History of the universe

The universe is expanding at an accelerating rate, and there is a cosmic horizon. We shall consider more of the mind-boggling consequences. In this chapter, we study the history of universal expansion. In the next, we construct the de Sitter universe (or, rather, deconstruct the de Sitter universe, which is apparently the one we live in).

A bit of review. Einstein's field equations describe the structure of spacetime.

$$G_{\mu\nu} = T_{\mu\nu} \tag{32.1}$$

Spacetime curvature (the left side of the equation) is determined by the mass-energy-momentum density (the right side of the equation). In terms of our present discussion

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \tag{32.2}$$

The left side, Hubble's constant squared, measures curvature. (Think about that.) ρ on the right represents mass-energy-momentum density. (We are lumping, to convey the key ideas.) Note that G on the right is Newton's constant, not the Einstein tensor of the field equation.

We identify three components to the energy density of the universe: matter, radiation, and dark energy. Matter includes the ordinary (baryonic) stuff of stars and planets plus dark matter, of which there is four times more than ordinary baryonic matter. We can detect dark matter by its gravitational effects but don't yet know its composition. Radiation includes all the photons zipping around out there, on the order of ten billion for every particle of ordinary (baryonic) matter.

Dark energy is more subtle. It comprises about three-quarters of all the mass-energy in the universe, and we don't know what it is. It pervades all the nooks and crannies of all the universe. Even in a universe completely devoid of matter and radiation, there would be dark energy.

Most physicists think dark energy is vacuum energy associated with zero-point quantum fluctuations. By Heisenberg's uncertainty principle, the vacuum is jiggling with quantum jitters. The smallest scales of spacetime cannot settle down to absolute zero energy. Add up all the quantum energy of all the vacuum, and there's an awful lot of it.

And it gravitates. (But by repulsion – hints of this later in this chapter, and more in the next.)

Add all these density components in the field equation. (We drop the constant coefficients, to concentrate on the physical parameters.)

$$\left(\frac{\dot{a}}{a}\right)^2 = \rho_m + \rho_{rad} + \rho_{vac} \tag{32.3}$$

where the terms on the right represent matter density, radiation density, and vacuum density respectively. From this equation, we can derive the history of the universe. We consider the contributing terms one at a time.

Matter density is inversely proportional to volume. We assume that the total baryonic matter in the universe is constant over time. Starting with an initial matter density, ρ_0 , the density at later times is

$$\rho_m = \frac{\rho_0}{a^3} \tag{32.4}$$

Back to the field equation, we can find the scale factor as a function of time in a matter-dominated universe.

$$\left(\frac{\dot{a}}{a}\right)^2 = \rho_m = \frac{\rho_0}{a^3} \tag{32.5}$$

$$\left(\frac{da}{dt}\right)^2 = a^2\left(\frac{\rho_0}{a^3}\right) = \frac{\rho_0}{a} \tag{32.6}$$

$$\frac{da}{dt} = \sqrt{\frac{\rho_0}{a}} \tag{32.7}$$

$$\int a^{1/2} da = \int \sqrt{\rho_0} dt \tag{32.8}$$

$$a_m \approx t^{2/3} \tag{32.9}$$

In a matter-dominated universe, the scale increases as the two-thirds power of time.

In a radiation-dominated universe, energy density falls off as the fourth power of the scale. (Photons are not only becoming more dilute as the volume increases but they are also losing energy as their wavelength stretches.)

$$\rho_{rad} = \frac{\rho_0}{a^4} \tag{32.10}$$

We find, by a calculation similar to that above,

$$a_{rad} \approx \sqrt{t} \tag{32.11}$$

Now the vacuum energy. Curiously, vacuum energy density remains constant as the universe expands. Same vacuum, same energy density. Just more of it. In empty spacetime (no mass or radiation),

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \rho_{vac} \tag{32.12}$$

forever. We calculate.

$$\left(\frac{da}{dt}\right)^2 = a^2\rho_{vac} \tag{32.13}$$

$$\frac{da}{dt} = a\sqrt{\rho_{vac}} \tag{32.14}$$

$$\int \frac{da}{a} = \int \sqrt{\rho_{vac}}dt \tag{32.15}$$

$$ln(a) = \sqrt{\rho_{vac}}t = Ht \tag{32.16}$$

$$a_{vac} = e^{Ht} \tag{32.17}$$

We found this before. It's reassuring that it pops out again. The scale factor in a vacuum-dominated universe is an exponential function of time. The universal expansion accelerates. Somehow the vacuum is pushing entire galaxies and vast clusters of galaxies – ponderous collections of hundreds of billions of stars and clouds of gas and dust and even more dark matter – accelerating those gigantic structures faster and faster away from each other, filling the space in between with dark energy.

Now put it all together. The newborn universe, at the Big Bang, was radiation-dominated, filled with gamma rays (and all the loops and propagators of myriad Feynman diagrams in the soup of energy). As it cooled and baryonic matter precipitated out, the density of the early universe became matter-dominated. But as its expansion accelerates into forever time, vacuum energy takes over. More on the consequences next.

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