and the m term represents the internal potential (interaction) energy. If we work in units such that the string mass = 1 and the velocity of a string in the boosted frame is approximately the speed of light, also = 1, then $p_z \cong 1$ (with appropriate units). So we can rewrite the intrinsic string energy:

$$E - p_z \equiv E_{str} = \frac{{p_x}^2 + {p_y}^2}{2} + \frac{m^2}{2} \tag{2.6}$$

Note that the binding energy is proportional to m^2 , not m. This was one of the signals encouraging early string theorists, modeling hadrons as if quarks were bound by strings, that they might be on the right track. Experimental data indicated that angular momentum associated with quark interactions was proportional to mass squared, not, as expected, linearly proportional to mass.

Note also that the binding energy, proportional to m^2 , is the lowest energy state of the string, i.e. the intrinsic energy of the string itself without energy due to internal motion. We will return to this important observation when we quantize the string.

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