The closed string spectrum

Have we banished the tachyon? What particles pop out of the closed string spectrum?

Turns out the spectrum looks pretty good. No tachyon, and the spectrum offers an unexpected surprise. As before, we'll build the spectrum one energy level at a time starting with the ground state $|0\rangle$.

We will need additional symbols to account for extra variables on the closed string. Let $a_1^+(R)$ represent the creator that adds one unit of energy to the *x*-component of R-moving modes in the first quantum level. $b_2^-(L)$ annihilates two units of energy from the y-component of L-moving modes, etc. Here goes.

Given the strictures of level-matching, we cannot produce a closed string with only one unit of energy above the ground state. $a_1^+(R)|0\rangle$ would produce such a state, but it's not allowed because there's no matching unit of energy from L-moving components. Similarly $b_1^+(L)|0\rangle$ lacks a matching R-moving component.

The lowest occupied state, then, lies at the second energy level. It can be produced by combinations of creators such as $a_1^+(R)b_1^+(L)|0\rangle$ or $a_1^+(L)b_1^+(R)|0\rangle$. (Note that the rules of level-matching do not allow other $E = 2$ states such as $a_2^+(R)|0\rangle$.) If we write the spectrum as circular polarization states, possible lowest-energy states include:

$$\left(a_1^+(R) + ib_1^+(R)\right)\left(a_1^+(L) + ib_1^+(L)\right)|0\rangle \tag{9.1}$$

$$\left(a_1^+(R) + ib_1^+(R)\right)\left(a_1^+(L) - ib_1^+(L)\right)|0\rangle \tag{9.2}$$

$$\left(a_1^+(R) - ib_1^+(R)\right)\left(a_1^+(L) + ib_1^+(L)\right)|0\rangle \tag{9.3}$$

$$\left(a_1^+(R) - ib_1^+(R)\right)\left(a_1^+(L) - ib_1^+(L)\right)|0\rangle \tag{9.4}$$

Each of these four distinct states has two spin components, in parentheses **( )** and **( )** .

Think a bit about those $+$ and $-$ signs. Back to the definition of the creators.

$$a_1^+(R) = \left(\frac{\sqrt{n}x}{2} - \frac{ip}{\sqrt{n}}\right) \tag{9.5}$$

creates the *x*-component of a wave moving rightward around the string. The negative of the creator

$$-a_1^{\dagger}(R) = \left(\frac{-\sqrt{n}x}{2} + \frac{ip}{\sqrt{n}}\right) \tag{9.6}$$

flips the phase of the wave. The four combinations above account for all possible waves in the first energy level on the closed string.

Physicists identify each of the four states (9.1 - 4) with a particle. Since there are only two spin components and no spin zero state in this collection, the particles must be mass-less. The first and fourth states, mass-less spin two particles, are gravitons.

Gravitons! From the closed string model, out pop gravitons, the quanta that carry the force of gravity. Here is a quantum theory that includes gravity! String theory began as an attempt to model the strong interactions. It turns out that it includes – in fact it requires – the force of gravity.

Return to Table of Contents