de Sitter space

“Empty” space is filled with dark energy, probably the effect of zero-point quantum fluctuations. Such a universe expands exponentially over time. That’s our universe as it ages. As matter and radiation get diluted with expansion, dark energy takes over.

We are describing a de Sitter universe, first proposed by Willem de Sitter as a solution to Einstein’s field equations. We’ve seen the metric.

$$ds^2 = -dt^2 + a^2 dx^2 = -dt^2 + e^{2Ht} dx^2 \tag{33.1}$$

Let’s take a closer look. We would like to map this exponential function onto a more familiar geometry.

Identify a new “time” parameter

$$T = -\frac{1}{H} e^{-Ht} \tag{33.2}$$

And note, for future reference,

$$e^{-Ht} = a^{-1} = -HT \tag{33.3}$$

So

$$a = -\frac{1}{HT} \tag{33.4}$$

Why bother? What’s the point of this new way of defining time? Well, it’s convenient for graphing – we’ll see. First note that in the most dim and distant past, t an infinity ago, T also evaluates to forever long ago $(-\infty)$. But in the far away distant future, $T \to 0$. The T coordinate allows us to map the entire history of the universe onto a half-plane.

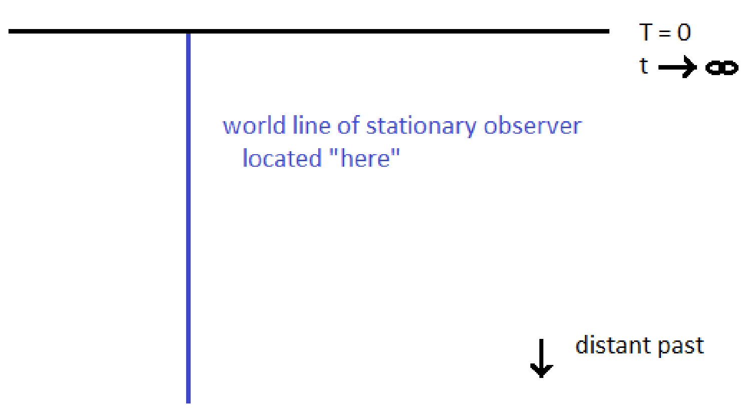

Figure 33.1. The universe, re-parameterized.

We plug our new T parameter into the metric.

$$\frac{dT}{dt} = e^{-Ht} \tag{33.5}$$

$$dt = e^{Ht} dT \tag{33.6}$$

$$ds^2 = -e^{2Ht} dT^2 + e^{2Ht} dx^2 = e^{2Ht}(-dT^2 + dx^2) \tag{33.7}$$

Onward, applying the identities in (33.4) ,

$$ds^2 = e^{2Ht}(-dT^2 + dx^2) = a^2(-dT^2 + dx^2) = \left(\frac{1}{HT}\right)^2 (-dT^2 + dx^2) \tag{33.8}$$

The last term on the right looks like the metric of good ol' Minkowski space. And as in Minkowski space, light rays follow $45°$ paths across the spacetime diagram (because for light $ds = 0$, so $dx^2 = dT^2$.)

We draw the graph. Two observers, Alice and Bob, at rest in different regions of the universe, follow their spacetime trajectories through this de Sitter space.

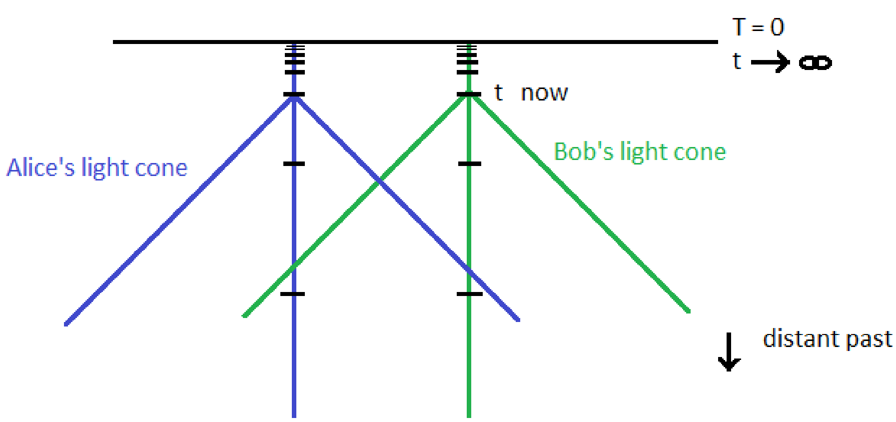

Figure 33.2. World lines and horizons of two observers in de Sitter space.

Study this. Space extends forever left and right. The T scale, on the other hand, rises from the distant past but becomes compressed at the top, where $T = 0$. Tick marks on the T axis squeeze closer and closer together, squeezing all future time against the top of the graph.

Alice can only see the regions of the universe contained in her light cone, and she sees different regions than Bob. Note that, even in the infinite future, neither Alice nor Bob can ever see the whole universe. Their light cones represent their cosmic horizons, from beyond which they cannot communicate. Note, also, that the distance to the cosmic horizon is constant. Doesn't look like it, at first glance, but the appearance is an artifact of the compressed T axis.

That's our universe. Future astronomers will look out into emptiness. No stars. No nebulae. As the galaxies are swept beyond the cosmic horizon, red-shifted into invisibility by the accelerated expansion, they will blink out one by one and leave us in blackness.

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