So far, we have described the properties and basic interactions of open and closed strings, and we've investigated the energy spectrum of open strings based on a mass-and-spring model. We work in a boosted frame (the "light cone" coordinates) in order to slow down the strings' internal motions. Next we quantize the open string. After all, strings live at the quantum scale, and their physics demands a quantum description. In the open string spectrum we will find photons, an encouraging sign that we might be on the right track for a model of known particles and forces. But we'll also find tachyons, which doom the theory. In closed strings, however, the tachyons disappear, and, amazingly, closed strings produce gravitons plus some other familiar critters in the particle zoo. What started as a theory to describe the strong force turns out to be a theory of quantum gravity.

Quantizing the string

We have derived an equation for string energy based on superposition of modes in light cone coordinates and another based on the mass-and-spring model. Now we'll quantize the string.

Our goal is to determine the energy spectrum of the Neumann open string, to find out whether or not it describes familiar particles. Energy equations are convenient because there are so many of them. They all describe the same energy, but each gives us a different perspective as to what's going on.

Start with the classical strings-as-springs equation.

$$E = \frac{1}{2}\left(m\left(\frac{dx}{dt}\right)^2 + k(\Delta x)^2\right) \tag{6.1}$$

Energy per unit mass

$$\frac{E}{m} = \frac{1}{2}\left(\left(\frac{dx}{dt}\right)^2 + \frac{k}{m}(\Delta x)^2\right) \tag{6.2}$$

The coefficient $\frac{k}{m}$ is the square of the oscillation frequency, ω^2 . Makes sense: k measures spring tension, and m is inertia. A spring at high tension oscillates at higher frequency, and greater inertia slows down the oscillation.

Now compare the basic quantum equation

$$E = \hbar\omega \tag{6.3}$$

Energy is proportional to frequency, ω , quantized by Planck's constant.

Next identify $\omega = n$, where n is the string oscillation mode.

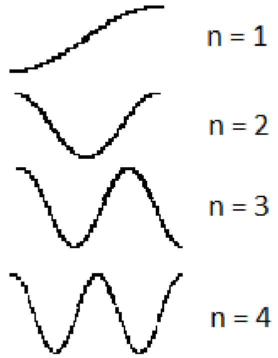

Figure 6.1. First four modes of an open string.

For convenience, we let $\hbar = 1$ and $m = 1$, giving energy per unit length of string. Then

$$E = n = \frac{1}{2}\left(\left(\frac{\partial x}{\partial t}\right)^2 + \omega^2 \Delta x^2\right) = \frac{1}{2}\left((\dot{x}^2) + n^2 x^2\right) = \frac{1}{2}(n^2 x^2 + \dot{x}^2) \tag{6.4}$$

To quantize this classical spring equation, we substitute the momentum operator p for the time derivative and rewrite the equation with complex factors.

$$E = \frac{1}{2}\left((nx + ip)(nx - ip)\right) = n \tag{6.5}$$

This describes a string in equilibrium at $E = n$. (Complex numbers conveniently express magnitudes and phase angles representing quantum states.)

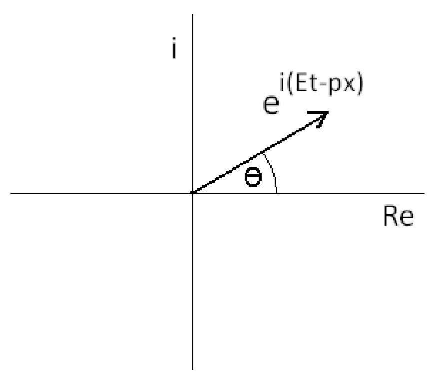

Figure 6.2. Wavefunction as complex variable.

The energy operator in quantum mechanics $\left(E = -i\hbar\frac{\eth}{\partial t}\right)$ measures the time evolution of a system. In a system at equilibrium, energy is stationary, so any new mode created by a quantum fluctuation must be offset by the annihilation of a mode or combination of modes with equal energy. We would like to refine our tools to be able to create or annihilate one unit of energy at a time. We do so by normalizing the equation.

$$E = \frac{1}{2}\left(\frac{1}{\sqrt{n}}(nx + ip)\frac{1}{\sqrt{n}}(nx - ip)\right) = 1 \tag{6.6}$$

We identify $\frac{1}{\sqrt{n}}(nx - ip)$ as the creation operator a^+ , and $\frac{1}{\sqrt{n}}(nx + ip)$ is the annihilation operator a^- . We check these assignments with the commutation relation

$$\begin{aligned}[a^-, a^+]\,|0\rangle &= [a^- a^+ - a^+ a^-]\,|0\rangle \\ &= \frac{1}{n}[(nx + ip)(nx - ip) - (nx - ip)(nx + ip)]\,|0\rangle = |0\rangle\end{aligned} \tag{6.7}$$

This commutator does indeed preserve the state of the system (the ground state, $|0\rangle$, in this example).

We are ready to build the energy spectrum of the open string. We start with the ground state $E = m^2 = 0$ and build the spectrum by adding (with the creation operator) one unit of energy at a time. (Think of plucking additional guitar strings to create chords.)

$$E_0 = |0\rangle \tag{6.8}$$

$$E_1 = {a_1}^+|0\rangle$$

$$E_2 = {a_1}^+{a_1}^+|0\rangle$$

or

$$E_2 = {a_2}^+|0\rangle$$

etc.

All promising so far – but we have a problem! This model of open strings creates tachyons and an unstable vacuum. That's not good.

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