Closed strings and level matching

Open strings reproduce some of the features of the Standard Model. Their spectrum includes photons, and their behavior mimics the strong force. But tachyons are unacceptable. How do we get rid of tachyons and restore a stable vacuum? Here we take a first look at closed strings, which provide a possible resolution to the tachyon problem.

Here are the rules for closed strings.

- End-to-end along the open string was π units. For the closed string, we define one circuit around the loop as 2π , measured in units of σ . Our choice is arbitrary but convenient. It's 2π radians around a circle.
- Closed strings carry waves traveling both directions, designated R-moving and L-moving.

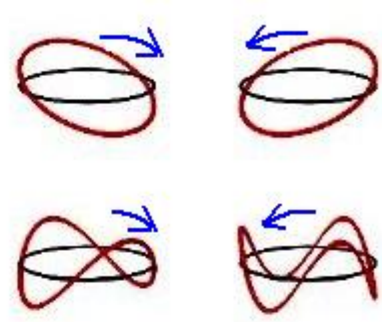

Figure 8.1. First three modes on a closed string showing R- and L- components. Top two figures show first mode R- and L-. R- component of second mode and L-component of third mode are shown below. Credits http://universe-review.ca/R15-18-string.htm

- The wavefunctions for L- and R- movers must be such that $\psi(0) = \psi(2\pi)$. That is, the waves must fit in integer multiples of wavelength around the loop.

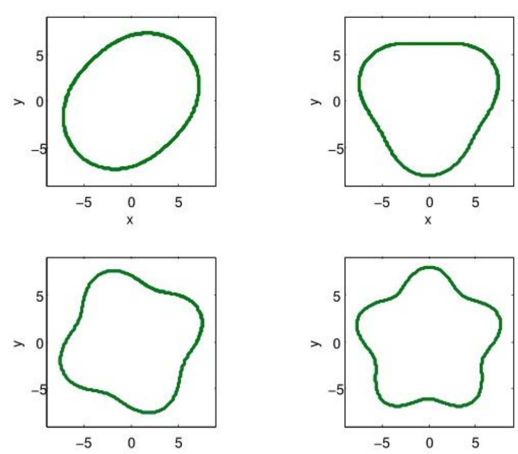

Figure 8.2. First four oscillation modes of a closed string.

- The strings are rotationally symmetric. There is no preferred location on the string; all points are indistinguishable. (When we model the string, we will arbitrarily assign a $\sigma = 0$, but this is only for convenience. We could choose any point as the origin.)
- The energy carried by L-moving waves must equal the energy carried by R-movers. This is called level-matching. In fact, level-matching is a consequence of the symmetry requirement. We shall prove this shortly.

We build our model for closed strings. We need equations of motion around loops. Complex numbers provide the tools. Instead of the equation for displacement on open strings,

$$x = \sum_{n=0}^{\infty} x_n cos(n\sigma) \tag{8.1}$$

for closed strings we substitute

$$x = \sum_{n=0}^{\infty} x_n e^{in\sigma} + \sum_{n=0}^{\infty} x_{-n} e^{-in\sigma} + x_0 \tag{8.2}$$

The first term on the right describes R-movers, the second describes L-movers, and x_0 represents the center-of-mass. We don't have to worry about boundary conditions, because there are no boundaries on the loop.

From the equation for spring energy, total energy in such a string is

$$E = \int_0^{2\pi} \left(\left(\frac{\partial x}{\partial \tau} \right)^2 + \left(\frac{\partial x}{\partial \sigma} \right)^2 \right) d\sigma \tag{8.3}$$

We separate the argument into R- and L-moving components.

$$E = \frac{1}{2} \int_0^{2\pi} \left(\left(\frac{\partial x}{\partial \tau} + \frac{\partial x}{\partial \sigma} \right)^2 + \left(\frac{\partial x}{\partial \tau} - \frac{\partial x}{\partial \sigma} \right)^2 \right) d\sigma \tag{8.4}$$

We will be interested in the energy difference between those components.

$$E_R - E_L = \frac{1}{2} \int_0^{2\pi} \left(\left(\frac{\partial x}{\partial \tau} + \frac{\partial x}{\partial \sigma} \right)^2 - \left(\frac{\partial x}{\partial \tau} - \frac{\partial x}{\partial \sigma} \right)^2 \right) d\sigma = \frac{1}{2} \int_0^{2\pi} 4 \left(\frac{\partial x}{\partial \tau} \frac{\partial x}{\partial \sigma} \right) d\sigma \tag{8.5}$$

There is no energy difference if

$$\frac{1}{2} \int_0^{2\pi} 4 \left(\frac{\partial x}{\partial \tau} \frac{\partial x}{\partial \sigma} \right) d\sigma = 0 \tag{8.6}$$

We will show that this is the case. Our argument is based on symmetry. If $\psi(0) = \psi(2\pi)$, then

$$\int_0^{2\pi} d\psi = 0 \tag{8.7}$$

Around the loop, ψ takes on equal numbers of positive and negative values. We deconstruct this equation.

$$\int_0^{2\pi} d\psi = \int_0^{2\pi} \frac{d\psi}{d\sigma} d\sigma = \int_0^{2\pi} \frac{d\psi}{dx}\frac{dx}{d\sigma} d\sigma = 0 \tag{8.8}$$

But $\frac{d\psi}{dx}$ is the momentum operator acting on ψ.

$$\frac{d\psi}{dx} = \frac{\partial}{\partial x}\psi = \frac{\partial x}{\partial \tau} \tag{8.9}$$

We substitute back into the integral.

$$\int_0^{2\pi} \frac{d\psi}{dx}\frac{dx}{d\sigma} d\sigma = \int_0^{2\pi} \frac{\partial x}{\partial \tau}\frac{\partial x}{\partial \sigma} d\sigma = 0 \tag{8.10}$$

This is the heart of equation (8.5), and it equals zero. We have shown that there can be no energy difference between the R- and L- moving components. Symmetry demands that the energy contributions from the two sets of waves must be the same. This is level-matching.

We can see this. If the energies are different, there must be R-moving waves with different wavenumbers than the L-movers. In that case, an outside observer could see wave motion around the loop. The symmetry is broken. On the other hand, if the R- and L- wavenumbers are the same, we could not detect any motion around the loop, only stationary waves. The loop would remain symmetric.

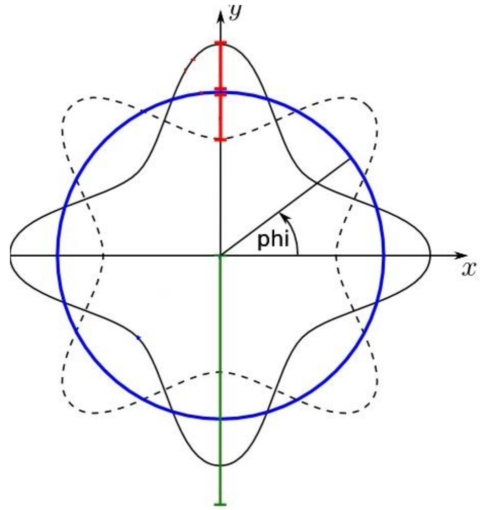

Figure 8.3. If solid waves are traveling R at $\frac{d\varphi}{dt}$ and dashed waves are traveling L at $\frac{-d\varphi}{dt}$, an outside observer would see a 4-pointed star alternating with a circle but would not detect any R or L motion.

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