Math conventions

Mathematicians scold physicists for lack of rigor. While the mathematicians insist on strict conventions of formal proof, physicists use math as the language with which to express (and test) their ideas. (They often let mathematicians fill in the proofs – or despair if mathematical rigor destroys a beautiful hypothesis.) String theory has provided a fertile ground for both disciplines. Sometimes physics leads the way, opening surprising new fields of mathematics. Sometimes the formal math provides new insights into the physics.

In these chapters, I shall adopt a sometimes casual attitude toward the maths. We are most interested in discerning key variables that affect the world, and I shall ignore some of the details. Details can clutter the picture, making it hard to follow Ariadne's thread through the maze.

Of course there's a danger. Details might be critical. Certainly, if we perform experiments to test our ideas (which, after all, is the heart of science), we must calculate with the precision available to our measuring apparatus.

With those caveats in mind, here are some of the shortcuts we'll take in the following chapters.

To emphasize a physical relationship, I sometimes drop constant coefficients from the argument. For example, a black hole radius is

$$R = \frac{2GM}{c^2}$$

but the main point is

$$R \sim M$$

The radius of a black hole depends on its mass.

For convenience (and also because nature, at its heart, is simple – at least we prefer to think it so), I will often set the natural constants equal to one.

$$c = h = G = 1$$

Nothing wrong here. The assigned (complicated and non-intuitive) values are arbitrary. In standard units, for example,

$$h = 6.626 \times 10^{-34}\, {kg \cdot m^2}/_{sec}$$

plus more decimals. The funny value results because Napoleon's science advisors chose familiar earth-based systems for the definitions of mass and length and time. Had we defined nature's fundamental unit of action knowing what we know now, scientists working in the quantum realm would save themselves a lot of key punching and lots of errors just by choosing $h = 1$.

Similarly, working backward from the definitions above, I will occasionally redefine the standard international units of measurement (meters, kilograms, seconds) in Planck units. For example, the smallest black hole has radius one Planck length, event horizon area $= 4\pi {l_P}^2$ (i.e. proportional to the square of the Planck length), mass equal to one Planck mass, and light crosses the black hole in one unit of Planck time. Expressed in Planck units, the minimal black hole is the fundamental kernel of mass, length, and time.

So what are these Planck units? We can find their equivalents in standard units most easily by dimensional analysis. For example, the units of force are expressed

$$[F] = \frac{ML}{T^2} = \frac{GM^2}{L^2}$$

where M represents units of mass, L units of length, and T units of time. The square brackets indicate that the equation gives the unit composition of force, with the general definition in the middle and gravitational force on the right. Solving for G

$$[G] = \frac{L^3}{MT^2}$$

Similarly, we can find the units of Planck's constant from $h = Et$ or $h = px$.

$$[h] = \frac{ML^2}{T}$$

Units for the speed of light are easy.

$$[c] = \frac{L}{T}$$

Solving this system of equations for L we find the standard international units for Planck length. (Plug in the units to see that this makes sense.)

$$l_P = \sqrt{\frac{Gh}{c^3}} \cong 1.616 \times 10^{-35} m$$

We can find the SI units of Planck time and Planck mass with similar calculations.

Finally among our maths conventions, I shall sometimes abbreviate spacetime coordinates as follows. Where I should properly write

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu \equiv -{dx_0}^2 + {dx_1}^2 + {dx_2}^2 + {dx_3}^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

I will simply summarize

$$ds^2 = -dt^2 + dx^2$$

with the understanding that the last term includes all directions of spatial displacements.

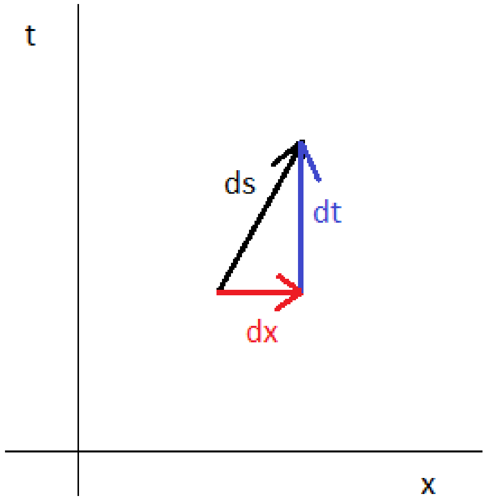

Figure 1. dx is shorthand for the 3-dimensional vector displacement.

There we have it. I shall introduce other key math ideas as we go along. The universe is found in the equations, and we can ask questions of the universe by solving the equations.

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