Dirichlet and Neumann boundaries

We are considering open strings. We have calculated the energy of a string in light cone coordinates, and we have also calculated the energy using the mass-and-spring model. Turns out, though, not all open strings are the same. We have to consider what's happening at their endpoints.

There are two possible boundary conditions. Either the ends are fixed to some other structure, so they can't move. Or the ends are flopping around, moving across the spacetime background. Fixed ends are called Dirichlet boundary conditions, after the French mathematician Johan Peter Gustav Lejeune Dirichlet. Free ends are called Neumann, after Carl Neumann.

We define the two types of open strings in terms of x (displacement against the background) as a function of the string parameters, σ and τ. Dirichlet strings meet the following criteria.

$$x(0) = x(\pi) = 0 \tag{4.1}$$

and

$$\frac{dx}{dt} = 0 \text{ at } x = 0 \text{ and } x = \pi \tag{4.2}$$

i.e. the ends are anchored. They don't move.

But in a stringy universe, what can strings stick to? They attach to branes. We'll deduce the existence of branes and explain what they are in a later chapter. Suffice to say, for the moment, they are required by the theory: if Dirichlet strings exist, so must branes.

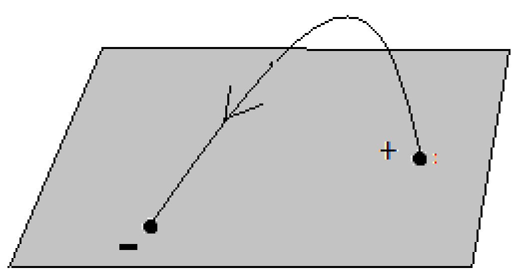

Figure 4.1. String ends attached to a Dirichlet brane.

Neumann strings, our present focus, have the following property. At the endpoints,

$$\frac{dx}{d\sigma} = 0 \tag{4.3}$$

That is, Neumann strings must be "flat" at the endpoints, with no tension pulling them up or down. To see why, consider the following. Strings have tension, like stretched rubber bands. If there was any slope at the ends, with component of tension pulling the string up or down, string tension would accelerate the endpoints indefinitely, to velocities greater than the speed of light. That's not allowed.

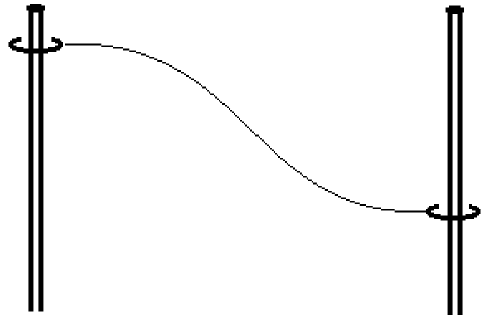

Figure 4.2. Neumann strings have zero slope at their ends, as if they were attached to friction-free rings sliding on end poles.

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